BASIC ELECTRICITY

NAVY TRAINING COURSES

NAVPERS 10086

BA

BASIC ELECTRICITY

Prepared by

BUREAU OF NAVAL PERSONNEL

NAVY TRAINING COURSES

NAVPERS 10086

UNITED STATES

GOVERNMENT PRINTING OFFICE

WASHINGTON : 1956

For sale by the Superintendent of Documents, U. S. Government Printing Office
Washington 25, D. C. - Price $2.25

THE UNITED STATES NAVY

GUARDIAN OF OUR COUNTRY

The United States Navy is responsible for maintaining control of the sea and is a ready force on watch at home and overseas, capable of strong action to preserve the peace or of instant offensive action to win in war.

It is upon the maintenance of this control that our country's glorious future depends; the United States Navy exists to make it so.

WE SERVE WITH HONOR

Tradition, valor, and victory are the Navy's heritage from the past. To these may be added dedication, discipline, and vigilance as the watchwords of the present and the future.

At home or on distant stations we serve with pride, confident in the respect of our country, our shipmates, and our families.

Our responsibilities sober us; our adversities strengthen us.

Service to God and Country is our special privilege. We serve with honor.

THE FUTURE OF THE NAVY

The Navy will always employ new weapons, new techniques, and greater power to protect and defend the United States on the sea, under the sea, and in the air.

Now and in the future, control of the sea gives the United States her greatest advantage for the maintenance of peace and for victory in war.

Mobility, surprise, dispersal, and offensive power are the keynotes of the new Navy. The roots of the Navy lie in a strong belief in the future, in continued dedication to our tasks, and in reflection on our heritage from the past.

Never have our opportunities and our responsibilities been greater.

PREFACE

Basic Electricity is written for men of the U. S. Navy and Naval Reserve whose duties require them to have a knowledge of the fundamentals of electricity.

This training course is therefore intended for use in conjunction with other required courses for candidates who desire to advance in any one of the following ratings: RD, SO, GM, FT, TM, MN, ET, RM, TE, EM, IC, AO, TD, AE, AT, CE, IM, DM, OM, UT, CM, GS, AQ, and GF.

Discussed at the beginning of this manual are the fundamental concepts of electricity—how all matter is essentially electrical in nature, and how current flow is the movement of free electrons in direction of an applied voltage. Primary and secondary cells are described as d-c (direct current) sources and these lead to the simple electric circuit. D-c circuit analysis is carried through the series, parallel, compound, and bridge networks. Electrical conductors and wiring techniques conclude the discussion on d-c circuits. Next, magnetism and magnetic circuits and inductance and capacitance are described; these form a basis for the chapter on electrical indicating instruments for measuring d-c circuit quantities.

The first half of this training course emphasizes d-c circuits and equipments with a treatment of d-c generators and motors. The latter portion of the manual emphasizes a-c (alternating current) circuit theory with the use of vectors and sine curves wherever possible. In this portion is discussed the basic a-c series circuit with *L* (inductance) and *R* (resistance), then *C* (capacitance) and *R*, and finally *L*, *R*, and *C*. This is followed by a treatment of the basic a-c parallel circuit with *L* and *R*, *C* and *R*, and finally *L*, *C*, and *R*. A-c theory is concluded with a treatment of a-c generators, transformers, motors, indicating instruments, and synchros. Series and parallel resonance is presented in the training course, *Basic Electronics*.

As one of the NAVY TRAINING COURSES, this book was prepared by the U. S. Navy Training Publications Center for the Bureau of Naval Personnel.

CONTENTS

BASIC ELECTRICITY

CHAPTER 1

FUNDAMENTAL CONCEPTS OF ELECTRICITY

INTRODUCTION

Today there is scarcely any technician associated with the Navy who does not use or come in contact with electrical or electronic equipment.

Many electrical items, that only a few years ago were considered luxuries are now regarded as essential to our normal way of life. Equipment such as the electric light, telephone, radio, television, electric refrigerator, electric stove, and automobile are accepted as a matter of course. Most people turn on a light switch without giving a thought to the vast electrical system to which it is connected or to what happens when the light comes on. The same attitude prevails when turning the switch on a radio, television, or other item. As long as the equipment functions properly, the results are accepted as a matter of course with little concern about what is actually taking place.

Prior to World War I, electricity in the U. S. Navy was still in its infancy. It was not until World War II that propulsion and fighting equipment aboard ships and aircraft became electrically controlled. Today, electricity aims guns, drops bombs, navigates ships, helps control engineering plants, and projects itself into Navy life at every turn.

With these changes it became necessary to revise the rating structure and the requirements for the advancement in rate and to increase the knowledge required for the operation, maintenance, and repair of electrical equipment.

This book is intended as a basic reference for ALL ENLISTED PERSONNEL of the Navy whose duties require them to have a knowledge of the fundamentals of electricity. It is designed to close the gap between the man with little or no knowledge of electricity who watches electrical machinery and equipment perform without

understanding the basic principles of its operation, and the experienced technician who understands what is actually happening. Whether his job involves work on fire control apparatus, radios, steering gear, or motors and generators, the technician should be familiar with the basic theory underlying the operation of the mechanism.

A knowledge of the principles of elementary mathematics is essential to an understanding of *Basic Electricity.* These principles include elementary arithmetic, algebra, geometry, and trigonometry. The algebra includes simple linear equations; the geometry emphasizes the relationship between the hypotenuse and the other sides of the right triangle; and the trigonometry stresses the three elementary functions of the angles of the right triangle—sine, cosine, and tangent. These functions are used in the resolution of vectors into their horizontal and vertical components and also in the summation of vectors in the chapters on alternating-current electricity.

WHAT IS ELECTRICITY?

The word "electric" is derived from the Greek word meaning AMBER. The ancient Greeks used the word to describe the strange forces of attraction and repulsion that were exhibited by amber after it had been rubbed with a cloth. The question, WHAT IS ELECTRICITY has been baffling the world's greatest scientists for many years. Although it is not known exactly what electricity is, by knowing what it does, it has been possible to develop theories that are proving productive; and the laws by which electricity operates are becoming more widely known and better understood. Today, all matter is considered to be essentially electrical in nature.

The Molecule

The objects that make up the world around us are said to be made of MATTER. (See fig. 1–1.) Matter is the physical substance of common experience. The conception of matter may be summarized in the following way.

The familiar forms of matter are of a particle nature in their last analysis. Consider, for example, a crystal of common table salt. If it were divided into very small particles, and then if one of these particles were divided again and again, finally a particle

would be reached that was so small that no further division could be made that would leave the material in the form of salt. This ultimate particle of salt is called a MOLECULE. Now it is known that salt is composed of two kinds of material—sodium and chlorine. The salt molecule is then the smallest possible physical

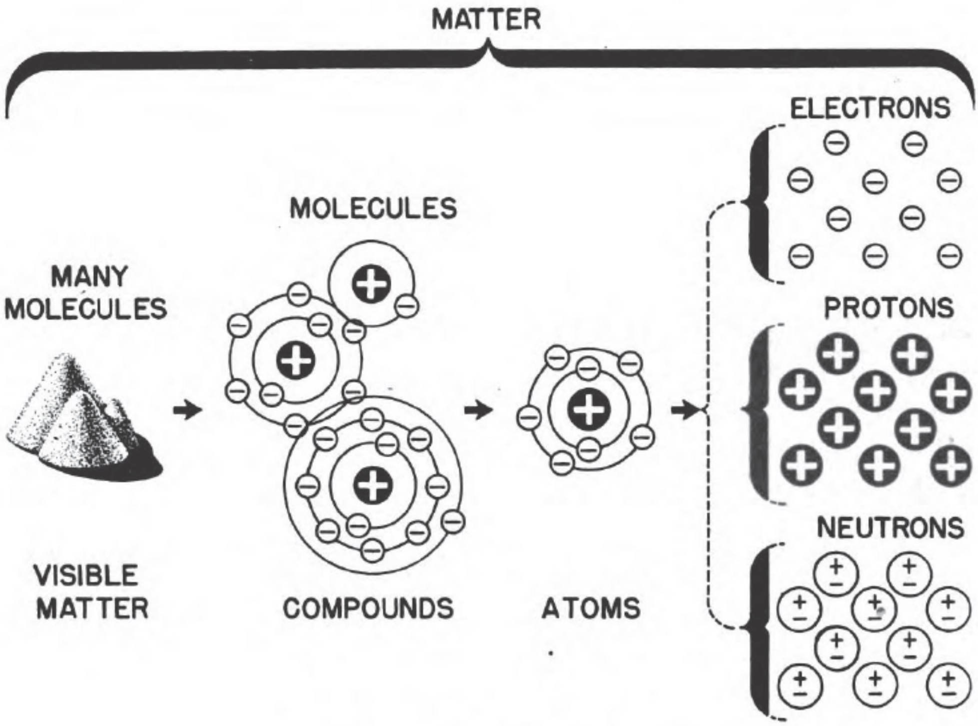


Figure 1–1.—Electricity and matter.

form of this compound (or chemical union) of the two constituent elements. The molecule is the particle that is involved in most of the CHEMICAL changes. The baking of bread, the explosion of dynamite, the changes involved in converting food into a component of blood—these are a few of the actions in which molecules are created and destroyed.

The Atom

In the study of chemistry it soon becomes apparent that the molecule is far from being the ultimate particle into which matter may be subdivided. The salt molecule may be decomposed into radically different substances—sodium and chlorine. These particles that make up molecules can be isolated and studied separately. They are called ATOMS.

The atom is the smallest particle that makes up that type of material called an ELEMENT. The element retains its characteristics when subdivided into atoms. More than 100 elements have been identified. They can be arranged into a table of increasing weight, and can be grouped into families of materials having similar properties. This arrangement is called the PERIODIC TABLE OF THE ELEMENTS.

The idea that all matter is composed of atoms dates back more than 2,000 years to the Greeks. Many centuries passed before the study of matter proved that the basic idea of atomic structure was correct. Physicists have explored the interior of the atom and discovered many subdivisions in it. The core of the atom is called the NUCLEUS. Most of the mass of the atom is concentrated in the nucleus. It is comparable to the sun in the solar system, around which the planets revolve. The nucleus contains PROTONS (positively charged particles) and NEUTRONS which are electrically neutral.

Most of the weight of the atom is in the protons and neutrons of the nucleus. Whirling around the nucleus are one or more smaller particles of negative electric charge. THESE ARE THE ELECTRONS. Normally there is one proton for each electron in the entire atom so that the net positive charge of the nucleus is balanced by the net negative charge of the electrons whirling around the nucleus. THUS THE ATOM IS ELECTRICALLY NEUTRAL.

The electrons do not fall into the nucleus even though they are attracted strongly to it. Their motion prevents it, as the planets are prevented from falling into the sun because of their centrifugal force of revolution.

The number of protons, which is usually the same as the number of electrons, determines the kind of element in question. Figure 1–2 shows a simplified picture of several atoms of different materials based on the conception of planetary electrons describing orbits about the nucleus. For example, hydrogen has a nucleus consisting of 1 proton, around which rotates 1 electron. The helium atom has a nucleus containing 2 protons and 2 neutrons with 2 electrons encircling the nucleus. Near the other extreme of the elements is curium (not shown in the figure), an element discovered in the 1940's, which has 96 protons and 96 electrons in each atom.

The Periodic Table of the Elements is an orderly arrangement of the elements in ascending atomic number (number of planetary

electrons) and also in atomic weight (number of protons and neutrons in the nucleus). The various kinds of atoms have distinct masses or weights with respect to each other. The element most closely approaching unity (meaning 1) is hydrogen whose atomic weight is 1.008 as compared with oxygen whose atomic weight is 16. Helium has an atomic weight of approximately 4, lithium 7, fluorine 19, and neon 20, as shown in figure 1–2.

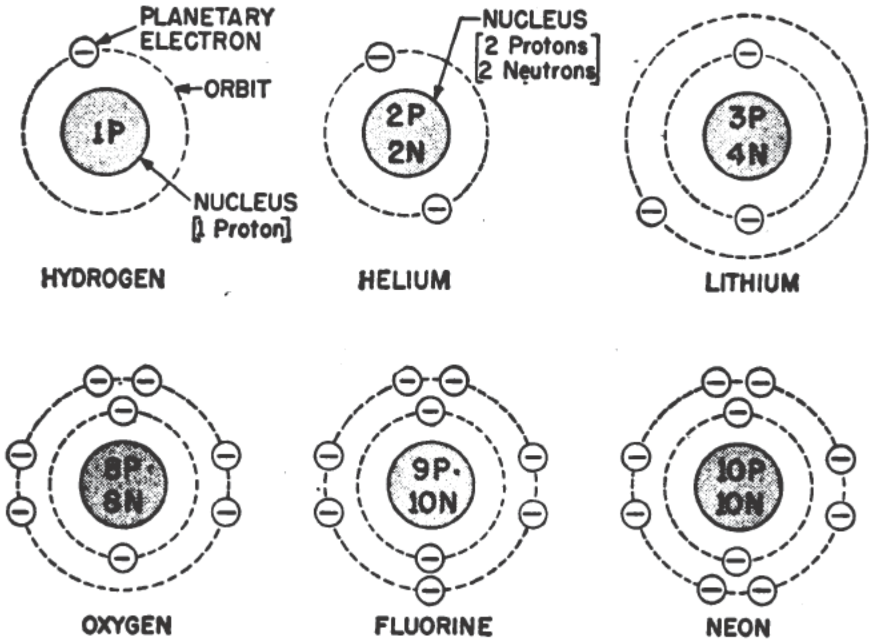


Figure 1–2.—Atomic structure of different elements.

The electrons in the outer orbits of certain elements are easily separated from the positive nucleii of their parent atoms and caused to flow in metals, in vacuums, or in tubes containing gas. Electrons have many important characteristics. The weight of an electron is very small compared with that of a proton or neutron (about 1/1845 of the weight of the proton of the lightest atom—that of hydrogen). The electron has a weight of 9×10^{-28} gram and a negative charge of 1.6×10^{-19} coulomb. This combination makes the electron an extremely active particle with many possibilities for practical use.

Ionization

Ordinarily an atom is most likely to be in that state in which the internal energy is at a minimum, called the NORMAL state.

If the internal energy of the atom is raised above that of the normal state, it is said to be EXCITED. Excitation may be produced in a number of ways, such as collision of the atom with high-speed positive or negative particles which may give up all or part of their energy to the atom during the collision. The excess energy absorbed by an atom may become sufficient to cause loosely bound outer electrons to leave the atom against the force that acts to hold them within. An atom that has lost or gained one or more electrons is said to be IONIZED. If the atom loses electrons it becomes positively charged and is referred to as a POSITIVE ION. Conversely, if the atom gains electrons, it becomes negatively charged and is referred to as a NEGATIVE ION. Actually then, an ion is a small particle of matter or group of such particles having a net positive or negative charge.

Free Electrons

When an orbital electron is removed from an atom it is called a FREE ELECTRON. Some of the electrons of certain metallic atoms are so loosely bound to the nucleus that they are comparatively free to move from atom to atom. Thus a very small force or amount of energy will cause such electrons to be removed from the atom and become free electrons. It is these free electrons that constitute the flow of an electric current in electrical conductors.

CONDUCTORS AND INSULATORS

Substances that permit the free motion of a large number of electrons are called CONDUCTORS. Copper wire is considered a good conductor because it has many free electrons. Electrical energy is transferred through conductors by means of the movement of free electrons that migrate from atom to atom inside the conductor. Each electron moves a very short distance to the neighboring atom where it replaces one or more electrons by forcing them out of their orbits. The replaced electrons repeat the process in other nearby atoms until the movement is transmitted throughout the entire length of the conductor. The greater the number of electrons that can be made to move in a material under the application of a given force the better are the conductive qualities of that material. A good conductor is said to have a low opposition or low resistance to the current (electron) flow.

In contrast to good conductors, some substances such as rubber, glass, and dry wood have very few free electrons. In these materials large amounts of energy must be expended in order to break the electrons loose from the influence of the nucleus. Substances containing very few free electrons are called POOR CONDUCTORS, NONCONDUCTORS, OR INSULATORS. Actually, there is no sharp dividing line between conductors and insulators, since electron motion is known to exist to some extent in all matter. Electricians simply use the best conductors as wires to carry current and the poorest conductors as insulators to prevent the current from being diverted from the wires.

Listed below are some of the best conductors and best insulators arranged in accordance with their respective abilities to conduct or to resist the flow of electrons.

Conductors	*Insulators*
Silver	Dry air
Copper	Glass
Aluminum	Mica
Brass	Rubber
Zinc	Asbestos
Iron	Bakelite

STATIC ELECTRICITY

One of the fundamental laws of electricity is that LIKE CHARGES REPEL EACH OTHER AND UNLIKE CHARGES ATTRACT EACH OTHER. Thus, there is a force of attraction in the atom between the positive nucleus and the negative electrons revolving about the nucleus in the planetary elliptical orbits.

The word STATIC means "standing still" or "at rest." Originally static electricity was considered electricity at rest because the experimenters of long ago thought that electrical energy produced by friction did not move. A simple experiment can easily be performed to produce static discharges. If a dry comb is run vigorously through the hair several times, and a cracking or popping sound is heard, it indicates that static discharges are taking place. Charges are first built up on the hair and the comb by the transfer of electrons from one to the other due to the friction between them. The discharge that follows is the rapid movement of electrons in the opposite direction from the comb to the hair as the charges neutralize each other. It is possible to see these discharges in the dark as tiny sparks.

Charged Bodies

In such an experiment strands of hair may stand out at angles because the loss of electrons has caused the hair to become positively charged and like charges repel each other. The comb, on the other hand, has gained electrons and thus acquired a negative charge.

If the negatively charged comb is held near a small piece of paper, the paper will be attracted to it and will cling for a short time. The negative charge on the comb will repel free electrons on the paper to the far side leaving the side nearest the comb positively charged. The unlike charges on the comb and on the nearest side of the paper account for the attractive force that draws the paper into contact with the comb. During contact some of the excess electrons move from the comb to the paper, thus giving the paper a negative charge. Since like charges repel each other the paper is repelled from the comb. Thus the paper is first attracted to the comb and then repelled by it.

Summarizing, a CHARGED body is one that has more or less than the normal number of electrons. It may be either positively or negatively charged. A positively charged body is one in which some of the electrons have been removed from the atoms and there is a deficiency of electrons, or fewer electrons than protons. A negatively charged body is one in which there are more than the normal number of electrons in each atom—that is, there are more electrons than protons. A body in which there is an equal number of electrons and protons in each atom is an uncharged body.

Removing electrons from a body involves physically attaching them to another body and then moving the other body some distance away. The second body will have an excess of electrons, and thus will be negatively charged. The first body will have a deficiency in electrons and thus will be positively charged. This principle can be illustrated by rubbing glass with silk. Some of the electrons are rubbed off the glass onto the silk, leaving the glass with a positive charge (deficiency of electrons) and the silk with a negative charge (surplus of electrons). So long as the silk and the glass are not brought into contact, they will retain the charges. However, when they are allowed to touch, the surplus of electrons on the silk will move onto the glass and neutralize the charge on the two bodies.

Coulomb's Law of Charges

It has been shown experimentally that charged bodies attract each other when they have unlike charges and repel each other when they have like charges. Thus electrons and protons attract each other, electrons repel other electrons, and protons repel other protons. The forces of attraction or repulsion change with the magnitude of the charges and also with the distance between them. This relation is dealt with in the law of forces discovered by a French scientist named Charles A. Coulomb, which states that CHARGED BODIES ATTRACT OR REPEL EACH OTHER WITH A FORCE THAT IS DIRECTLY PROPORTIONAL TO THE PRODUCT OF THE CHARGES ON THE BODIES AND INVERSELY PROPORTIONAL TO THE SQUARE OF THE DISTANCE BETWEEN THEM.

The charge on one electron or proton might be used as the unit of electric charge, but it would not be practical because of its very small magnitude. The practical unit of charge is the COULOMB, which is equivalent to the charge on 6,280,000,000,000,000,000 electrons. A more convenient way of expressing this number is 6.28×10^{18} electrons.

Electric Fields

The space between and around charged bodies in which their influence is felt is called an ELECTRIC FIELD OF FORCE. The electric field requires no physical or mechanical connecting link; it can exist in air, glass, paper, or a vacuum. ELECTROSTATIC FIELDS and DIELECTRIC FIELDS are other names used to refer to this region of force.

Fields of force spread out in the space surrounding their point of origin and, in general, DIMINISH IN PROPORTION TO THE SQUARE OF THE DISTANCE FROM THEIR SOURCE. An example of the force of gravity is the field of force that penetrates the space surrounding the earth and acts through free space to cause all unsupported objects in that region to fall to the earth. Newton discovered the law of gravitation, which states that EVERY OBJECT ATTRACTS EVERY OTHER OBJECT WITH A FORCE THAT IS DIRECTLY PROPORTIONAL TO THE PRODUCT OF THE MASSES OF THE OBJECTS AND INVERSELY PROPORTIONAL TO THE SQUARE OF THE DISTANCE BETWEEN THEM.

Note the similarity between the law of gravity and the law of attraction of charged bodies. The gravitational fields hold the

universe together, for with no gravitational field the planets, including the earth, would fly off into space instead of revolving around the sun. The moon would cease to revolve around the earth; and, because of the earth's rotation about its own axis, objects from its surface would fly out into space. Similarly, electrons revolving at high velocity around the positive nucleus of the atom are held in their orbits by the force of attraction of the positive nucleus. We conclude that a field of force must exist between the electrons and nucleus.

Relatively speaking, there are enormous spaces between the electrons and nucleus, even in the densest atoms. For example, if a copper penny were enlarged to the size of the earth's orbit around the sun (approx. 186,000,000 miles in diameter), the electrons in the penny would be the size of baseballs and the average distance between them would be about 3 miles.

Lines of Force

In diagrams, lines are used to represent the direction and intensity of the electric field of force. The intensity of the field (field strength) is indicated by the density (number of lines per unit area), and the direction of the field is indicated by arrowheads on the lines pointing in the direction in which a small test charge will move or tend to move when acted upon by the field of force.

A small test charge, either positive or negative, can be used to test the direction in which the force acts because the force of the dielectric field will act on either. Arbitrarily, however, it has been agreed to use a small positive charge when determining the direc-

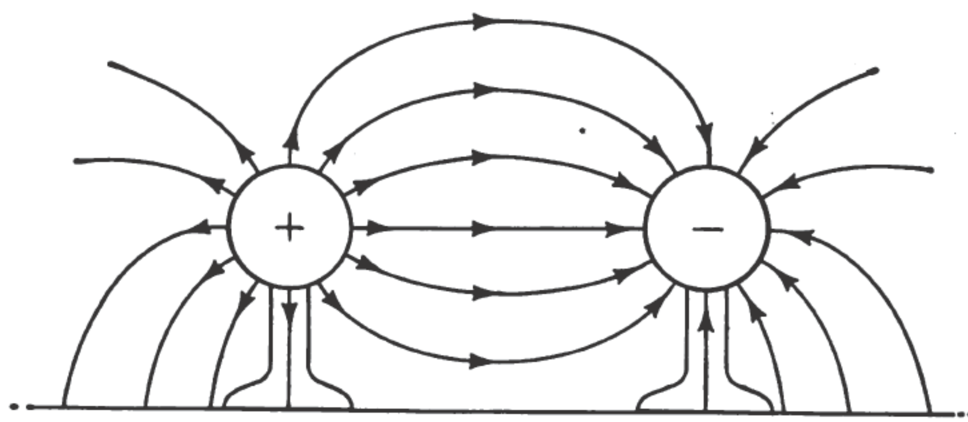


Figure 1–3.—Direction of electric field about positive and negative charges.

tion of the field. The test shows that the direction of the field about a POSITIVE charge is AWAY from the charge because a positive test charge is REPELLED; and that the direction about a NEGATIVE charge is TOWARD the charge because the positive test charge it ATTRACTED toward it. Thus, in figure 1–3, the direction of the field between the positive and the negative charges is from positive to negative.

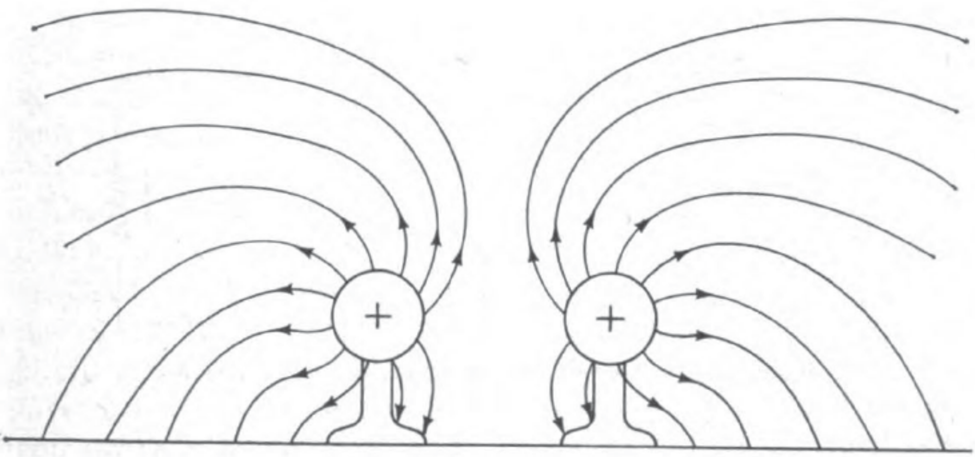


Figure 1–4.—Electric field between two positively charged bodies.

The electric field about like charges is shown in figure 1–4. It can be seen that the lines of force apparently repel each other. Also, in figure 1–3, the lines of force between the two charged bodies are not parallel. They bend outward at the center as if they were repelling each other. In figure 1–4, the lines of force located in the region between the two charges apparently repel each other.

In both figures the lines terminate on material objects and extend from a positive charge to a negative charge. They are regarded as imaginary lines in space along which a real force acts. In both examples, the direction in which the force acts is that in which a small positive test charge placed in the field will move—that is, from the positive charge to the negative charge.

ELECTRIC CURRENT

Up to this point reference has been made to static electricity, or electricity at rest. The free electrons in a conductor are moving constantly and changing positions in a vibratory manner.

If a source of supply (battery or d-c generator) is connected to the two ends of an electric circuit, as shown in figure 1–5, the free electrons almost instantly begin to move along the wires axially in one direction around the circuit.

The direction of flow is considered to be out from the negative terminal of the source, up through the load, and back through the positive terminal. This direction is opposite to the old established convention which assumes the direction of current flow

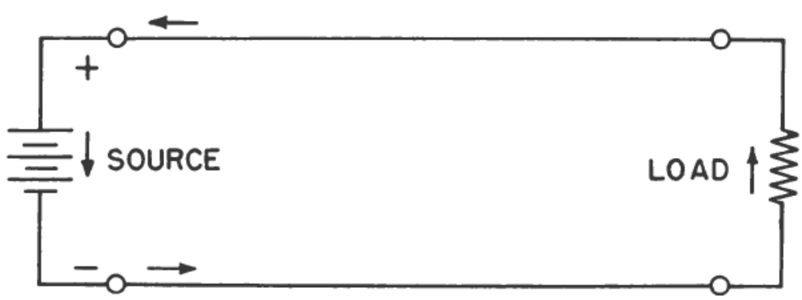


Figure 1–5.—Current (electron) flow around a circuit.

to be from positive to negative. The direction of electron flow is chosen as a basis for stating the rules governing circuit analysis in this training course because it is the direction of electron flow in conductors and electron tubes that comprises to a large extent the actual current flow (movement of free electrons) with which the technician will deal. THE READER IS CAUTIONED TO REGARD CAREFULLY, THE SYMBOLS OF CURRENT FLOW USED ON RECTIFIERS. These symbols generally refer to the old established convention which is opposite to that used in this training course.

Although the counterclockwise movement of the free electrons around the circuit is relatively slow, the speed of response is very fast. Thus, when the source is connected to the circuit, electrons start moving through the load almost immediately. The time interval between the instant the circuit is connected to the source and the instant current starts to flow in the load may be found by dividing the distance between the source and the load by a constant that is approximately equal to the speed of light. Thus if the load is 0.186 mile from the source, the time interval is 0.186 divided by 186,000 miles per second, or 0.000001 second. This interval is more conveniently expressed as 1 microsecond. Thus, the response is almost instantaneous and current starts to flow in all parts of the circuit when the source is connected to it.

The drift or flow of electrons through the circuit is called an ELECTRIC CURRENT. In order to determine the amount of current, a unit of measure must be adopted with which to work. The term that defines unit current flow is called the AMPERE and is named after the French philosopher Andre Marie Ampere who discovered in 1823 the relation between the direction of current in a wire and the direction of the magnetic field around it. The symbol for the ampere is I.

A flow of 1 ampere is equivalent to the flow of 6.28×10^{18} electrons per second past a fixed point in the circuit. The ampere is analogous to the rate of flow of water through a pipe in gallons per second.

Unit quantity of electricity is moved through an electric circuit when 1 ampere flows for 1 second. This unit is equivalent to 6.28×10^{18} electrons. It is called the COULOMB after Charles A. Coulomb who invented the torsion balance for proving the law of the inverse square, which he applied to the forces between charged bodies. The symbol for the coulomb is Q. The rate of flow of current in amperes and the quantity of electricity moved through a circuit are related by the common factor of time. Thus, the quantity of electric charge, in coulombs, moved through a circuit is equal to the product of the current in amperes, I, and the duration of flow in seconds, t. Expressed as an equation,

$$Q = It.$$

For example, if a current of 2 amperes flows through a circuit for 10 seconds the quantity of electricity moved through the circuit is 2×10, or 20 coulombs. Conversely, current flow may be expressed in terms of coulombs and time in seconds. Thus, if 20 coulombs are moved through a circuit in 10 seconds, the average current flow is $\frac{20}{10}$, or 2 amperes. Note that the current flow in amperes implies the rate of flow of coulombs per second without indicating either coulombs or seconds. Thus a current flow of 2 amperes is equivalent to a rate of flow of 2 coulombs per second.

DIFFERENCE IN POTENTIAL

The force that causes free electrons to move in a conductor as an electric current is called (1) an ELECTROMOTIVE FORCE (emf), (2) a VOLTAGE, or (3) a DIFFERENCE IN POTENTIAL. When

a difference in potential exists between two charged bodies that are connected by a conductor, electrons will flow along the conductor from the negatively charged body to the positively charged body until the two charges are equalized and the potential difference no longer exists.

An analogy of this action is shown in the two water tanks connected by a pipe and valve in figure 1–6. At first the valve is closed and all the water is in tank *A*. Thus, the water pressure across the valve is a maximum. When the valve is opened, the water flows through the pipe from *A* to *B* until the water level becomes the same in both tanks. Then the water stops flowing in the pipe because there is no longer a difference in water pressure between the two tanks.

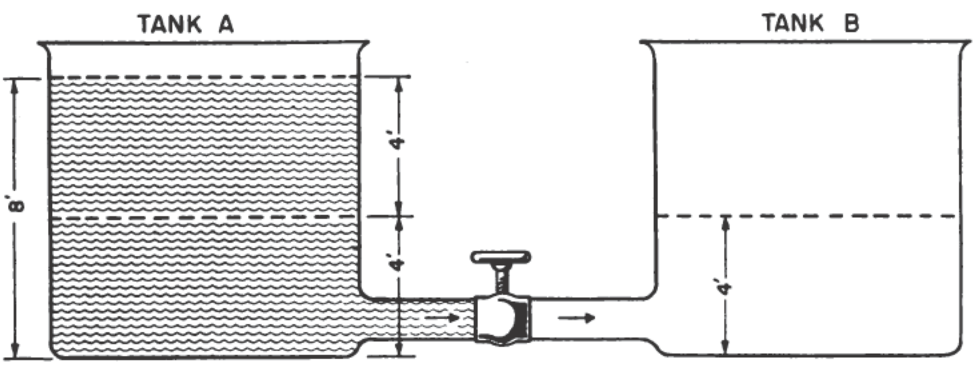


Figure 1–6.—Water analogy of electric difference in potential.

There are several methods of obtaining a voltage. The most common are BATTERIES and GENERATORS. The Italian scientist Alessandro Volta (1745–1827) invented the first electric battery. The unit of electric pressure, the VOLT, is named in his honor.

A voltage may be produced in various ways: (1) mechanically by moving a conductor through a magnetic field so that it cuts the lines of force; (2) chemically, for example, by inserting carbon and zinc electrodes into a solution of sulfuric acid and water, (3) by friction as when a rubber rod is rubbed with a woolen cloth, (4) by the application of heat to the junction of two dissimilar substances, such as copper and iron, (5) by the piezoelectric effect described in the training manual, *Basic Electronics,* NavPers 10087, and (6) by the photovoltaic effect in which a light ray strikes a substance like copper oxide. The first two methods are used in generators and batteries, respectively; the

third is used in generators of high-voltage static electricity; the fourth in devices such as thermocouples for the measurement of temperatures; the fifth in crystal microphones and phonograph pickups; and the sixth in light meters.

Current flow through an electric circuit is directly proportional to the difference in potential across the circuit, just as the flow of water through the pipe in figure 1–6 is directly proportional to the difference in water level in the two tanks.

A fundamental law of current electricity is that the CURRENT IS DIRECTLY PROPORTIONAL TO THE APPLIED VOLTAGE.

RESISTANCE

Electrical resistance is that quality of an electric circuit that opposes the flow of current through it. The simple electric circuit of figure 1–5 possesses resistance in varying degree in all parts of it—that is, in the source, in the load, and in the connecting wires. The size and material of the wires are such as to keep the electrical resistance low so that current can flow as easily through them as water flows through the pipe between the tanks in figure 1–6 when the valve is opened. If the water pressure remains constant, the flow of water in the pipe will vary with the opening of the valve. The smaller the opening the greater the opposition to the flow and the smaller will be the rate of flow.

In the electric circuit, the larger the diameter of the wires the lower will be their electrical resistance (opposition) to the flow of current through them. In the water analogy, pipe friction opposes the flow of water between the tanks. This friction is similar to electrical resistance. The resistance of the pipe to the flow of water through it depends upon (1) the length of the pipe, (2) the diameter of the pipe, and (3) the nature of the inside walls (rough or smooth). Similarly, the electrical resistance of the conductors depends upon (1) the length of the wires, (2) the diameter of the wires, and (3) the material of the wires (copper, aluminum, etc.).

Temperature also affects the resistance of electrical conductors to some extent. In most conductors (copper, aluminum, iron, etc.) the resistance increases with temperature. Carbon is an exception. In carbon the resistance decreases with temperature increase. Certain alloys of metals (manganin and constantan) have resistance that does not change appreciably with temperature.

The relative resistance of several conductors of the same length and cross section is given in the following list with silver as a standard of 1 and the remaining metals arranged in an order of ascending resistance:

Silver	1.0
Copper	1.08
Gold	1.4
Aluminum	1.8
Platinum	7.0
Lead	13.5

QUIZ

1. What particle is involved in most chemical changes?
2. What is the smallest particle that makes up a material called an ELEMENT?
3. In what portion of the atom is most of its weight concentrated?
4. What is the relation between the number of electrons and the number of protons in an atom in its normal state?
5. How many protons and electrons are contained in the hydrogen atom?
6. How many protons and electrons are contained in the curium atom?
7. What is the weight and charge of the electron?
8. What is the relative weight of an electron compared to a proton?
9. What is the relation between the number of electrons and the number of protons in an ionized atom?
10. What is an ion?
11. What constitutes the flow of an electric current in solid conductors?
12. With regard to the number of free electrons, what is the difference between a good conductor and a good insulator?
13. Is there a sharp dividing line between conductors and insulators?
14. What is the relative number of electrons on a charged body compared to the number when the body is uncharged?
15. A positively charged body will attract a (a) ________ charged body and repel a (b) ________ charged body.
16. A negatively charged body will attract a (a) ________ charged body and repel a (b) ________ charged body.
17. If the distance between two charged bodies is doubled, how is the force of attraction or repulsion affected?
18. What is the space between and around charged bodies called?
19. How is the direction in which an electrostatic line of force acts commonly determined?

20. What are electrostatic lines of force?
21. The flow of 6.28×10^{18} electrons past a fixed point in a circuit in 0.5 second is equivalent to how much current flow in amperes?
22. What is the unit of quantity of electricity?
23. How many ampers flow in a circuit when 20 coulombs move through it in 5 seconds?
24. If the voltage across a given resistor is doubled, how is the current affected?
25. What will be the net effect on the flow of current in an electrical circuit if the size of the connecting wires is reduced and the electrical pressure remains constant?
26. For most conductors used in electrical circuits, what is the effect on the current flow of an increase in conductor temperature?

CHAPTER 2

BATTERIES

VOLTAIC CELLS

When two dissimilar elements such as carbon and zinc (or copper and zinc) are immersed into a solution of sulfuric acid and water, the acid will attack the zinc more readily than the carbon, and a potential difference will appear between the two elements.

This arrangement is called a VOLTAIC CELL. A voltaic cell is a device designed to convert chemical energy into electric energy. The elements are known as the ELECTRODES, and the acid solution is called the ELECTROLYTE. If the elements must be discarded at the end of their useful life the cell is called a PRIMARY CELL. If the elements can be restored to their original condition by charging from an electric supply circuit the cell is called a SECONDARY CELL. The simple voltaic cell is a primary cell.

If a conductor is connected externally to the electrodes, electrons will flow under the influence of a difference in potential across the electrodes from the zinc (negative) through the external conductor to the carbon (positive) returning within the solution to the zinc. After a short period of time, the zinc will begin to waste away because of the "burning" action of the acid. If zinc is surrounded by oxygen it will burn (become oxidized) as a fuel. In this respect the cell is like a chemical furnace in which energy released by the zinc is transformed into electrical energy rather than heat energy. A simple primary cell is shown in figure 2–1. It consists of two electrodes (carbon and zinc) and a jar containing a dilute solution of sulfuric acid (H_2SO_4) and water (H_2O).

The voltage across the electrodes depends upon the materials comprising them and the composition of the solution. The difference of potential between carbon and zinc electrodes in a dilute solution of sulfuric acid and water is about 1.5 volts. In most practical primary cells the voltage does not exceed 2 volts.

The current that a primary cell may deliver depends upon the resistance of the entire circuit, including that of the cell itself. The internal resistance of the primary cell depends upon the size of the electrodes, the distance between them in the solution, and the resistance of the solution. The larger the electrodes and the

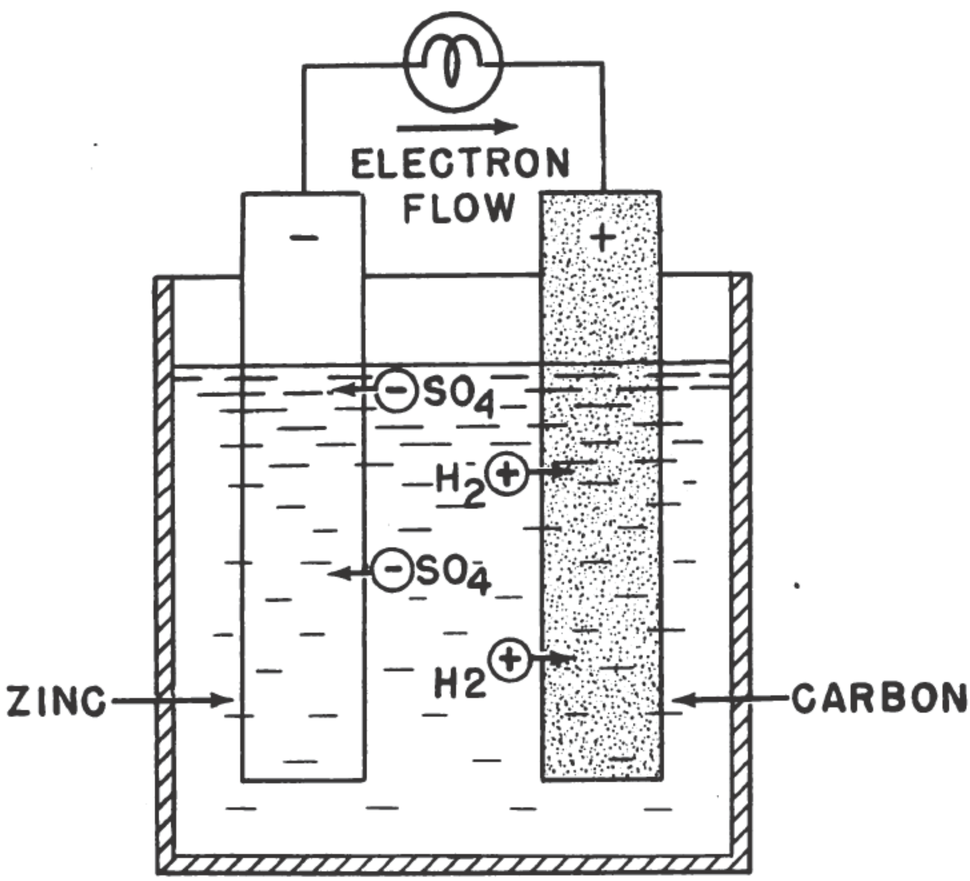


Figure 2–1.—Simple primary cell.

closer together they are in solution (without touching) the lower will be the internal resistance of the primary cell and the more current it will be capable of supplying to a load.

Action on Discharge

When current flows through a cell the zinc will gradually be dissolved in solution and the acid will be neutralized. A chemical equation is sometimes used to show the chemical action that takes place. The symbols in the equation represent the different materials that are used. The symbol for carbon is C and for zinc Zn.

The equation is quantitative and equates the number of parts of the materials used before and after the zinc is oxidized. It will be remembered from chapter 1 that all matter is composed of atoms and molecules, with the atom being the smallest part of an element and the molecule, the smallest part of a compound.

A COMPOUND is a chemical combination of two or more elements in which the physical properties of the compound are different from those of the elements comprising it. For instance, a molecule of water, H_2O, is composed of two atoms of hydrogen, H_2, and one atom of oxygen, O. Ordinarily, hydrogen and oxygen are gases, but when combined, as stated above, they form water, which normally is a liquid. On the other hand, sulfuric acid, H_2SO_4, and water, H_2O form a MIXTURE (not a compound) because the identity of both liquids is preserved when they are in solution together.

When a current flows through a primary cell having carbon and zinc electrodes and a dilute solution of sulfuric acid and water, the chemical reaction that takes place can be expressed as:

$$Zn + H_2SO_4 + H_2O \xrightarrow[\text{discharge}]{} ZnSO_4 + H_2O + H_2\uparrow.$$

The expression indicates that as current flows, a molecule of zinc combines with a molecule of sulfuric acid to form a molecule of zinc sulfate ($ZnSO_4$) and a molecule of hydrogen (H_2). The zinc sulfate dissolves in solution and the hydrogen appears as gas bubbles around the carbon electrode. As current continues to flow, the zinc is gradually consumed and the solution changes to zinc sulfate and water. The carbon electrode does not enter into the chemical changes taking place but simply provides a return path for the current.

In the process of oxidizing the zinc, the solution breaks up into positive and negative ions that move in opposite directions through the solution (fig. 2–1). The positive ions are hydrogen ions that appear around the carbon electrode (positive terminal). They are attracted to it by the free electrons from the zinc that are returning to the cell by way of the external load and the positive carbon terminal. The negative ions are SO_4 ions that appear around the zinc electrode. Positive zinc ions enter the solution around the zinc electrode and combine with the negative SO_4 ions to form zinc sulfate, $ZnSO_4$, a grayish-white substance that

dissolves in water. At the same time that the positive and negative ions are moving in opposite directions in the solution, electrons are moving through the external circuit from the negative zinc terminal, through the load, and back to the positive carbon terminal. When the zinc is used up, the voltage of the cell is reduced to zero. There is no appreciable difference in potential between zinc sulfate and carbon in a solution of zinc sulfate and water.

Polarization

The chemical action that takes place in the cell (fig. 2–1) while the current is flowing causes hydrogen bubbles to form on the surface of the positive carbon electrode in great numbers until the entire surface is surrounded. This action is called POLARIZATION. Some of these bubbles rise to the surface of the solution and escape into the air. However, many of the bubbles remain until there is no room for any more to be formed.

The hydrogen tends to set up an electromotive force in the opposite direction to that of the cell, thus increasing the effective internal resistance, reducing the output current, and lowering the terminal voltage.

A cell that is heavily polarized has no useful output. There are several ways to prevent polarization from occurring or to overcome it after it has occurred. The very simplest method might be to remove the carbon electrode and wipe off the hydrogen bubbles. When the electrode is replaced in the electrolyte, the emf and current are again normal. This method is not practicable because polarization occurs rapidly and continuously in the simple voltaic cell. A commercial form of voltaic cell, known as the DRY CELL, employs a substance rich in oxygen as a part of the positive carbon electrode, which will combine chemically with the hydrogen to form water, H_2O. One of the best depolarizing agents used is manganese dioxide (MnO_2), which supplies enough free oxygen to combine with all of the hydrogen so that the cell is practically free from polarization.

The chemical action that occurs may be expressed as

$$2MnO_2 + H_2 \rightarrow Mn_2O_3 + H_2O.$$

The manganese dioxide combines with the hydrogen to form water and a lower oxide of manganese. Thus the counter emf

of polarization does not exist in the cell, and the terminal voltage and output current are maintained normal.

Local Action

When the external circuit is opened, the current will cease to flow, and theoretically all chemical action within the cell will stop. However, commercial zinc contains many impurities, such as iron, carbon, lead, and arsenic. These impurities form many small cells within the zinc electrode in which current flows between the zinc and its impurities. Thus the zinc is oxidized even though the cell itself is on open circuit. This wasting away of the zinc on open circuit is called LOCAL ACTION. For example, a small local cell exists on a zinc plate containing impurities of iron, as shown in figure 2-2. Electrons flow between the zinc and iron and the solution around the impurity becomes ionized. The

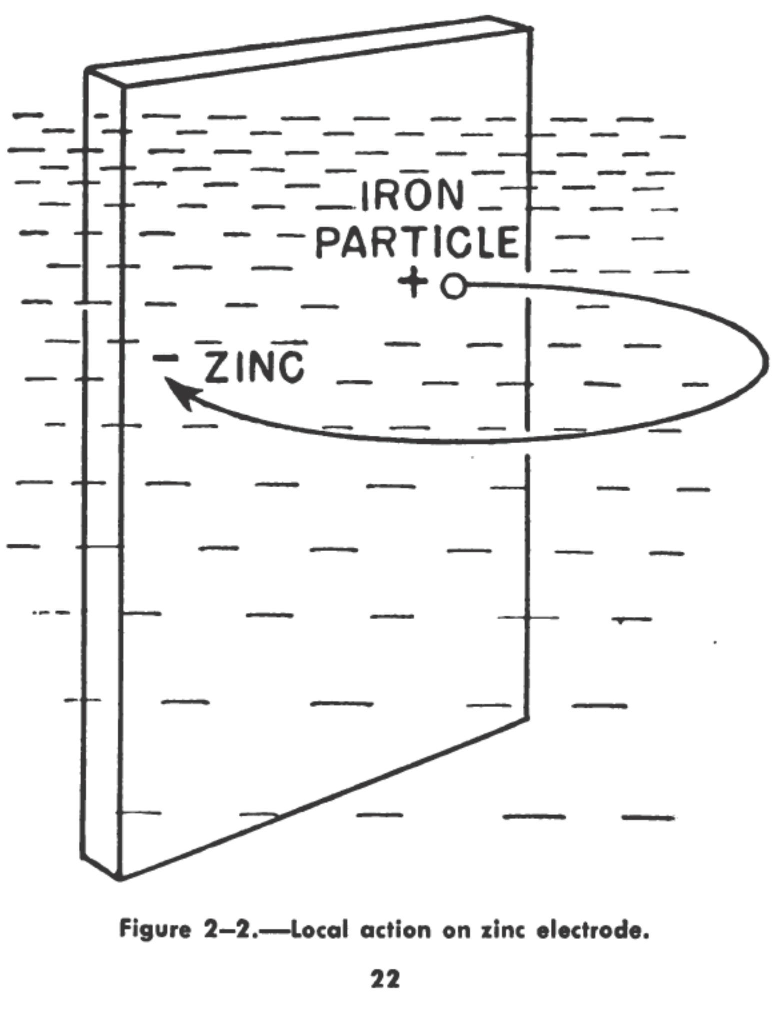


Figure 2-2.—Local action on zinc electrode.

negative SO_4 ions combine with the positive Zn ions to form $ZnSO_4$. Thus the acid is depleted in solution and the zinc consumed.

Local action may be prevented (1) by using pure zinc (which is not practical), (2) by coating the zinc with mercury, or (3) by adding a small percentage of mercury to the zinc during the manufacturing process. The treatment of the zinc with mercury is called AMALGAMATING the zinc. Since mercury is 13.6 times as heavy as an equal volume of water, small particles of impurities having a lower relative weight than that of mercury will rise (float) to the surface of the mercury. The removal of these impurities from the zinc prevents local action. The mercury is not readily acted upon by the acid, and even when the cell is delivering current to a load, the mercury continues to act on the impurities in the zinc, causing them to leave the surface of the zinc electrode and float to the surface of the mercury. This process greatly increases the life of the primary cell.

Dry (Primary) Cells

There are two types of primary cells, the WET CELL and the DRY CELL. Although many types of wet cells were used in the past, they are not prevalent today because of the more convenient so-called dry cell. Because wet cells must remain in an upright position in order not to spill the electrolyte, they are not readily transportable.

LECLANCHE CELL.—The dry cell (also called the LeClanche cell) is much more commonly used than the wet cell. Actually this cell is not dry in the sense that there is no solution present. It is called a dry cell because the construction is such that the chemicals will not spill; the electrolyte is made in the form of a paste and the cell is sealed so that the water inside will not leak out. Figure 2–3 shows a cut-away view of a common type of dry cell.

The container is usually zinc and serves as the negative electrode. The zinc container is lined with a layer of blotting paper, which prevents the zinc from touching the positive electrode. The positive electrode is a carbon rod, located centrally in the cell and surrounded by, and in contact with, a paste of ground carbon, manganese dioxide, sal ammoniac (ammonium chloride),

and zinc chloride. The paste is moistened with water. (If the water evaporates, the cell will become inoperative.)

The sal ammoniac (NH_4Cl) is the electrolyte. The zinc chloride ($ZnCl_2$) reduces local action, and the manganese dioxide is the depolarizing agent. The ground carbon decreases

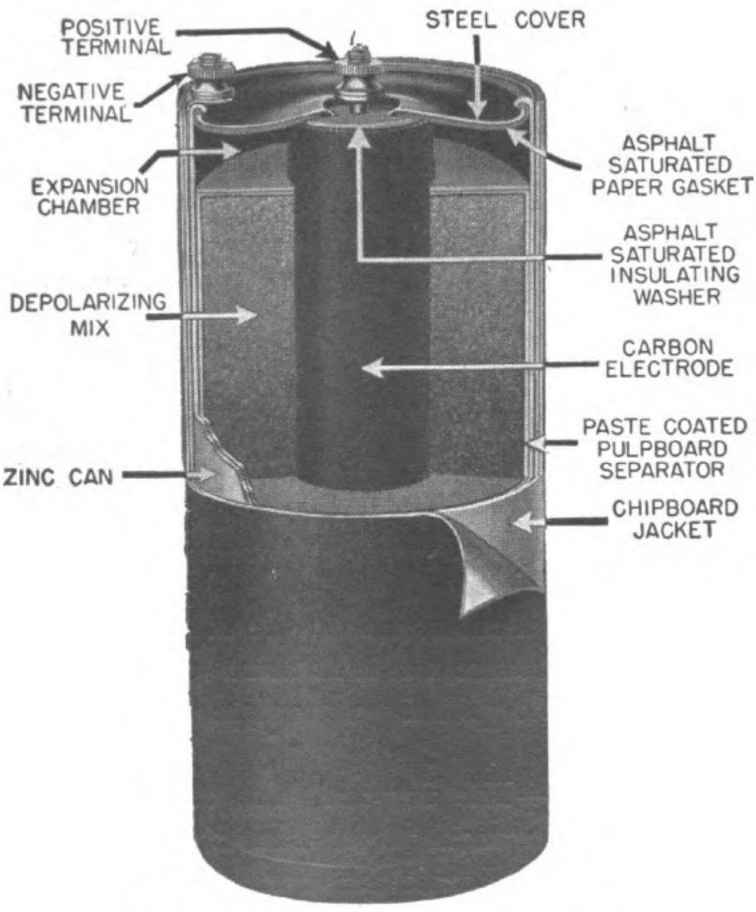


Figure 2–3.—Cut-away view of dry cell.

the internal resistance of the cell by increasing the area of the carbon electrode in contact with the electrolyte. The chemical actions taking place within the cell are expressed as follows:

$$Zn+2NH_4Cl+2MnO_2 \xrightarrow[\text{discharge}]{} Mn_2O_3+H_2O+2NH_3\uparrow+ZnCl_2$$

The left side of the expression represents the materials within the cell when it is new. The right side of the equation represents the materials in the cell when it is completely discharged. The

expression indicates that as current flows, the zinc, manganese dioxide, and ammonium chloride combine to produce a lower form of manganese dioxide, water, ammonia, and zinc chloride. If the cell is allowed to discharge too rapidly a gas will be formed by the ammonia in quantities too great to be absorbed by the water, thus causing the cell to swell or crack. The emf of this type of cell depends upon the materials used and is approximately 1.5 volts irrespective of the size.

Thus the smallest flashlight cells have the same open circuit voltage as the largest types of dry cells, such as the No. 6 dry cell. Dry cells are best suited for intermittent use, such as in flashlights, portable test equipment, walkie-talkie radios, and small direct-current motors. The polarizing agent in dry cells is slow acting, because of the paste electrolyte. If considerable current is drawn continuously, the cell will become polarized and the terminal voltage will fall so that the cell appears to be completely discharged. However, if allowed to remain inoperative at intervals

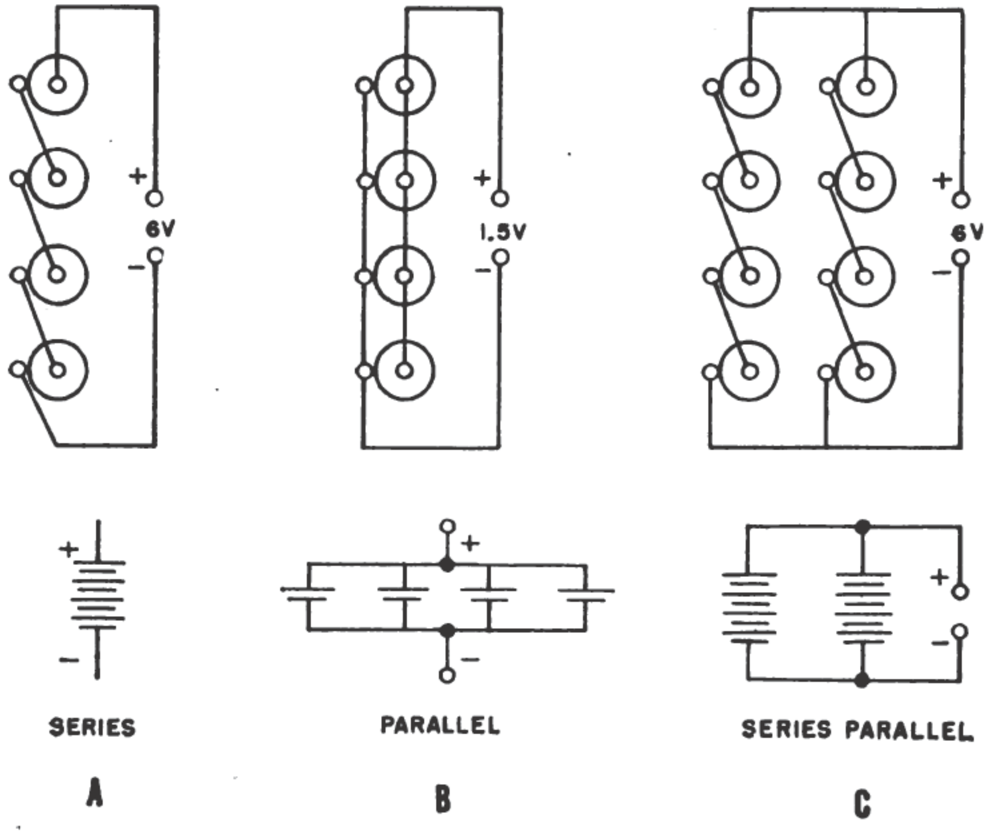


Figure 2–4.—Methods of connecting primary cells.

the cell will depolarize and the terminal voltage will rise to the normal value.

Many electrical devices require higher voltage or higher current than a single cell is able to furnish. Therefore, it is often necessary to connect several cells together to increase either the current, or voltage, or both. Such a grouping of cells is called a BATTERY. The manner in which the cells are connected depends upon the need. Connecting the cells in series increases the voltage output. Connecting them in parallel increases the current output. Figure 2–4 shows several plan views of No. 6 dry cells connected in series, in parallel, and in series-parallel and also the standard battery symbols corresponding to these connections.

Cells are arranged in series with their voltages in addition by connecting the positive (center) terminal of one cell to the negative (outer) terminal of the next cell, as shown in figure 2–4, A. The total voltage of the battery is equal to the sum of the voltages of the individual cells. Some batteries are made by combining 15, 30, or 45 cells in series to produce 22½, 45, or 67½ volts, respectively. This type of battery is referred to as a B-battery. The current output of a series of cells is equal to that of one cell.

Cells are arranged in parallel by connecting all of the positive terminals together to form one common terminal of the battery, and then connecting all of the negative terminals together to form the negative terminal of the battery, as shown in figure 2–4, B. The current output of a group of parallel cells is equal to the sum of the individual currents in each cell. The voltage output of the parallel group is equal to that of one cell. In the parallel group each cell must have the same voltage and internal resistance. Otherwise a cell with higher voltage and lower internal resistance would force current through the lower voltage cell and carry all of the load.

Another method of arranging the cells is to connect them in series-parallel, as shown in figure 2–4, C. The 4 cells in each group are connected in series and two groups are connected in parallel. This method increases the battery voltage to 4 times that of one cell and the current rating to twice that of one cell.

RM DRY CELL.—During recent years, the demand for a dry cell that would stand up under tropical conditions of heat and

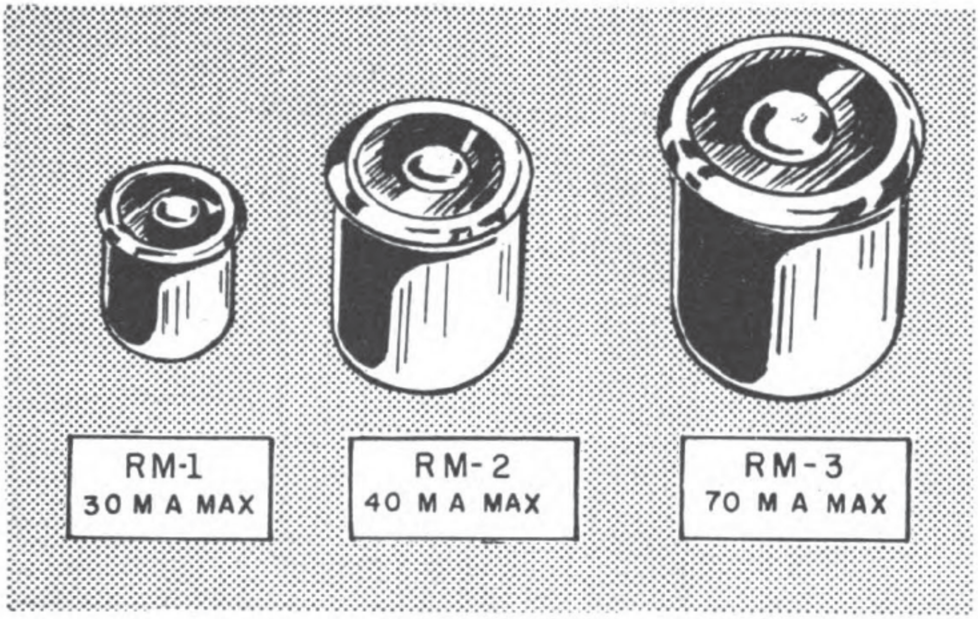


Figure 2–5.—RM cells of various sizes.

humidity led to the development of the RM cell. This cell is also called the RUBEN cell and the MERCURY cell.

The RM cell is much more expensive to manufacture than the LeClanche cell (dry cell); however, it produces 4 to 7 times as much current and its output does not fall off as rapidly.

The RM cells, (fig. 2–5) are in the form of cylinders ½ to 1 inch in diameter and approximately 1 inch in length.

Electric current in the RM cell is produced by a chemical reaction between zinc and mercuric oxide. The positive electrode is a mixture of mercuric oxide and ground carbon held in a small iron-can container which encloses the remainder of the cell. The negative electrode is a small pellet of zinc extending through the top of the can at the center. The electrolyte is a solution of potassium hydroxide.

The voltage of the RM cell when not in use is 1.34 volts, but under normal current drain the voltage will drop to between 1.31 and 1.24 volts. Although, the closed-circuit voltage is approximately 0.2 volt below that of a regular dry cell; the RM cell maintains a more constant voltage under load for its entire useful life. The main uses of the RM cells are in walkie-talkie and other portable electronic equipments.

RESERVE CELL.—A reserve cell is a cell in which the elements are kept dry until the time of use, at which time the electrolyte

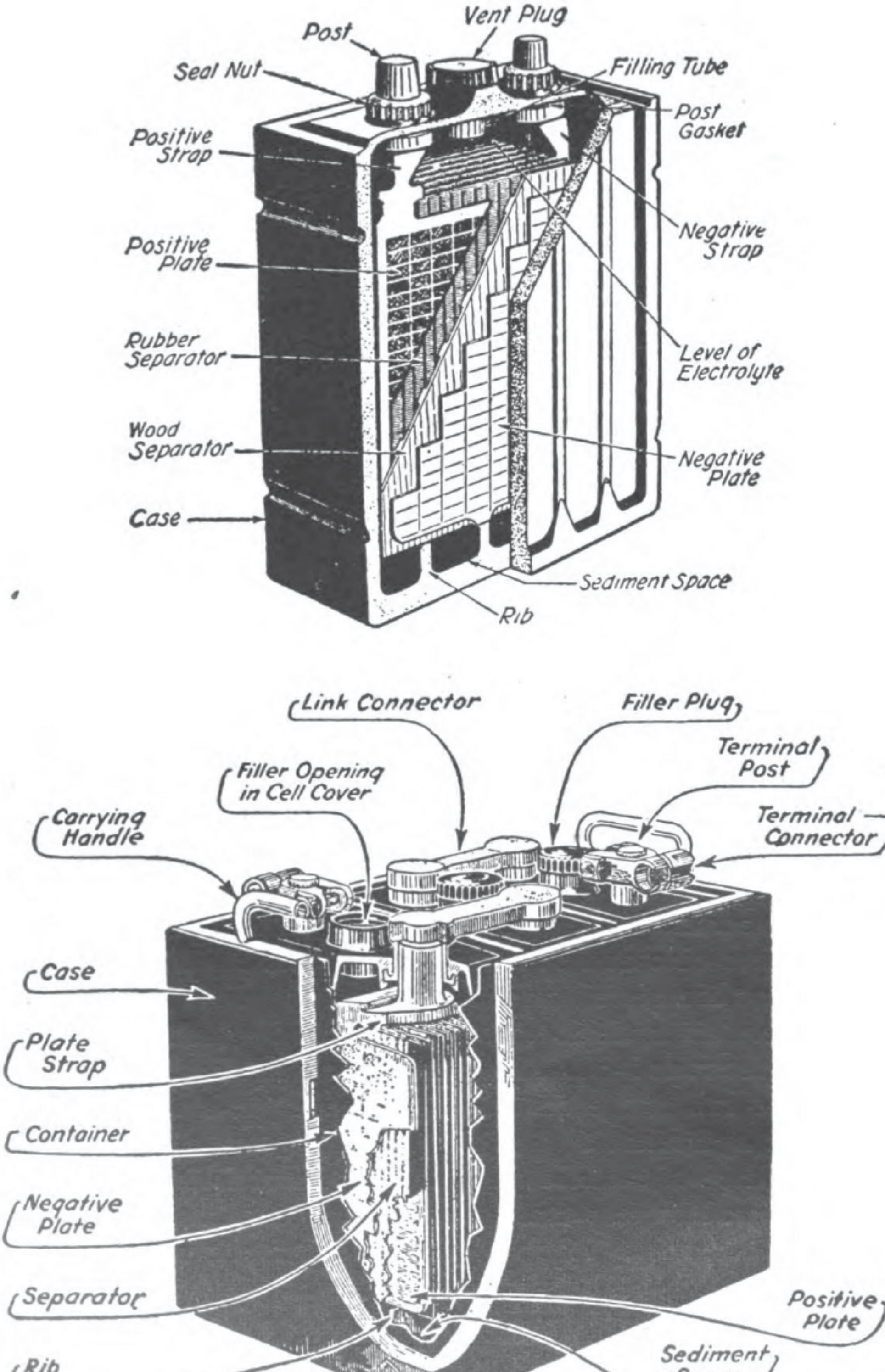


Figure 2–6.—Lead-acid storage battery.

is admitted and the cell starts producing current. The rigorous requirements of the proximity fuse led to the development of this type of cell. In the proximity fuse the battery consists of 63 annular (or ring-shaped) zinc and carbon electrodes. A glass container is mounted in the center of the electrodes. The shock of the acceleration produced by firing the shell breaks the glass container and forces the electrolyte between the electrodes, thus activating the cell.

The shelf life of primary reserve batteries is indefinitely long even under adverse tropic or arctic conditions. This life is attained by incorporating the depolarizer in the liquid electrolyte so that the elements of the cell are subject to no chemical action until the time of use.

SECONDARY CELLS

As mentioned previously a secondary cell is one in which the elements can be restored to their original condition by charging from an electric supply circuit. Thus when a charging current flows through the cell in the opposite direction to that of the discharge current the solution and the electrodes are restored to their original condition. The secondary cell is used in battery form (2 or more cells connected together) in automobiles, motor boats, submarines, etc. Secondary cells are also referred to as STORAGE CELLS or ELECTRIC ACCUMULATORS.

Lead-Acid Cell

There are two types of storage cells in general use today. They are (1) the lead-acid cell, and (2) the nickel-iron-alkaline (Edison) or nickel-cadmium alkaline cell. The lead-acid battery is the type most widely used in the Navy. A 6-volt lead-acid battery with a cut-away view of the cells is shown in figure 2–6.

CHEMICAL ACTION.—The nature of the chemical reactions that take place in the lead-acid cell is rather involved, but the following brief description will give some idea of what occurs during a cycle of discharge and charge.

When a cell is fully charged (fig. 2–7, A), the active material of the positive plate is in the form of lead peroxide, PbO_2, and of the negative plate pure sponge lead, Pb. The specific gravity of the electrolyte (relative weight compared with the weight of the

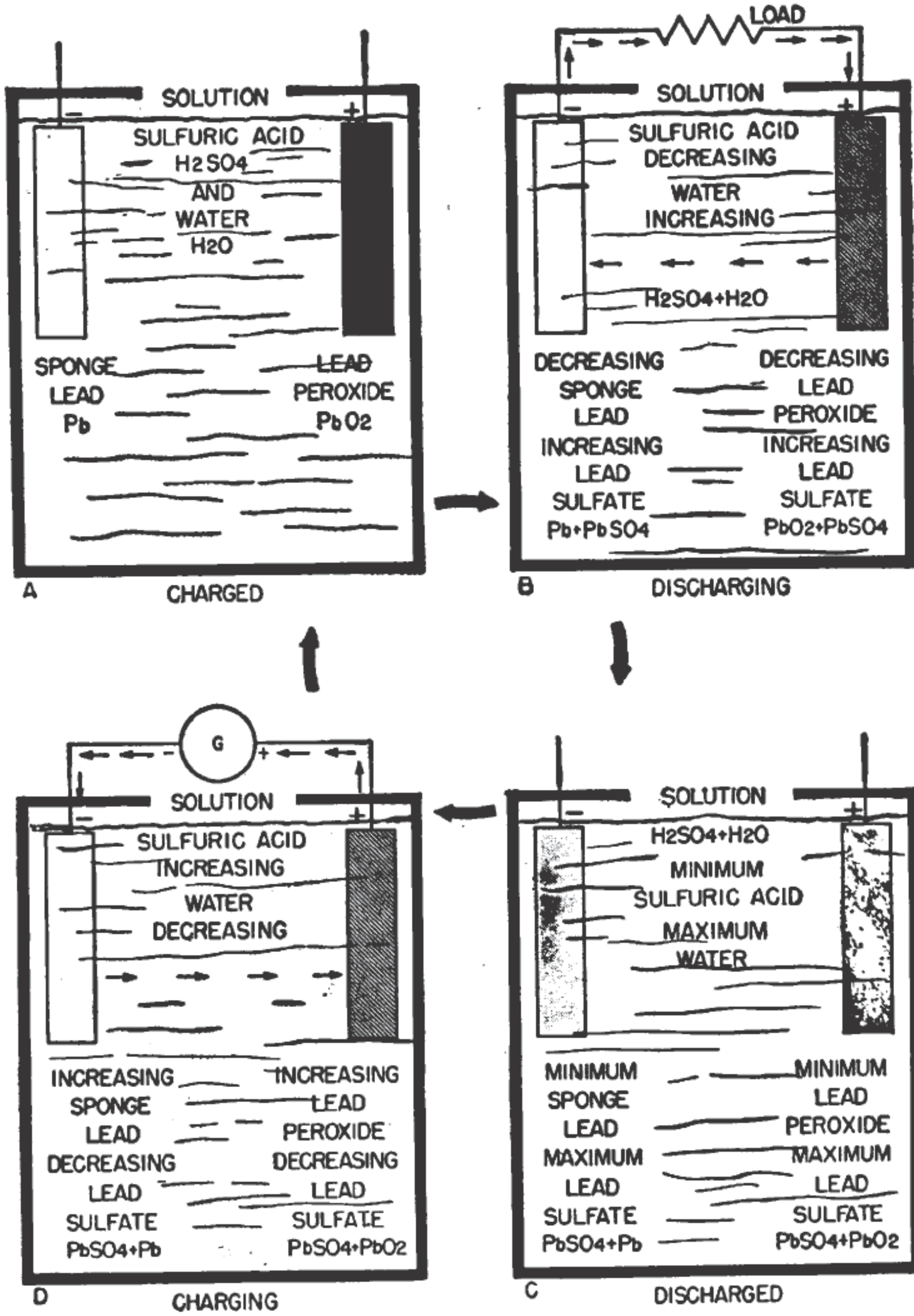


Figure 2–7.—Chemical action in lead-acid cell.

same volume of water) is at its maximum. Chemical energy is stored in the cell in this condition. The open-circuit voltage (no connected load) is a little more than 2 volts.

If an external circuit is closed between the positive and negative terminals of the cell, current begins to flow because of the action of the electrolyte upon the active material. The chemical energy is thus transformed into electrical energy and the cell is said to be discharging (fig. 2–7, B). The electrolyte reacts with the lead on the negative plate and the lead peroxide on the positive plate to form lead sulfate on both the positive and negative plates. The action is represented by the following chemical equation,

$$Pb + PbO_2 + 2H_2SO_4 \underset{\text{charging}}{\overset{\text{discharging}}{\rightleftarrows}} 2PbSO_4 + 2H_2O.$$

The left side of the expression represents the cell in the charged condition and the right side represents the cell in the discharged condition.

In the charged condition the positive plate contains lead peroxide, PbO_2; the negative plate is composed of sponge lead, Pb; and the solution contains sulfuric acid, H_2SO_4. In the discharged condition both plates contain lead sulfate, $PbSO_4$, and the solution contains water, H_2O. As the discharge progresses, the acid content of the electrolyte becomes less and less because it is used in forming lead sulfate, and the specific gravity of the electrolyte decreases. A point is reached where so much of the active material has been converted into lead sulfate that the cell can no longer produce sufficient current to be of practical value. At this point the cell is said to be discharged (fig. 2–7, C). Since the amount of sulfuric acid combining with the plates at any time during discharge is in direct proportion to the ampere-hours (product of current in amperes and time in hours) of discharge, the specific gravity of the electrolyte is a guide in determining the state of discharge of the lead-acid cell.

If the discharged cell is properly connected to a direct-current charging source the voltage of which is slightly higher than that of the cell, current will flow through the cell, in the opposite direction to that of discharge, and the cell is said to be charging (fig. 2–7, D). The effect of the current will be to change the lead sulfate on both the positive and negative plates back to its original active form of lead peroxide and sponge lead, respectively. At

the same time, the sulfate is restored to the electrolyte with the result that the specific gravity of the electrolyte increases. When all the sulfate has been restored to the electrolyte the specific gravity will be a maximum. The cell is then fully charged and is ready to be discharged again.

It should always be remembered that the addition of sulfuric acid to a discharged lead-acid cell does not recharge the cell. Adding acid only increases the specific gravity of the electrolyte and does not convert the lead sulfate on the plates back into active material (sponge lead and lead peroxide) and consequently does not bring the cell back to a charged condition. A charging current must be passed through the cell to do this.

As a cell charge nears completion, hydrogen gas, H_2, is liberated at the negative plate and oxygen gas, O_2, is liberated at the positive plate. This action occurs because the charging current is greater than the amount that is necessary to reduce the small remaining amount of lead sulfate on the plates. Thus, the excess current ionizes the water in the electrolyte. This action is necessary to assure full charge to the cell.

PASTED PLATES.—Lead-acid storage batteries use a variety of plates ranging from the spun-lead (Plante) type to the lighter pasted construction. The Plante-type plates require repeated charging and discharging to develop the active material to the proper depth. The process is slow and expensive. In 1881 Camille Faure in France made a radical improvement in the construction of storage batteries by developing a pasted plate that was manufactured with much less expense.

The pasted plate is most commonly used in portable lead-acid batteries today. The plates are formed by applying special lead-oxide pastes to a grid made of lead-antimony alloy. The grid holds in place the active material with which the spaces in the grid are filled and distributes the current evenly to all parts of the plate.

When the paste is dry, the plates are given a forming charge. They are formed by immersing them in electrolyte and passing current through them in the proper direction to change the paste to lead peroxide for the positive plates and to sponge lead for the negative plates. This type of plate takes less time to manufacture and is relatively light in weight compared to the Plante spun-lead plates, which have a more rugged and durable construction.

IRONCLAD PLATE.—The IRONCLAD PLATE is used extensively in submarine batteries because it has a long life. The positive ironclad plate (fig. 2–8, A) is quite different from the pasted plate. It consists of several slotted hard-rubber tubes filled with active material that is similar to the material used in the pasted plate. Extending through the center of each tube is a core of lead antimony, which is firmly anchored in the paste by means of radial fins that are equally spaced along the core. The core, which is lead-burned to the top and bottom of the frame, serves primarily as a conductor for the electric current and strengthens the plate. Lead-antimony cores form the grids that comprise the conducting network between the hard-rubber tubes of the positive plate.

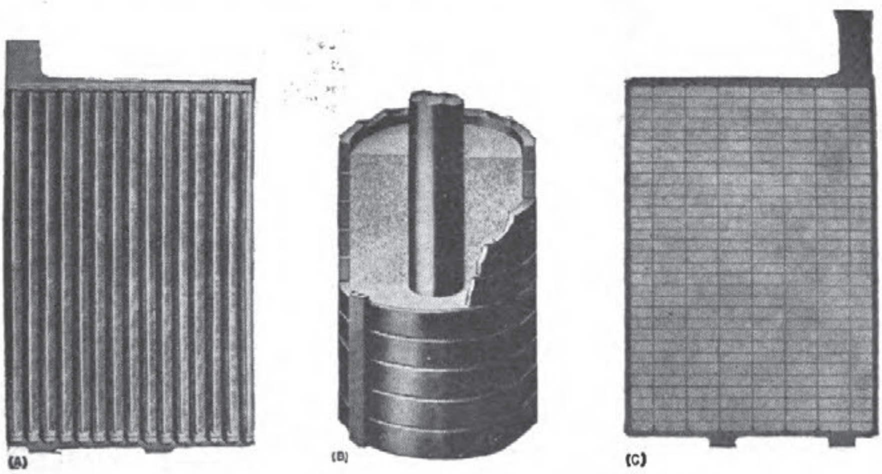


Figure 2–8.—Ironclad plate construction.

The hard-rubber tubes contain narrow horizontal slots to permit the passage of electrolyte through them so that it may come into contact with the active material inside the tubes (fig. 2–8, B).

The slotted tubes prevent the washing away, or shredding, of the active material. Each tube has two full-length ribs—one on each side—which keep the separators (spacers) from covering the slots and preventing the electrolyte from circulating freely. The ironclad negative plates are essentially the same as the negative pasted plates (fig. 2–8, C).

GOULD PLATE.—The GOULD PLATE is a pasted plate that differs very little from the ordinary positive and negative pasted plates. A glass-wool mat directly over the positive plate holds the active

material in place. Rubber or wood separators are used as in other batteries. The glass-wool mat prevents shedding and makes the Gould battery almost as durable as the ironclad battery. Hence the Gould battery is used extensively in both submarines and surface vessels.

Cell Element

The plates are formed into positive and negative GROUPS. When these groups are assembled they become a CELL ELEMENT (fig. 2–9). The number of negative plates is always one more

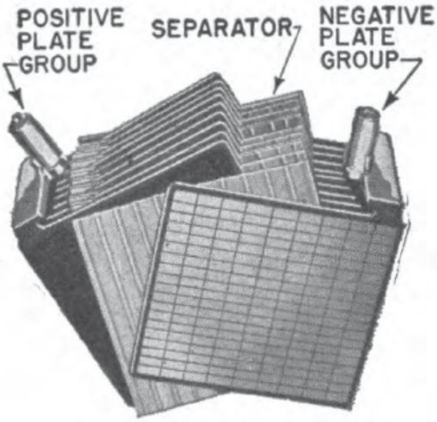


Figure 2–9.—Partly assembled element.

than the number of positive plates so that both sides of each positive plate are acted upon chemically. The active material on the positive plates expands and contracts as the battery is charged and discharged. The expansion and contraction must be kept the same on both sides of the plates to prevent buckling.

Separators of wood, rubber, or glass are placed between the positive and negative plates to act as insulators (fig. 2–9). These separators are grooved vertically on one side and are smooth on the other. The grooved side is placed next to the positive plate to permit free circulation of the electrolyte around the active material.

An assembled lead-acid cell with the positive and negative terminals projecting through the cell cover is shown in figure 2–10. A hole fitted with a filler cap is provided in each cell cover to permit filling and testing. The filler cap has a vent hole to allow the gas that forms in the cell during charge to escape.

The ordinary 6-volt portable storage battery consists of 3 cells assembled in a molded hard-rubber (monobloc) case. Metal cannot be used because of the acid electrolyte. Each cell is contained in an acid-proof compartment within the case. The cells are connected in series by means of lead-alloy connectors that are

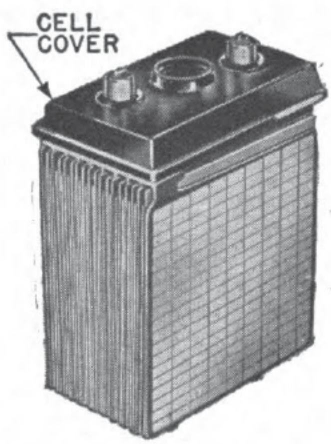


Figure 2–10.—Assembled lead-acid cell.

attached to the terminal posts of adjacent cells by a lead-burning process. The space between the case and the edges of the cell covers is filled with an acid-proof battery-sealing compound, or pitch. This compound is a blend of bituminous materials that are processed so that they remain solid at high temperatures and do not crack at low temperatures.

The cells of a battery are filled with ELECTROLYTE, which is a mixture of concentrated sulfuric acid and distilled water. The electrolyte conducts the electric current between the positive and negative plates inside the battery and reacts chemically with the active material of the plates. The solution of sulfuric acid and distilled water is shipped and stored in large glass bottles called CARBOYS.

Edison Cell

The Edison cell has positive electrodes of nickel-oxide flakes and negative electrodes of powdered iron. The electrolyte is a dilute solution of potassium hydroxide. This cell has a higher kilowatt-hour capacity per pound than the lead-acid cell and can stand indefinitely either in a charged or discharged condition

without damage. The Edison cell, however, gasses on discharge and the voltage per cell, which is 1.2 volts under load, is only about half that of the lead-acid cell. The Edison cell has a high internal resistance, which increases when the cell is cold. Because of these disadvantages the Edison cell is not used in the Navy.

Nickel-Cadmium Cell

The nickel-cadmium cell is a recently developed storage cell for use in aircraft. While it is essentially an alkaline cell, it has overcome most of the disadvantages of the Edison storage battery. The nickel-cadmium alkaline battery is made up of 20 cells in series to supply 24 volts and is encased in a stainless-steel container. The electrolyte is in the form of a paste rather than a liquid and the cells are hermetically sealed. After the cells are sealed they do not require any maintenance care, such as adding water or testing the specific gravity of the electrolyte. This battery will deliver the same load current as the lead-acid battery of the same specifications, and it will supply this current for twice the length of time—an important factor in starting aircraft engines.

Specific Gravity

The ratio of the weight of a certain volume of liquid to the weight of the same volume of water is called the SPECIFIC GRAVITY of the liquid. The specific gravity of pure water is 1.000. Sulfuric acid has a specific gravity of 1.835; thus sulfuric acid is 1.835 times as heavy as water. The specific gravity of a mixture of sulfuric acid and water varies with the strength of the solution from 1.000 to 1.835.

As a storage battery discharges, the sulfuric acid is depleted and the electrolyte is gradually converted into water. As mentioned previously in this chapter, this action provides a guide in determining the state of discharge of the lead-acid cell. The electrolyte that is usually placed in a lead-acid battery has a specific gravity of 1.350 or less. Generally, the specific gravity of the electrolyte in Navy portable batteries is adjusted between 1.210 and 1.220. On the other hand the specific gravity of the electrolyte in submarine batteries when charged is from 1.250 to 1.265. A change of 0.001 is called a change of 1 point (part) in 1,000.

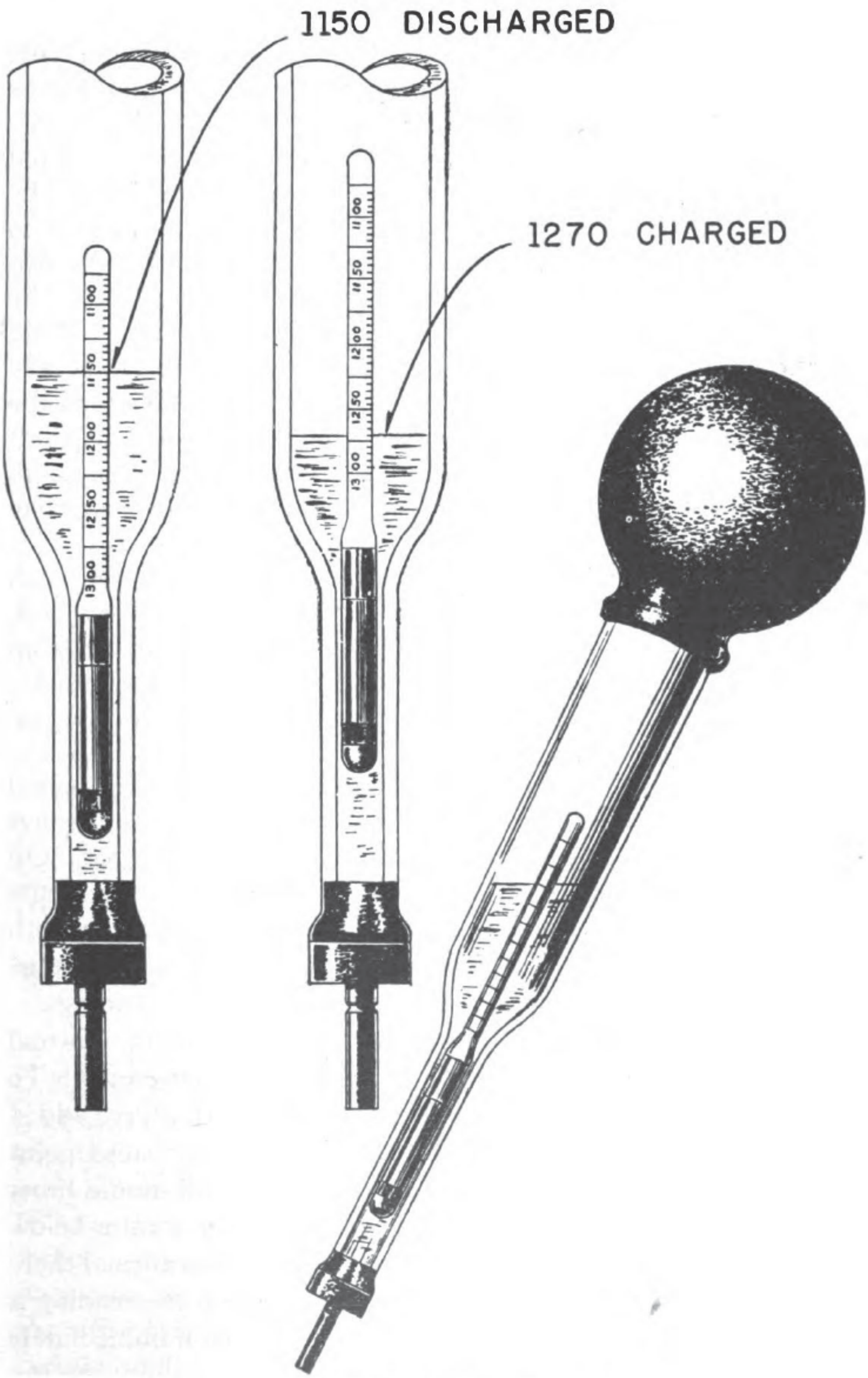


Figure 2-11.—Type-B hydrometer.

HYDROMETER.—The specific gravity of the electrolyte is measured with a hydrometer. In the syringe-type hydrometer (fig. 2–11), a part of the battery electrolyte is drawn up into a glass tube by means of a rubber bulb at the top.

The hydrometer float consists of a hollow glass tube weighted at one end and sealed at both ends. A scale calibrated in specific gravity is laid off axially along the body (stem) of the tube. The hydrometer float is placed inside the glass syringe and the electrolyte to be tested is drawn up into the syringe, thus immersing the hydrometer float into the solution. When the syringe is held approximately in a vertical position, the hydrometer float will sink to a certain level in the electrolyte. The extent to which the hydrometer stem protrudes above the level of the liquid depends upon the specific gravity of the solution. The reading on the stem at the surface of the liquid is the specific gravity of the electrolyte in the syringe.

The Navy uses two types of hydrometer bulbs, or floats, each having a different scale. The type-A hydrometer is used with submarine batteries and has two different floats with scales from 1.060 to 1.240 and from 1.120 to 1.300. The type-B hydrometer is used with portable and radio storage batteries and has a scale of from 1.100 to 1.300.

CORRECTIONS.—The specific gravity of the electrolyte is affected by its temperature. The electrolyte expands and becomes less dense when heated and its specific gravity reading is lowered. On the other hand, the electrolyte contracts and becomes more dense when cooled and its specific gravity reading is raised. In both cases the electrolyte may be from the same fully charged storage cell. Thus, the effect of temperature is to distort the readings.

All Navy standard storage batteries use 80° F. as the normal temperature to which specific gravity readings are corrected. To correct the specific gravity reading of a storage battery, add 1 point to the reading for each 3° F. above 80° and subtract 1 point for each 3° F. below 80°. The electrolyte in a cell should be at the normal level when the reading is taken. If the level is below normal the reading is too high. If the level is above normal there is too much water, the electrolyte is weakened, and the reading is too low. A hydrometer reading is inaccurate if taken immediately after water is added because the water tends to remain at the top of the cell. When water is added, the battery should be charged

for at least an hour to mix the electrolyte before a hydrometer reading is taken.

ADJUSTING SPECIFIC GRAVITY.—Only authorized personnel on a repair ship or at a shore station should add acid to a battery. Acid with a specific gravity above 1.350 is never added to a battery. If the specific gravity of a cell is more than 1.220, it can be reduced to within the 1.210-to-1.220 limits by removing some of the electrolyte and adding distilled water. The battery is charged for 1 hour to mix the solution, and then hydrometer readings are taken. The adjustment is continued until the desired true readings are obtained.

Mixing Electrolytes

The electrolyte of a fully charged battery usually contains about 38 percent sulfuric acid by weight, or about 27 percent by volume. In preparing the electrolyte only pure distilled water is used with sulfuric acid that meets Navy specifications. New batteries may be delivered with containers of concentrated sulfuric acid of 1.835 specific gravity or electrolyte of 1.400 specific gravity, both of which must be diluted with distilled water to make electrolyte of the proper specific gravity. The container used for diluting the acid should be made of glass, earthenware, rubber, or lead.

When mixing electrolyte, ALWAYS POUR ACID INTO WATER—never pour water into acid. Pour the acid slowly and cautiously to prevent excessive heating and splashing. Stir the solution continuously with a nonmetallic rod to mix the heavier acid with the lighter water and to keep the acid from sinking to the bottom. When concentrated acid is diluted, the solution becomes very hot.

Treatment of Acid Burns

If acid or electrolyte from a lead-acid battery comes into contact with the skin, the affected area should be washed as soon as possible with large quantities of fresh water, after which a salve such as vasoline, boric acid, or zinc ointment should be applied. If none of these salves are available, clean lubricating oil will suffice. When washing, large amounts of water should be used, since a small amount of water might do more harm than good in spreading the acid burn.

Acid spilled on clothing may be neutralized with dilute ammonia or a solution of baking soda and water.

Capacity

The capacity of a battery is measured in ampere-hours. As mentioned before, the ampere-hour capacity is equal to the product of the current in amperes and the time in hours during which the battery is supplying this current. The ampere-hour capacity varies inversely with the discharge current. The size of a cell is determined generally by its ampere-hour capacity. The capacity of a cell depends upon many factors, the most important of which are (1) the area of the plates in contact with the electrolyte, (2) the quantity and specific gravity of the electrolyte, (3) the type of separators, (4) the general condition of the battery (degree of sulfating, plates buckled, separators warped, sediment in bottom of cells, and so forth), and (5) the final limiting voltage.

Rating

Navy storage batteries are rated according to their rate of discharge and ampere-hour capacity. All batteries, except those used for radio and sound systems, are rated according to a 10-hour rate of discharge—that is, if a fully charged battery is completely discharged during a 10-hour period, it is discharged at the 10-hour rate. Thus if a battery can deliver 40 amperes continuously for 10 hours, the battery has a rating of 40 x10, or 400 ampere-hours. Thus the 10-hour rating is equal to the average current that a battery is capable of supplying without interruption for an interval of 10 hours.

All Navy standard batteries deliver 100 percent of their available capacity if discharged in 10 hours or more, but they will deliver less than their available capacity if discharged at a faster rate. The faster they discharge, the less ampere-hour capacity they have.

The low-voltage limit, as specified by the manufacturer, is the limit beyond which very little useful energy can be obtained from a battery. Generally, the low-voltage limit for a battery at different rates of discharge varies only slightly with the size and the make of the battery.

At the conclusion of a 10-hour discharge rate the closed-circuit voltmeter reading is about 1.75 volts per cell and the specific gravity of the electrolyte is about 1.060. At the end of a charge, the closed-circuit voltmeter reading, while the battery is being

charged at the finishing rate, is between 2.4 and 2.6 volts per cell. The specific gravity of the electrolyte corrected to 80° F. is between 1.210 and 1.220. In climates of 40° F. and below, authority may be granted to increase the specific gravity to 1.280.

Test Discharge

The test discharge is the best method of determining the capacity of a battery. Most battery switchboards are provided with the necessary equipment for giving test discharges. If proper equipment is not available, a tender, repair ship, or shore station may make the test. To determine the battery capacity, a battery is normally given a test discharge once every 6 months. Test discharges are also given (1) whenever any cell of a battery after charge cannot be brought within 10 points of full charge, or (2) when one or more cells is found to have less than normal voltage after an equalizing charge.

A test discharge must always be preceded by an equalizing charge. Immediately after the equalizing charge, the battery is discharged at its 10-hour rate until either (1) the total battery voltage drops to a value equal to 1.75 times the number of cells in series, or (2) the voltage of any individual cell drops to 1.65 volts, whichever occurs first. The rate of discharge should be kept constant throughout the test discharge. Because Navy standard batteries are rated at the 10-hour capacity, the discharge rate for a 175 ampere-hour battery is 175÷10, or 17.5 amperes. If the temperature of the electrolyte at the beginning of the charge is not exactly 80° F., the time duration of the discharge must be corrected for the actual temperature of the battery. The table for this correction is contained in article 62–264 of the *Bureau of Ships Manual.*

A battery of 100-percent capacity discharges at its 10-hour rate for 10 hours before reaching its low-voltage limit. If the battery or one of its cells reaches the low-voltage limit before the 10-hour period has elapsed, the discharge is discontinued immediately and the percentage of capacity is determined from the equation

$$C=\frac{H_a}{H_t}\times 100, \tag{2-1}$$

where C is the percentage of ampere-hour capacity available, H_a the total hours of discharge, and H_t the total hours for 100-

percent capacity. The date for each test discharge should be recorded on the storage battery record sheet.

For example, a 150-ampere-hour 6-volt battery delivers an average current of 15 amperes for 10 hours. At the end of this period the battery voltage is 5.25 volts. On a later test the same battery delivers an average current of 15 amperes for only 7 hours. The discharge was stopped at the end of this time because the voltage of the middle cell was found to be only 1.65 volts. The percentage of capacity of the battery is now $\frac{7}{10} \times 100$, or 70 percent. Thus, the ampere-hour capacity of this battery is reduced to $0.7 \times 150 = 105$ ampere hours.

State of Charge

After a battery is discharged completely from full charge at the 10-hour rate, the specific gravity drops about 150 points to about 1.060. The number of points that the specific gravity drops per ampere-hour can be determined for each type of battery. For each ampere-hour taken out of a battery a definite amount of acid is removed from the electrolyte and combined with the plates.

For example, if a battery is discharged from full charge to the low-voltage limit at the 10-hour rate and if 100 ampere-hours are obtained with a specific gravity drop of 150 points, there is a drop of $\frac{150}{100}$, or 1.5 points per ampere-hour delivered. If the reduction in specific gravity per ampere-hour is known, the drop in specific gravity for this battery may be predicted for any number of ampere-hours delivered to a load. For example, if 70 ampere-hours are delivered by the battery at the 10-hour rate or any other rate or collection of rates, the drop in specific gravity is 70×1.5, or 105 points.

Conversely, if the drop in specific gravity per ampere-hour and the total drop in specific gravity are known, the ampere-hours delivered by a battery may be determined. For example, if the specific gravity of the previously considered battery is 1.210 when the battery is fully charged and 1.150 when it is partly discharged, the drop in specific gravity is $1{,}210 - 1{,}150$, or 60 points, and the number of ampere-hours taken out of the battery is $\frac{60}{1.5}$, or 40

ampere-hours. Thus, the number of ampere-hours expended in any battery discharge can be determined from the following three items: (1) The specific gravity when the battery is fully charged; (2) the specific gravity after the battery has been discharged; and (3) the reduction in specific gravity per ampere-hour.

Voltage alone is not a reliable indication of the state of charge of a battery except when the voltage is near the low-voltage limit on discharge. During discharge the voltage falls. The higher the rate of discharge the lower will be the terminal voltage. Open-circuit voltage is of little value because the variation between full charge and complete discharge is so small—only about 0.1 volt per cell. However, abnormally low voltage does indicate injurious sulfation or some other serious deterioration of the plates.

Types of Charges

A battery should not be recharged every time the specific gravity drops a few points, because excessive gases are produced if the battery is overcharged and this action may cause premature loosening of the active material on the plates. When the specific gravity of a battery on intermittent or standby service drops to 1.180 from a fully charged reading of from 1.210 to 1.220, it should normally be placed on charge. A fully discharged battery having a specific-gravity reading of 1.060 should be immediately and completely recharged.

The following types of charges may be given to a storage battery, depending upon the condition of the battery: (1) Initial charge, (2) normal charge, (3) equalizing charge, (4) floating charge, and (5) emergency charge.

INITIAL CHARGE.—When a new battery is shipped dry, the plates are in an uncharged condition. After the electrolyte has been added, it is necessary to convert the plates into the charged condition. This is accomplished by giving the battery a long low-rate INITIAL CHARGE. The charge is given in accordance with the manufacturer's instructions, which are shipped with each battery. If the manufacturer's instructions are not available, reference should be made to the detailed instruction in articles 62–221 through 62–223 of the *Bureau of Ships Manual*.

NORMAL CHARGE.—A normal charge is a routine charge that is given in accordance with the nameplate data during the ordi-

nary cycle of operation to restore the battery to its charged condition. The following steps should be observed:

1. Determine the starting and finishing rate from the nameplate data. If this information is not available, use the charging rate specified in article 62–244 of the *Bureau of Ships Manual.*
2. Add water, as necessary, to each cell.
3. Connect the battery to the charging panel and make sure the connections are clean and tight.
4. Turn on the charging circuit and set the current through the battery at the value given as the starting rate.
5. Check the temperature and specific gravity of pilot cells hourly.
6. When the battery begins to gas freely, reduce the charging current to the finishing rate.

A normal charge is complete when the specific gravity of the pilot cell, corrected for temperature, is within 5 points (0.005) of the specific gravity obtained on the previous equalizing charge.

EQUALIZING CHARGE.—An equalizing charge is an extended normal charge at the finishing rate. It is given periodically to ensure that all the sulfate is driven from the plates and that all the cells are restored to a maximum specific gravity. The equalizing charge is continued until the specific gravity of all cells, corrected for temperature, shows no change for a 4-hour period. Readings of all cells are taken every half hour.

FLOATING CHARGE.—A battery may be maintained at full charge by connecting it across a charging source that has a voltage maintained within the limits of from 2.13 to 2.17 volts per cell of the battery. In a floating charge the charging rate is determined by the battery voltage rather than by a definite current value. The voltage is maintained between 2.13 and 2.17 volts per cell with an average as close to 2.15 volts as possible.

EMERGENCY CHARGE.—An emergency charge is used when a battery must be recharged in the shortest possible time. The charge starts at a much higher rate than is normally used for charging. It is seldom used because spare batteries are provided for all battery-operated equipment and it may be harmful to the battery.

Charging Rate

The charging rate of every Navy storage battery is given on the battery nameplate. If the available charging equipment does not have the desired charging rates, the nearest available rates should be used. However, the rate should never be so high that violent gassing occurs. NEVER ALLOW THE TEMPERATURE OF THE ELECTROLYTE IN ANY CELL TO RISE ABOVE 125° F.

Charging Time

The charge must be continued until the battery is fully charged. Frequent readings of specific gravity should be taken during the charge. These readings should be corrected to 80° F. and compared with the reading taken before the battery was placed on charge. If the rise in specific gravity in points per ampere-hour is known, the approximate time in hours required to complete the charge is

$$\frac{\text{rise in specific gravity in points to complete charge}}{\dfrac{\text{rise in specific gravity}}{\text{in points per ampere-hour}} \times \text{charging rate in amperes.}}$$

Gassing

When a battery is being charged, a portion of the energy is dissipated in the electrolysis of the water in the electrolyte. Thus, hydrogen is released at the negative plates and oxygen at the positive plates. These gases bubble up through the electrolyte and collect in the air space at the top of the cell. If violent gassing occurs when the battery is first placed on charge, the charging rate is too high. If the rate is not too high, steady gassing, which develops as the charging proceeds, indicates that the battery is nearing a fully charged condition. A mixture of hydrogen and air can be dangerously explosive. No smoking, electric sparks, or open flames should be permitted near charging batteries.

QUIZ

1. How does a secondary cell differ from a primary cell?
2. The internal resistance of a cell depends upon what three factors?

3. Identify the following as mixtures or compounds:
   (a) H_2SO_4 and H_2O.
   (b) $ZnSO_4$.
   (c) H_2O.
4. Negative ions move toward which electrode (carbon or zinc) in a discharging primary cell?
5. What is polarization?
6. What are the effects of polarization on the effective internal resistance, the terminal voltage, and the output current?
7. What is the function of manganese dioxide as used in dry cells?
8. What is local action?
9. How can local action be prevented in a zinc electrode?
10. What electrolyte is used in the LeClanche dry cell?
11. In a dry cell identify the materials of the positive and negative electrodes.
12. If 10 dry cells are connected like the cells in figure 2–4, A, what is the approximate terminal voltage of the group?
13. What are series or parallel groupings of cells called?
14. If 10 dry cells are connected like the cells in figure 2–4, B and each cell is capable of delivering a maximum current of 35 amperes, what is the maximum output current of the group?
15. A cell in which the elements are kept dry until the time of use is called a ______________________.
16. Name the two types of storage cells in general use today.
17. In a lead-acid cell, what are the active materials of the positive and negative plates?
18. When a lead-acid battery discharges, what happens to the specific gravity of the electrolyte?
19. If sulfuric acid is added to a cell that is discharged, will the cell be recharged?
20. What is the principal naval application of the iron-clad plate storage battery?
21. What is the purpose of the vent hole in the storage cell filler cap?
22. The mixture of sulfuric acid and water comprising the solution of the lead-acid cell is called the ______________________.
23. The voltage of an Edison cell is greater or less than a lead-acid cell?
24. The nickel-cadmium battery will deliver the same load current as a lead-acid battery of the same specifications for what relative length of time?

25. If a specific gravity reading is taken above 80° F. (the normal temperature) should temperature corrections (specific gravity points) be added or subtracted from the observed reading?
26. Why should a battery be placed on charge for an hour after water is added before testing the specific gravity?
27. How should electrolyte be mixed?
28. The ampere-hour rating of a battery is a measure of its ________.
29. All Navy standard batteries are rated according to a standard ____________ hour rate of discharge, and their ____________ capacity.
30. A battery that has been subjected to an equalizing charge is discharged at a constant rate for 10 hours. If the battery is a 185 ampere-hour battery, what is the discharge rate?
31. A standard 300 ampere-hour 6-volt battery discharges at an average rate of 30 amperes for 10 hours. At the end of this time the battery voltage has fallen to 5.25 volts. On a later test the battery delivers an average current of 30 amperes for only 8 hours, the discharge being stopped due to low cell voltage. What is the percentage of capacity of the battery?
32. A 3-cell battery is discharged from full charge to the low-voltage limit of 5.25 volts at the 10-hour rate. 150 ampere hours are obtained along with a specific gravity drop of 125 points. If 60 ampere hours are delivered by the battery at the 10-hour rate, what is the drop in specific gravity?
33. Initially a battery delivers 150 ampere hours with a drop in specific gravity of 150 points. On a later test the battery delivers 100 ampere hours. If the specific gravity is 1.280 at the beginning of the test, estimate the specific gravity at the end of the test.
34. What is the nature and purpose of the initial charge given to a battery?
35. What is the purpose and nature of NORMAL charge?
36. An equalizing charge is given periodically to ensure that ________ ____________________.
37. The floating charge rate is determined by the battery ________ ____________________.
38. The charging rate of all Navy batteries is given on the ________ of the battery.
39. Violent gassing of a charging battery when first placed on charge indicates ____________________.

CHAPTER 3

THE SIMPLE ELECTRIC CIRCUIT

INTRODUCTION

Electrical energy can be transmitted through conducting circuits efficiently and with relatively simple supervisory control. Today's vast electrical networks supplying factories and homes as well as cities and farms with electric power in ever-increasing amounts are convincing evidence of the superiority of this means of transmission.

The transmission of electrical energy always takes place through a closed electric circuit; and in its simplest form this circuit consists of a two-terminal generating source, a two-terminal load, and connecting wires, as shown in figure 3–1. The study of even

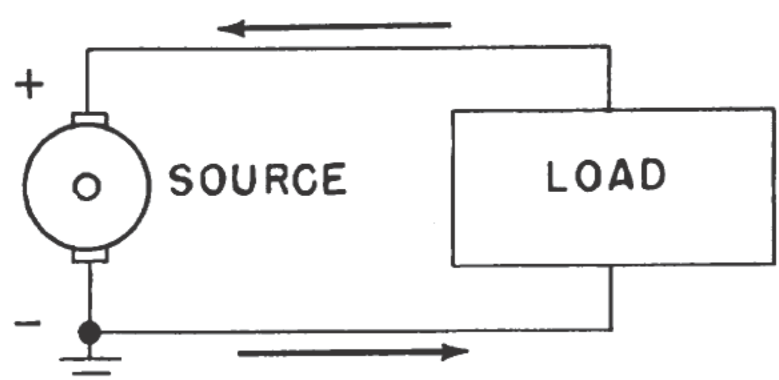


Figure 3–1.—Simple electric circuit.

the simplest electric circuit immediately involves practical electrical units such as the volt (E), ampere (I), ohm (R), and watt (W). It is the purpose of this chapter to introduce the relationship between these units, especially the relation between E, I, and R, widely known as Ohm's law, and to describe the closely associated units of power and energy. Rules for solving problems relating to the simple two-terminal load device are described and examples are given.

OHM'S LAW

The 19th century, German philosopher, Georg Simon Ohm, proved by experiment the constant proportionality between electric current and voltage in the simple electric circuit and published the results of his findings in 1826. Ohm's law is fundamentally linear and therefore simple. It is exact and applies to d-c circuits and devices in its basic form. In a modified form it may also be applied to a-c circuits. The unit of electrical resistance is called the OHM (designated by the Greek letter omega—Ω) and finds wide application in the sources, loads, and conductors of electrical and electronics circuits.

Meaning of Ohm's Law

Ohm's law may be stated in words, as follows: THE INTENSITY OF THE CURRENT IN AMPERES IN ANY ELECTRIC CIRCUIT IS EQUAL TO THE DIFFERENCE IN POTENTIAL IN VOLTS ACROSS THE CIRCUIT DIVIDED BY THE RESISTANCE IN OHMS, OF THE CIRCUIT.

Expressed as an equation, the law becomes

$$I=\frac{E}{R}, \tag{3–1}$$

where I is the intensity of the current in amperes, E the difference in potential in volts, and R the resistance in ohms.

If any two of these quantities are known, the third may be found by applying equation (3–1).

For example, if the voltage across the load in figure 3–1 is 120 volts and the effective resistance of the load is 20 ohms, the current through the load will be $\frac{120}{20}$, or 6 amperes. If the effective resistance of the load remains constant at 20 ohms, in accordance with Ohm's law, the current will double if the voltage doubles, or halve if the voltage halves. In other words the CURRENT THROUGH THE LOAD WILL VARY DIRECTLY WITH THE VOLTAGE ACROSS THE LOAD. Thus if the voltage across the load is reduced to zero, the current will equal $\frac{0}{20}$, or 0 amperes. If the voltage across the load is increased in steps of 10 volts starting at zero and continuing to 120 volts, the current through the load becomes: $\frac{10}{20}=0.5$ ampere; $\frac{20}{20}=1.0$ ampere; $\frac{30}{20}=1.5$ ampere; and so forth.

The relation between current and voltage in this example is shown in figure 3–2 as a graph in which the voltage is plotted horizontally along the X axis to the right of the origin and the corresponding values of current are plotted vertically along the Y axis above the origin. The graph is a straight line, the equation of which is $I=\frac{E}{20}$. The constant, 20, represents the resistance in ohms of the circuit and is assumed not to change with the current in this example. This graph illustrates an important characteristic of the basic law—namely, the CURRENT VARIES DIRECTLY WITH THE APPLIED VOLTAGE IF THE RESISTANCE IS CONSTANT.

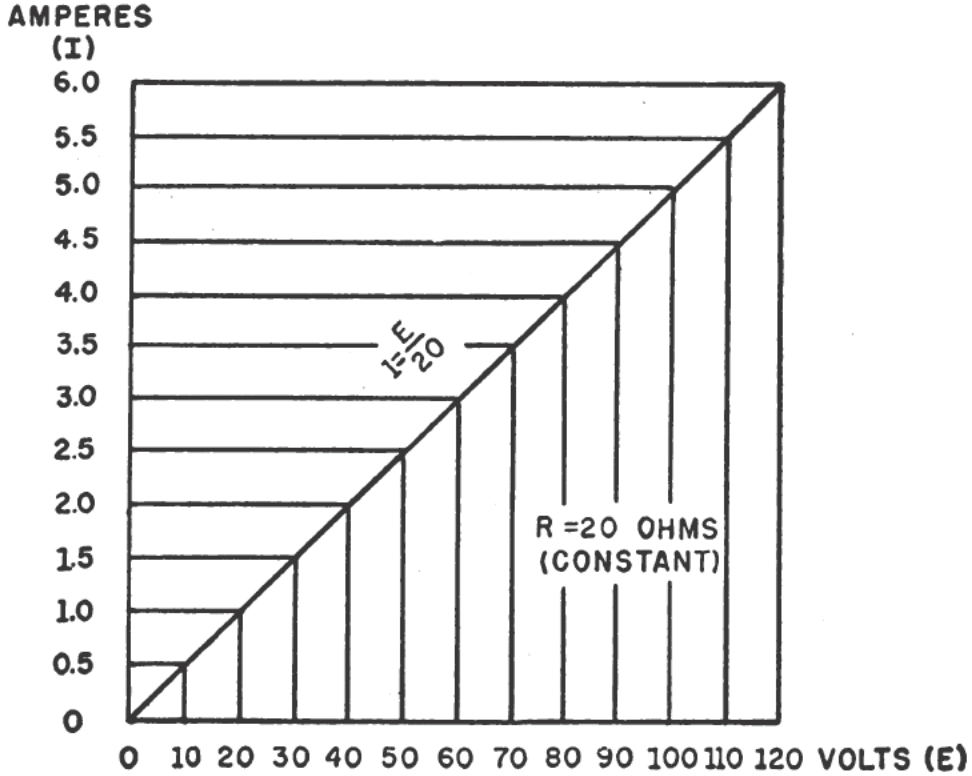


Figure 3–2.—Graph of voltage vs. current in a constant-resistance circuit.

If the voltage across the load in figure 3–1 is maintained at a constant value of 120 volts, the current through the load will depend solely upon the effective resistance of the load. For example, if the resistance is 120 ohms, the current will be $\frac{120}{120}$, or 1 ampere. If the resistance is halved, the current will be doubled; if the resistance is doubled, the current will be halved. In other words, the current will vary inversely with the resistance.

If the resistance of the load is reduced in steps of 20 ohms starting at 120 ohms and continuing to 20 ohms, the current through the load becomes:

$$\frac{120}{100}=1.2 \text{ amperes}; \frac{120}{80}=1.5 \text{ amperes}; \frac{120}{60}=2 \text{ amperes};$$

and so forth. The relation between current and resistance in this example is expressed as a graph (fig. 3–3), the equation of which is $I=\frac{120}{R}$. The numerator of the fraction represents a constant value of 120 volts in this example. As R approaches a small value the current approaches a very large value. The example illustrates a second equally important relation in Ohm's law—namely, that the CURRENT VARIES INVERSELY WITH THE RESISTANCE.

If the current through the load in figure 3–1 is maintained constant at 5 amperes the voltage across the load will depend upon the resistance of the load and will vary directly with it. The relation between voltage and resistance is shown in the graph of figure 3–4. Values of resistance are plotted, horizontally along the X axis to the right of the origin, and corresponding values of voltage are plotted vertically along the Y axis above

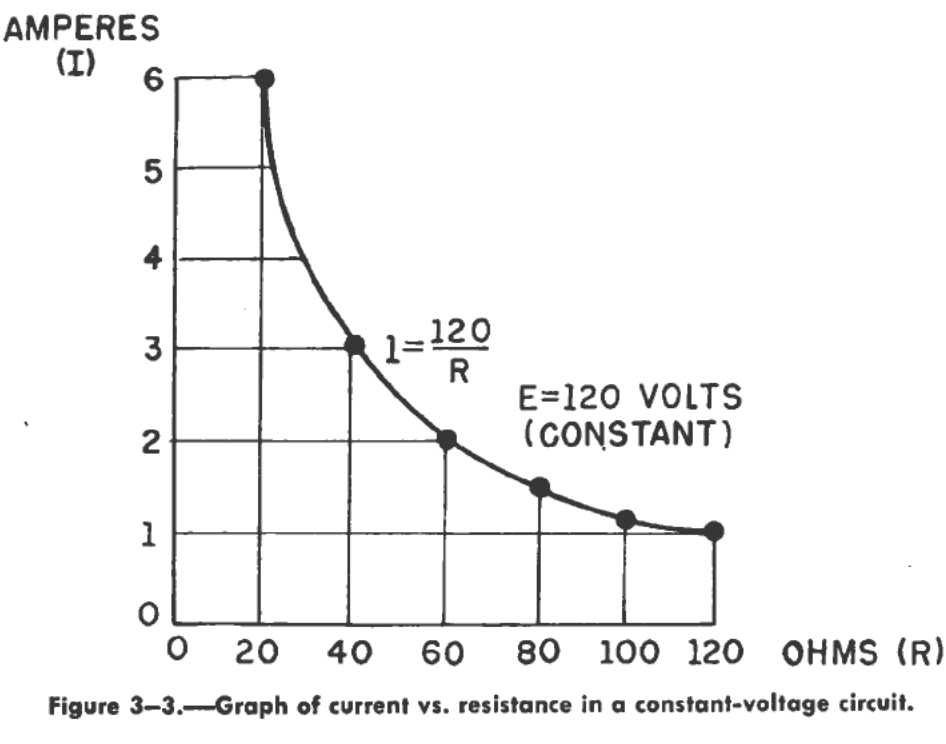


Figure 3–3.—Graph of current vs. resistance in a constant-voltage circuit.

the origin. The graph is a straight line having the equation $E=5R$. The coefficient 5 represents the assumed current of 5 amperes which is constant in this example. Thus a third important relation is illustrated—namely, that the VOLTAGE across

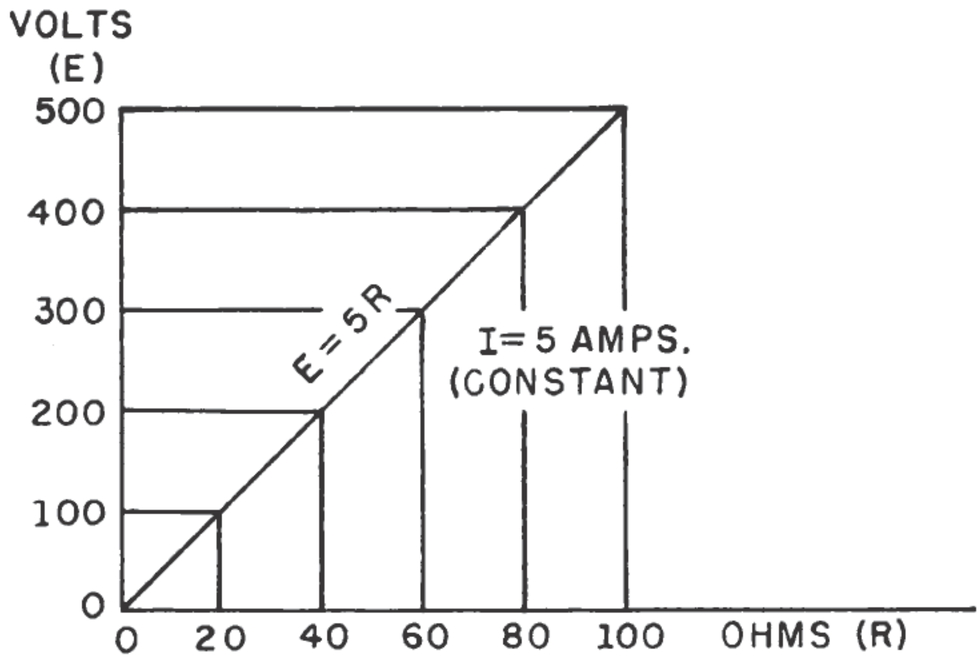


Figure 3–4.—Graph of voltage vs. resistance with constant current.

a device VARIES DIRECTLY WITH THE EFFECTIVE RESISTANCE of the device provided the current through the device is maintained constant.

Applying Ohm's Law

Equation (3–1) may be transposed to solve for the resistance if the current and voltage are known, or to solve for the voltage if the current and resistance are known. Thus $R=\frac{E}{I}$ and $E=IR$. For example, if the voltage across a device is 50 volts and the current through it is 2 amperes, the resistance of the device will be $\frac{50}{2}$, or 25 ohms. Also, if the current through a wire is 3 amperes and the resistance of the wire is 0.5 ohm, the voltage drop across the wire will be 3×0.5, or 1.5 volts.

Equation (3–1) and its transpositions may be obtained readily with the aid of figure 3–5. The circle containing *E, I,* and *R*

is divided into two parts with E above the line and IR below it. To determine the unknown quantity, first cover that quantity with a finger. The location of the remaining uncovered letters in the circle will indicate the mathematical operation to be performed. For example to find I, cover I with a finger. The uncovered letters indicate that E is to be divided by R, or $I=\frac{E}{R}$. To find E, cover E. The result indicates that I is to be multiplied by R, or $E=IR$. To find R, cover R. The result indicates that E is to be divided by I, or $R=\frac{E}{I}$.

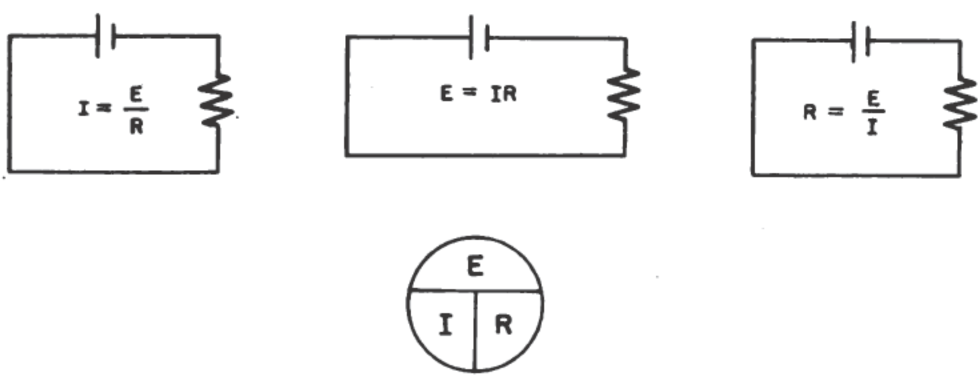


Figure 3–5.—Ohm's law in diagram form.

The beginning student is cautioned not to rely wholly on the use of this diagram when transposing simple formulas but rather to use it to SUPPLEMENT his knowledge of the algebraic method. Algebra is a basic tool in the solution of electrical problems and the importance of knowing how to use it should not be underemphasized or bypassed after the student has learned a shortcut method such as the one indicated in this figure.

POWER AND ENERGY

In addition to the volt, ampere, and ohm, there are two units that frequently appear in electric circuit calculations. These are the unit of power and the unit of energy.

Power

Power is the rate of doing work. In a d-c electric circuit, power is equal to the product of the voltage and the current.

Expressing the power in watts (P), the current in amperes (I), and the emf in volts (E), the equation is

$$P = EI. \tag{3-2}$$

Thus the power delivered to a circuit varies directly with the applied voltage and the circuit current. If the value of current from equation (3–1) is substituted in equation (3–2), the equation for power may be expressed in terms of the circuit voltage and circuit resistance as,

$$\begin{aligned} P &= E\frac{E}{R} \\ &= \frac{E^2}{R}. \end{aligned} \tag{3-3}$$

From equation (3–3) it may be seen that the POWER IN WATTS DELIVERED TO A CIRCUIT VARIES DIRECTLY AS THE SQUARE OF THE APPLIED EMF IN VOLTS AND INVERSELY WITH THE CIRCUIT RESISTANCE IN OHMS.

A graph of equation (3–3) is shown in figure 3–6. Voltage is plotted along the X axis to the right of the origin and corresponding values of the power in watts are plotted vertically along the Y axis above the origin. The equation $P=\frac{E^2}{20}$ represents the relation between applied voltage and power in a simple electric circuit like that of figure 3–1, in which the circuit has a constant resistance of 20 ohms. The voltage is increased in steps of 20 volts beginning at 0 and ending at 100 volts. The corresponding values of power are calculated by substitution in equation (3–3) as:
$\frac{(0)^2}{20}=0$ watts; $\frac{(20)^2}{20}=20$ watts; $\frac{(40)^2}{20}=80$ watts; and so forth. The graph emphasizes the fact that the power in watts delivered to the circuit varies as the square of the applied voltage.

Equation (3–2) may be modified to express power in terms of current and resistance by substituting the value of E in terms of I and R into equation (3–2) from the relation $E=IR$ as follows:

$$\begin{aligned} P &= IRI \\ &= I^2R. \end{aligned} \tag{3-4}$$

From equation (3–4) it may be seen that the power in watts in a

circuit varies as the square of the circuit current in amperes and as the first power of the circuit resistance in ohms.

The graph of figure 3–6 is also the graph of the equation $P=20I^2$. In this case the circuit current is plotted horizontally along the X axis to the right of the origin beginning at 0 amperes, increasing in steps of 1 ampere, and continuing up to 5 amperes.

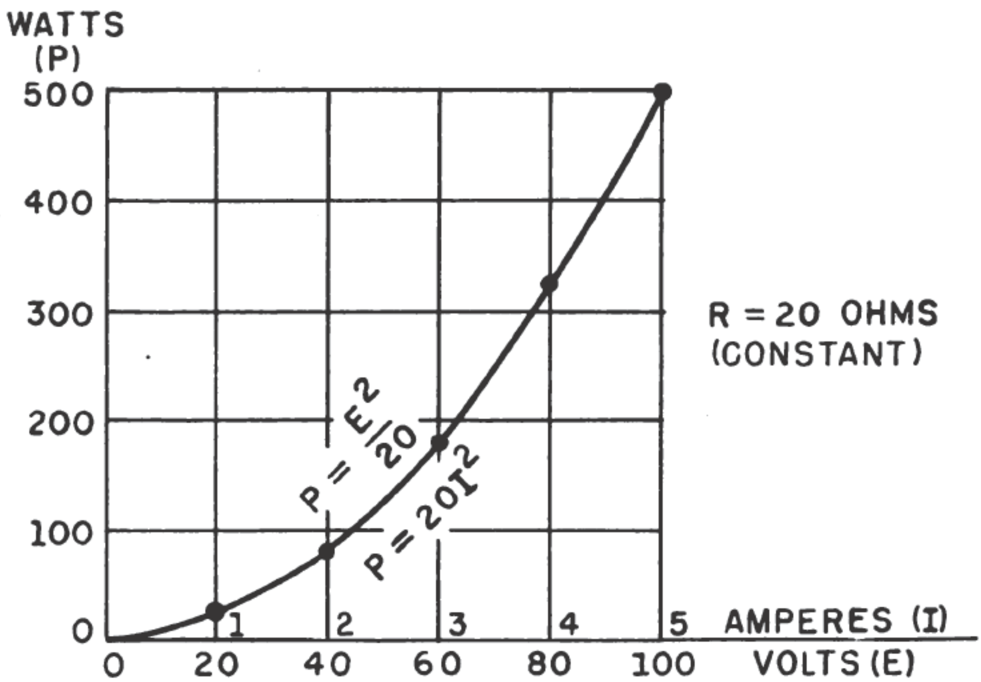


Figure 3–6.—Graph of voltage vs. power in a constant-resistance circuit.

Corresponding values of power are plotted along the Y axis above the origin. These values are found by substitution in equation (3–4) as: $(1)^2\times20=20$ watts; $(2)^2\times20=80$ watts; $(3)^2\times20=$ 180 watts; and so forth.

It is important to note that the graphs of equations (3–3) and (3–4) are not linear. The power varies either (1) as the square of the applied voltage or (2) as the square of the circuit current.

It is also important to note in the study of equation (3–3) that the power delivered to a circuit having a constant voltage varies inversely with the circuit resistance. Thus if the voltage supplied to a simple circuit like that of figure 3–1 has a constant value of 100 volts and the circuit resistance is decreased from 20 ohms to 10 ohms, the power delivered to the circuit is increased from

$\frac{(100)^2}{20}=500$ watts to $\frac{(100)^2}{10}=1{,}000$ watts. Thus if the circuit resistance is halved, the power is doubled.

Note that in this example the relationship is not linear. A graph of the equation $P=\frac{(100)^2}{R}$ is shown in figure 3–7 and represents the relation between power and resistance in a circuit having a constant applied voltage of 100 volts. Values of resistance are plotted along the X axis to the right of the origin and corresponding values of power are plotted along the Y axis above the origin. The resistance is varied in steps of 100 ohms starting at 100 ohms and ending at 1,000 ohms. Corresponding values of power are calculated as follows: $\frac{(100)^2}{100}=100$ watts; $\frac{(100)^2}{200}=50$ watts; $\frac{(100)^2}{300}=33.3$ watts; and so forth. The graph emphasizes the nonlinear relation between resistance and power in a circuit of constant voltage. Thus as the resistance approaches

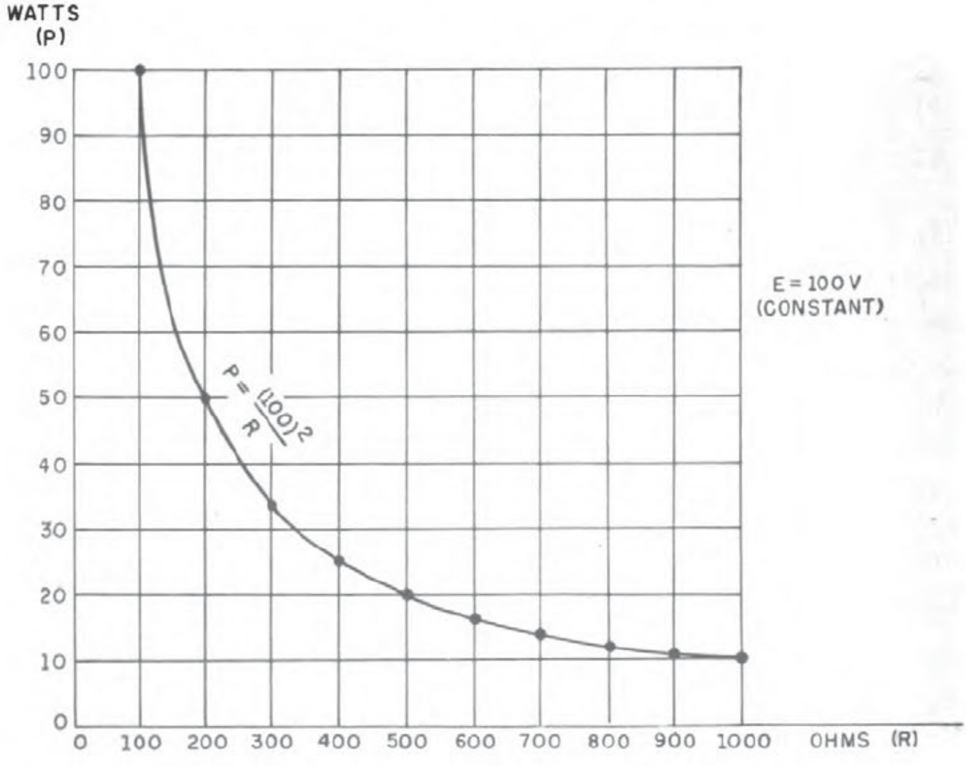


Figure 3–7.—Graph of power vs. resistance in a constant-voltage circuit.

a very small value, the power delivered approaches a very large value and vice versa.

In contrast to this relation the power in a constant-current circuit varies directly with the resistance of the circuit, as shown by the graph of figure 3–8. In this example the current is maintained constant at 5 amperes and the resistance of the circuit is increased in steps of 1 ohm beginning at 0 ohms and ending at 10 ohms. The resistance in ohms is plotted along the X axis to the right of the origin and corresponding values of power are plotted along the Y axis above the origin. These values are calculated as follows: $P=(5)^2\times 0=0$ watt; $5^2\times 1=25$ watts; $5^2\times 2=50$ watts; and so forth.

The graph emphasizes the linear relation between power and resistance in a circuit having a constant current.

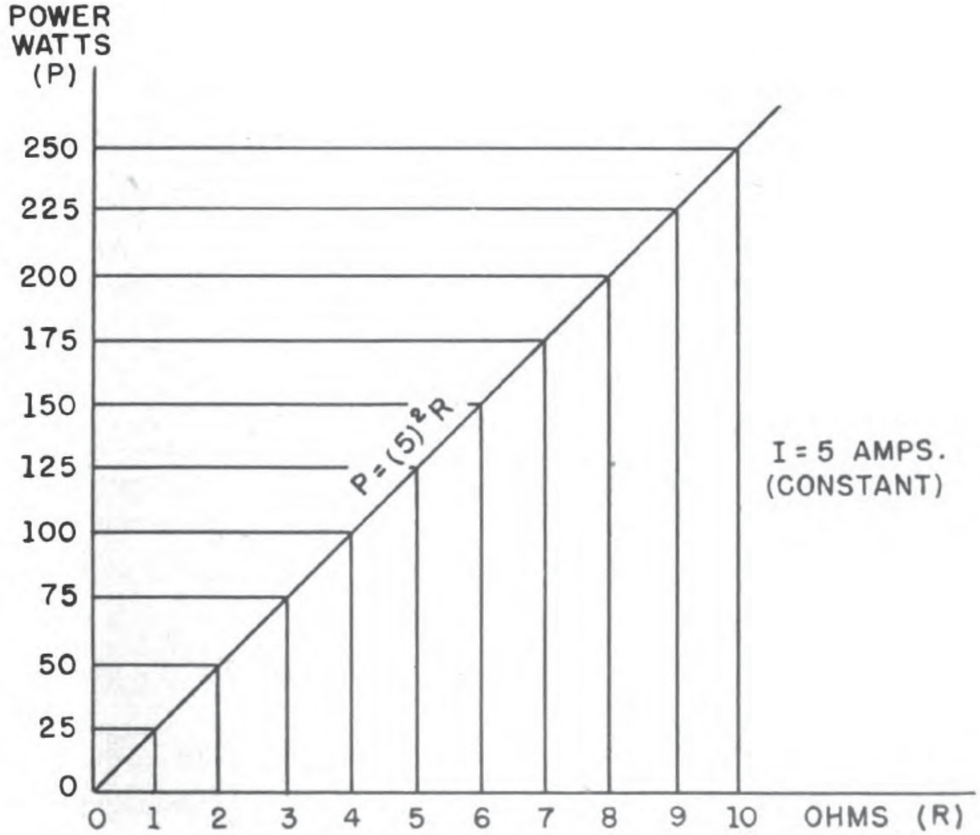


Figure 3–8.—Graph of resistance vs. power in a constant-current circuit.

Energy

Energy is defined as that which is capable of producing work. In mechanics, work is equal to the product of force and the distance through which the force acts. In practical electrical units energy is equal to the product of the power in watts and the time in hours. Since power is the rate of doing work, the multiplication of power by the time factor eliminates the rate and yields a product that is equal to the total energy involved over the period of time represented by the factor t.

An equation for energy is easily derived by multiplying both sides of equation (3–2) by the common factor of time, t, and equating the expression to the energy, W, as

$$\begin{aligned} W &= Pt \\ &= EIt. \end{aligned} \tag{3–5}$$

Similarily both sides of equations (3–3) and (3–4) may be multiplied by the time factor, t, and equated to the energy, W, as

$$W = \frac{E^2}{R}\, t, \tag{3–6}$$

and

$$W = I^2Rt. \tag{3–7}$$

In the energy equations (3–5), (3–6), and (3–7), E is in volts and I in amperes. If t is expressed in hours, W will be in watt-hours.

If t is expressed in seconds, W will be in watt-seconds or joules (1 joule is equal to 1 watt-second). Since $Q = It$ (where Q is in coulombs, I in amperes, and t in seconds) it is possible to substitute Q for It in equation (3–5) with the resulting expression for energy. Thus

$$W = QE, \tag{3–8}$$

where W is the energy in joules or watt-seconds, Q is the quantity in coulombs, and E is in volts.

Electrical energy is bought and sold in units of kilowatt-hours ($3{,}600 \times 10^3$ joules), and is totalized in large central generating stations in terms of megawatt-hours ($3{,}600 \times 10^6$ joules). For example, if the average demand over a 10-hour period is 70 megawatts, the total energy delivered is 70×10, or 700 megawatt-

hours. This amount of energy is equivalent to $700 \times 1{,}000 = 700{,}000$ kilowatt-hours, or $700 \times 3{,}600 \times 10^6 = 2{,}520{,}000 \times 10^6$ joules. The most practical unit to use depends in part upon the magnitude of the quantity of energy involved, and in this example the megawatt-hour is appropriate.

PROBLEMS

A wide variety of problems involving Ohm's law and the power relation are apparent with only a brief look at the electrical field.

Consider one of the most common examples—that of the incandescent electric light. Incandescent lamps for use in constant-potential circuits in homes and factories are rated in volts and watts. How much current do they take? How much resistance do they possess? If high-powered, they may draw several amperes; if low in wattage rating, they may take only a small fraction of an ampere. Specifically the current rating of a 100-watt 120-volt lamp is easily found by transposing equation (3–2) and solving for the current. Thus

$$I = \frac{P}{E} = \frac{100}{120} = 0.833 \text{ ampere.}$$

The resistance of this lamp during normal operation may be found by substituting the calculated value of current and the rated voltage in equation (3–1) and transposing for R—

$$R = \frac{E}{I} = \frac{120}{0.833} = 144 \text{ ohms.}$$

Thus a 100-watt 120-volt lamp draws a current of 0.833 ampere and has a hot resistance of 144 ohms. How much does it cost to use this lamp 4 hours each evening for 30 days at 3.2 cents per kilowatt-hour? Surprising little, as seen by the calculation:

$$\text{Total kilowatt-hours} = Pt = \frac{100 \times 4 \times 30}{1{,}000} = 12,$$

$$\text{Total cost} = 3.2 \times 12 = 38.4 \text{ cents.}$$

Consider the cost to operate a window air conditioner rated at 120 volts and 9.4 amperes in use 10 hours a day for 30 days. Applying equation (3–2), the power in kilowatts is

$$P=\frac{EI}{1,000}=\frac{120\times 9.4}{1,000}=1.128 \text{ kw.}$$

The total kilowatt-hours are $1.128\times 30\times 10$, or 338.4 kw-hr. At 3.2 cents per kw-hr the cost is $\$0.032\times 338.4$, or \$10.83.

Household electric stoves require special circuits as may be seen from the magnitude of the current drawn by a 7.5-kw 220-v range with all burners turned on full. The current is found by transposing equation (3–2) as follows:

$$I=\frac{P}{E}=\frac{7,500}{220}=34 \text{ amperes.}$$

Since most household branch circuits are not permitted to carry more than 20 amperes, the possibility of a 34-ampere load requires special circuits for the range. It is also important to note that the availability of the 220-volt service (rather than 110-volt service) permits the use of smaller conductors since current varies inversely with voltage for a given power rating. Thus a 7.5-kw range rated at 110 volts would draw a maximum of $\frac{220}{110}\times 34$, or 68 amperes.

To further illustrate the inverse ratio between current and voltage for a given power rating, consider the current drawn by a 100-watt incandescent lamp having a voltage rating of 6 volts. Transposing equation (3–2) the current is

$$I=\frac{P}{E}=\frac{100}{6}=16.6 \text{ amperes.}$$

This value of current is $\frac{120}{6}$, or 20 times the value required by a 120-volt lamp of the same wattage rating.

It is important to operate incandescent lamps within their voltage rating, since any appreciable over-voltage even for a short period reduces the life of the lamp a relatively large amount. For example, an increase in the voltage on a 25-watt 110-volt lamp from 110 volts to 140 volts will increase the current proportionately from $\frac{25}{110}=0.228$ amperes to $\frac{140}{110}\times 0.228=0.293$ ampere.

Since the power absorbed varies as the square of the applied voltage, increasing the applied voltage from 110 to 140 volts will increase the power to $\frac{(140)^2}{110}$, or 1.62 times the rated power. In a short time the lamp filament will burn out.

As another example illustrating the relation between the basic units, consider the current drawn by a 1,000-watt 120-volt electric flatiron. From equation (3–2) the current is found to be

$$I=\frac{P}{E}=\frac{1{,}000}{120}=8.33 \text{ amperes.}$$

The hot resistance of this flatiron may be found by substituting the known values of current and voltage in equation (3–1) and solving for R. Thus

$$R=\frac{E}{I}=\frac{120}{8.33}=14.4 \text{ ohms.}$$

The resistance of 14.4 ohms represents the operating resistance of the iron when it is carrying rated current. The resistance may also be calculated directly from equation (3–3) by substituting the values of voltage and power and transposing for R as follows:

$$R=\frac{E^2}{P}=\frac{(120)^2}{1{,}000}=14.4 \text{ ohms.}$$

The appropriate equation depends upon what is given and what is to be found.

For example, find the total energy in watt-hours delivered by a 12-volt storage battery supplying an average current of 10 amperes for a 10-hour period. Applying equation (3–5),

$$W=EIt=12\times10\times10=1{,}200 \text{ watt-hours.}$$

The product of amperes, hours, and volts is equal to watt-hours; and in this example, the battery is capable of furnishing electrical energy equivalent to 1,200 watt-hours, or 1.2 kilowatt-hours.

Another transposition of equation (3–3) may be used to solve the following problem: Find the voltage across, and the current through, a 100-ohm 500-watt resistor when it is carrying its rated load. Solving equation (3–3) for voltage in terms of the given values of power and resistance,

$$E=\sqrt{PR}=\sqrt{500\times100}=224 \text{ volts.}$$

Thus, a 500-watt 100-ohm load will have a voltage of 224 volts across it. The current through the load may be found by transposing equation (3–4) and substituting the given values as follows:

$$I=\sqrt{\frac{P}{R}}=\sqrt{\frac{500}{100}}=2.24 \text{ amperes.}$$

The power absorbed by the load may be checked by substituting the calculated value of current and the given value of resistance in equation (3–4), and solving as follows:

$$P=I^2R=(2.24)^2\times100=500 \text{ watts.}$$

The resistance of the load may be checked by transposing equation (3–1), substituting the calculated values of voltage and current, and solving for R as follows:

$$R=\frac{E}{I}=\frac{224}{2.24}=100 \text{ ohms.}$$

The student is urged to develop the habit of checking his work by using two or more equations to arrive at the same result. Two avenues of approach are more desirable than one since they tend to avoid mistakes that are frequently repeated in the single-approach method.

Current and voltage are important considerations in the operation of electric motors. Consider the power supplied to an industrial mill-motor when it is drawing 125 amperes from a 600-volt d-c circuit.

Applying equation (3–2),

$$P=EI=600\times125=75{,}000 \text{ watts.}$$

This amount of power may be expressed in equivalent horsepower (746 watts = 1 horsepower) as $\frac{75{,}000}{746}$, or 100.2 horsepower. If the requirements of the load are such that the horsepower remains constant, then the input current will vary inversely with the applied voltage. Suppose, for example, that the voltage falls

from 600 volts to 450 volts. The current required to maintain constant input power will be

$$I=\frac{P}{E}=\frac{75{,}000}{450}=166 \text{ amperes.}$$

This current represents an increase of 41 amperes, or 32.8 percent, and probably will cause overheating of the motor. The condition is peculiar to electric motors and rotating machinery in general and represents a situation in which the effective load resistance varies as the square of the applied voltage $\left(R=\frac{E^2}{P}\right)$.

The effective resistance offered by the motor to the flow of current when the voltage is 600 volts is

$$R=\frac{E}{I}=\frac{600}{125}=4.8 \text{ ohms,}$$

and when the voltage falls to 450 volts it is decreased to $\frac{450}{166}$, or 2.7 ohms. A check on the effective resistance at 600 volts may be obtained by transposing equation (3–3) and substituting the values of E and P as follows:

$$R=\frac{E^2}{P}=\frac{(600)^2}{75{,}000}=4.8 \text{ ohms.}$$

A similar check on the effective resistance at 450 volts is

$$R=\frac{E^2}{P}=\frac{(4.50)^2}{75{,}000}=2.7 \text{ ohms.}$$

The tendency of motors to increase their input current when the supply voltage decreases does not apply to electrical loads such as resistors, heating units, electric lights, and so forth, and the action described in the preceding example should not be assumed to apply indiscriminately without regard to the characteristics of the particular load being considered.

Applying the simple three-letter equations of this chapter to the solution of elementary problems in electricity is not difficult. However, the ease of solution may be misleading to the student with an inquiring mind. His attempt to prove his answers with practical equivalent circuits may reveal more than one variable in the elementary circuit.

For example, he finds that Ohm's law states that the current through an electric circuit is directly proportional to the applied voltage, yet when he increases the voltage (for example, on an incandescent lamp) he finds that the current increase is not in direct proportion to the voltage increase. The discrepancy is explained by the tendency of tungsten to increase its resistance with temperature. Thus an increase in filament voltage causes an

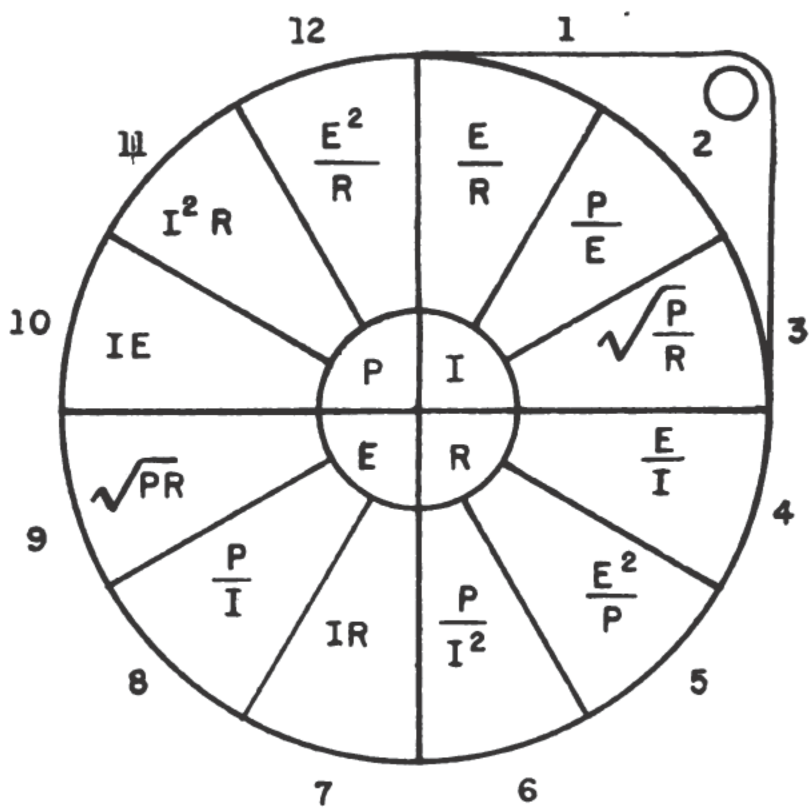


Figure 3–9.—Summary of basic equations using the volt, ampere, ohm, and watt.

increase in filament current and temperature. The increase in filament temperature causes an increase in filament resistance. Two variables now affect the circuit current instead of one. These variables are voltage and resistance instead of voltage alone.

For example, the current at 110 volts and 120 ohms is

$$I=\frac{E}{R}=\frac{110}{120}=0.917 \text{ ampere.}$$

TABLE 1.—*Relation between E, I, R, and P in basic d-c circuits*

Segment	Relation	Equation number
1	$I=\frac{E}{R}$	3–1
2	$I=\frac{P}{E}$	3–2 transposed
3	$I=\sqrt{\frac{P}{R}}$	3–4 transposed
4	$R=\frac{E}{I}$	3–1 transposed
5	$R=\frac{E^2}{P}$	3–3 transposed
6	$R=\frac{P}{I^2}$	3–4 transposed
7	$E=IR$	3–1 transposed
8	$E=\frac{P}{I}$	3–2 transposed
9	$E=\sqrt{PR}$	3–3 transposed
10	$P=EI$	3–2
11	$P=I^2R$	3–4
12	$P=\frac{E^2}{R}$	3–3

The current at 132 volts and 130 ohms is

$$I=\frac{132}{130}=1.015 \text{ amperes.}$$

The increase in voltage is 132 − 110, or 22 volts, and the percent increase is $\frac{22\times 100}{110}$, or 20 percent. The increase in resistance is 130 − 120, or 10 ohms, and the percent increase is $\frac{10\times 100}{120}$, or 8.33

percent. The increase in current is $1.015 - 0.917$, or 0.098 ampere, and the percent increase is $\frac{0.098 \times 100}{0.917}$, or 10.7 percent. Thus an increase in voltage of 20 percent is accompanied by an increase in current of only 10.7 percent. If the voltage had been the only variable the current would have followed it exactly—that is, an increase in voltage of 20 percent would have resulted in an increase in current of 20 percent. Instead, the current increased only 10.7 percent because the resistance increased 8.3 percent.

The equations themselves do not reveal the behavior of circuits. They do indicate, however, the numerical relation between the volt, ampere, ohm, and watt, and to that extent they contribute basic knowledge to the learner.

A summary of all the transpositions of equations (3–1) through (3–4) is given in the 12-segment circle of figure 3–9. The equations are stated in order from segment 1 to segment 12 and their relation to equations (3–1) through (3–4) is shown in the table on page 65.

The student is cautioned not to use figure 3–9 in place of the knowledge required to transpose these equations but to supplement it with a view to becoming able to solve for any of these quantities quickly in terms of the other known quantities.

QUIZ

1. Name the four most widely used practical electrical units that pertain to basic electric circuits.
2. Give the formula for current in terms of voltage and resistance.
3. In figure 3–2, an increase of 35 volts produces what increase in current?
4. Increasing the resistance from 100 ohms to 150 ohms across a constant potential of 150 volts will cause what numerical change in current?
5. Increasing the resistance from 5 ohms to 8 ohms in a constant-current circuit of 5 amperes will cause what numerical change in voltage?
6. Give formulas for the following:
   (a) Voltage in terms of current and resistance.
   (b) Resistance in terms of voltage and current.
7. Give a statement for power in terms of work and time.

8. Give formulas for power in terms of:
   (a) Current and voltage.
   (b) Voltage and resistance.
   (c) Current and resistance.
9. How much power is absorbed by a 20-ohm resistor when it is connected across a 60-volt supply?
10. How much power is absorbed by a 20-ohm resistor when the resistor current is 2 amperes?
11. Decreasing the resistance from 400 ohms to 200 ohms across a constant-potential source of 100 volts will cause what numerical change in the resistor power?
12. Give formulas for energy in terms of the following:
   (a) Current, voltage, and time.
   (b) Voltage, resistance, and time.
   (c) Current, resistance, and time.
   (d) Coulombs and volts.
13. An air conditioner draws 10 amperes from a 115-volt circuit for 6 hours a day for 25 days. How much energy in kilowatt hours is used?
14. A 1,200-watt heating unit is rated at 120 volts. What is its current rating?
15. What is the hot resistance of a 1,200-watt 120-volt heating unit?
16. How much energy in watt hours is delivered by a 6-volt storage battery supplying an average current of 5 amperes for an 8-hour period?
17. What is the voltage across a 50-ohm 250-watt resistor when it is operating at rated load?
18. What is the current in a 25-watt 100-ohm resistor when it is operating at rated load?
19. A 25-horsepower motor draws a current of 50 amperes from a 440-volt circuit. How many kilowatts does it take from the circuit?
20. An increase in the voltage on an incandescent 120-volt tungsten lamp filament from 40 volts to 80 volts was not accompanied by a proportionate increase in current. Explain.
21. Give two transpositions for each of the following formulas:
   (a) $E=IR$. (b) $P=EI$.
   (c) $P=I^2R$. (d) $P=\frac{E^2}{R}$.

CHAPTER 4

DIRECT-CURRENT SERIES AND PARALLEL CIRCUITS

INTRODUCTION

An electric circuit is a complete path through which electrons can flow from the negative terminal of the voltage source; through the connecting wires, or conductors; through the load, or loads; and back to the positive terminal of the voltage source. A circuit is thus made up of a voltage source, the necessary connecting conductors, and the effective load.

If the circuit is so arranged that the electrons have only ONE possible path, the circuit is called a SERIES CIRCUIT. If there are two or more paths supplied by a common voltage source, the circuit is called a PARALLEL CIRCUIT.

In any type of work that utilizes the effects of electron flow a knowledge of series and parallel circuits is desirable. None of the effects accompanying electron flow—for example, heating, lighting, or magnetic effects—would be possible without the use of electric circuits; and many electrical devices can be utilized more effectively if the operator has a knowledge of how they work. The purpose of this chapter is to give in simplified form conventional methods of calculating resistance in basic series and parallel circuits and to show how problems involving current, voltage, and power may be solved by the use of basic formulas. The next chapter is similar, except that more complex circuits are treated. In both chapters Ohm's law or Kirchhoff's laws are used as required to solve for various unknown values of current, voltage, or resistance.

SERIES CIRCUITS

Kirchhoff's Voltage Law Applied to Series Circuits

All conductors have resistance, and therefore a circuit made up of nothing but conductors would have some resistance, however

small it might be. In circuits containing long conductors through which an appreciable amount of current is drawn the resistance of the conductors becomes important. However, for the purposes of this chapter the resistance of the connecting wires is neglected, and only the resistance of each resistor element (which may be any device that has resistance) is considered.

In the simple series circuit shown in figure 4–1, the algebraic sum of the voltages around the circuit is equal to zero. This is an example of KIRCHHOFF'S LAW OF VOLTAGES. In order to solve problems applicable to this circuit and others like it uniformily the same, the following procedure is suggested:

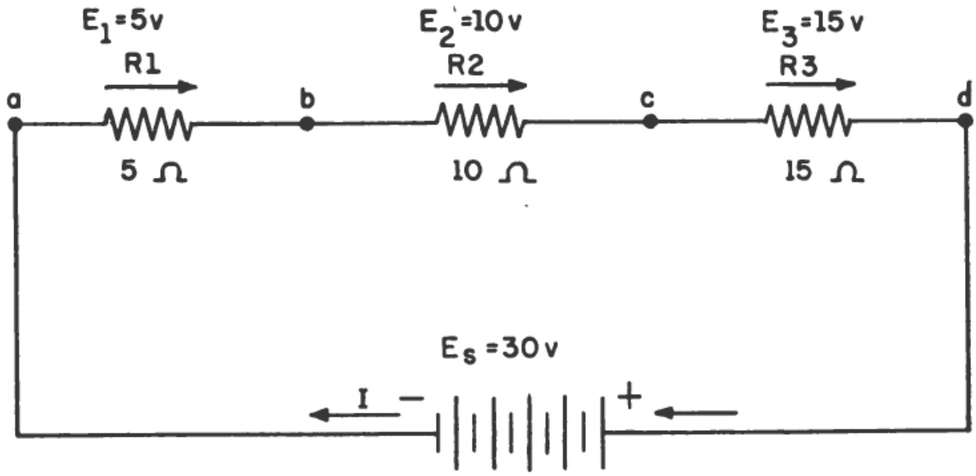


Figure 4–1.—Resistors in series.

1. Note the polarity of the source emf (E_s) and indicate the electron flow around the circuit. Electron flow is out from the negative terminal of the source, through the load, and back to the positive terminal of the source. In the example being considered, the arrows indicate electron flow in a clockwise direction around the circuit.
2. To apply Kirchhoff's law it is necessary to establish a voltage equation. The equation is developed by tracing around the circuit and noting the voltage absorbed (called voltage drop) across each part of the circuit encountered by the trace, and expressing the sum of these voltages according to the voltage law. It is important that the trace be made around a closed circuit, and that it encircle the circuit only once. Thus, a point is arbitrarily selected at which to start the trace. The trace is then made, and upon completion, the terminal point coincides with the starting point.

3. Sources of emf are preceded by a PLUS sign if in tracing through the source the first terminal encountered is positive; if the first terminal is negative, the emf is preceded by a MINUS sign.
4. Voltage drops along wires and across resistors (loads) are preceded by a minus sign if the trace is in the assumed direction of electron flow; if in the opposite direction the sign is plus.
5. If the assumed direction of electron flow is incorrect, the error is indicated by a minus sign preceding the current, as obtained in solving for circuit current. The magnitude of of the current will not be affected.

The preceding rules may be applied to the example of figure 4–1 as follows:

1. The left terminal of the battery is negative, the right terminal is positive, and electron flow is clockwise around the circuit.
2. The trace may arbitrarily be started at the positive terminal of the source and continued clockwise through the source to its negative terminal. From this point the trace is continued around the circuit to *a, b, c, d,* and back to the positive terminal, thus completing the trace once around the entire closed circuit.
3. The first term of the voltage equation is $+E_s$.
4. The second, third, and fourth terms are respectively, $-E_1$, $-E_2$, and $-E_3$. Their algebraic sum is equated to zero as follows:

$$E_s - E_1 - E_2 - E_3 = 0.$$

Transposing the voltage equation and solving for E_s,

$$E_s = E_1 + E_2 + E_3.$$

Since $E = IR$ from Ohm's law, the voltage drop across each resistor may be expressed in terms of the current and resistance of the individual resistor as follows:

$$E_s = IR_1 + IR_2 + IR_3,$$

where R_1, R_2, and R_3 are the resistances, respectively, of resistors $R1$, $R2$, and $R3$; E_s is the source voltage; and I is the circuit current.

E_s may be expressed in terms of the circuit current and total

resistance as IR_t. Substituting IR_t for E_s, the voltage equation becomes,

$$IR_t = IR_1 + IR_2 + IR_3.$$

Since there is only one path for current in the series circuit, the total current is the same in all parts of the circuit. Dividing both sides of the voltage equation by the common factor, I, an expression is derived for the total resistance of the circuit in terms of the resistances of the individual devices—

$$R_t = R_1 + R_2 + R_3.$$

Therefore, in series circuits the total resistance is the sum of the resistances of the individual parts of the circuit.

In the example of figure 4–1, the total resistance is

$$5 + 10 + 15 = 30 \text{ ohms.}$$

The total current may be found by applying the equation

$$I_t = \frac{E_t}{R_t} = \frac{30}{30} = 1 \text{ ampere.}$$

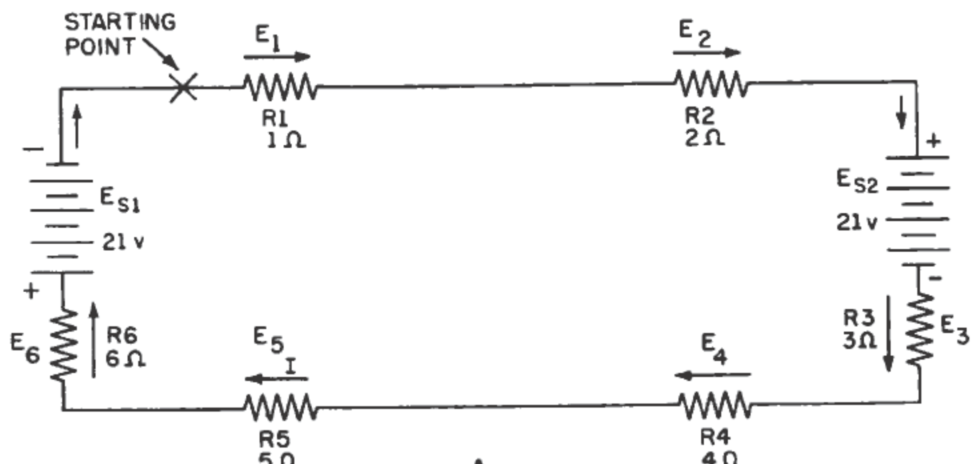
The power absorbed by resistor R_1 is I^2R_1, or $1^2 \times 5 = 5$ watts. Similarly the power absorbed by R_2 is $1^2 \times 10 = 10$ watts, and the power absorbed by R_3 is $1^2 \times 15 = 15$ watts. The total power absorbed is the arithmetic sum of the power of each resistor, or $5 + 10 + 15 = 30$ watts. The value is also calculated by $P_t = E_tI_t = 30 \times 1 = 30$ watts.

More than one voltage source may be used in a series circuit. If the polarities of the sources are such that they aid each other, they are simply added together, and the net result is the same as if a single source of higher voltage had been used. If the polarities of the sources are such that one or more of them tend to cancel or "buck out" the effect of the others, then the net result is the difference between the source voltages that would cause the circuit current to flow in one direction and the source voltages that cause it to flow in the opposite direction.

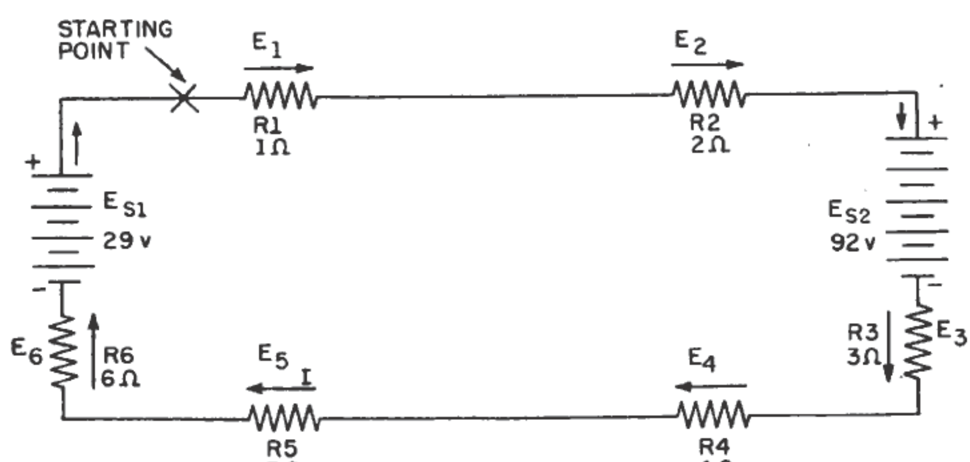
Actually, the current could be assumed to flow in either direction as long as the minus sign is placed before each voltage drop (when the circuit is traced in the direction of the assumed current flow) and the proper sign (the one encountered as the source is

approached in the circuit) is placed before all voltage sources. As mentioned previously, if the resultant circuit current has a negative sign it merely means that the assumed direction of current flow is incorrect.

Kirchhoff's voltage law is readily applied in solving series circuit problems. In many cases it is the most practical method that may be used. For example, in figure 4–2, Kirchhoff's voltage law is used to solve for current, *I;* and when this has been determined, the other unknown values may be determined by Ohm's law or the associated power fomulas.



A
SOURCES AIDING



B
SOURCES BUCKING

Figure 4–2.—Applications of Kirchhoff's voltage law to series circuits.

In figure 4–2, A, the two voltage sources are aiding each other; that is, the polarities are so connected that each source tends to make the current flow clockwise around the circuit. Starting at the left terminal of $R1$ and proceeding clockwise around the circuit, the voltage equation is,

$$-E_1-E_2+E_{s2}-E_3-E_4-E_5-E_6+E_{s1}=0.$$

Expressing the voltage drops around the circuit in terms of current and resistance, and the given values of source voltage, the equation becomes,

$$-I-2I+21-3I-4I-5I-6I+21=0,$$

from which

$$I=\frac{42}{21}=2 \text{ amperes.}$$

I is positive and therefore the assumed direction of current flow is correct, although in this case the direction of current flow was obvious.

The total power absorbed by the resistors in figure 4–2, A, is the sum of the power absorbed by each resistor.

The power in $R1$ is $I^2R_1=2^2\times 1=4$ watts.
The power in $R2$ is $I^2R_2=2^2\times 2=8$ watts.
The power in $R3$ is $I^2R_3=2^2\times 3=12$ watts.
The power in $R4$ is $I^2R_4=2^2\times 4=16$ watts.
The power in $R5$ is $I^2R_5=2^2\times 5=20$ watts.
The power in $R6$ is $I^2R_6=2^2\times 6=24$ watts.

Note that in the above calculations for power the current values in each is the same because these calculations apply to a series circuit. The total power absorbed is

$$4+8+12+16+20+24=84 \text{ watts.}$$

A check on the calculation is that the total power absorbed is equal to I_tE_t, or

$$2\times(21+21)=84 \text{ watts.}$$

In figure 4–2, B, the voltage sources are bucking. Because E_{s2} is larger than E_{s1}, E_{s2} determines the direction of current flow, and this direction is clockwise around the circuit. Starting at the

left terminal of $R1$ and proceeding clockwise around the circuit, the voltage equation is,

$$-E_1-E_2+E_{s2}-E_3-E_4-E_5-E_6-E_{s1}=0,$$

or

$$-I-2I+92-3I-4I-5I-6I-29=0,$$

from which

$$I=\frac{92-29}{21}=3 \text{ amperes.}$$

The power absorbed by $R1$ is $I^2R_1=3^2\times 1=9$ watts.
The power absorbed by $R2$ is $I^2R_2=3^2\times 2=18$ watts.
The power absorbed by $R3$ is $I^2R_3=3^2\times 3=27$ watts.
The power absorbed by $R4$ is $I^2R_4=3^2\times 4=36$ watts.
The power absorbed by $R5$ is $I^2R_5=3^2\times 5=45$ watts.
The power absorbed by $R6$ is $I^2R_6=3^2\times 6=54$ watts.
The total power is

$$9+18+27+36+45+54=189 \text{ watts.}$$

A check on the calculation is that the total power absorbed is equal to the product of the net voltage, acting in the direction of electron flow, and the current, or

$$EI=(92-29)\times 3=189 \text{ watts.}$$

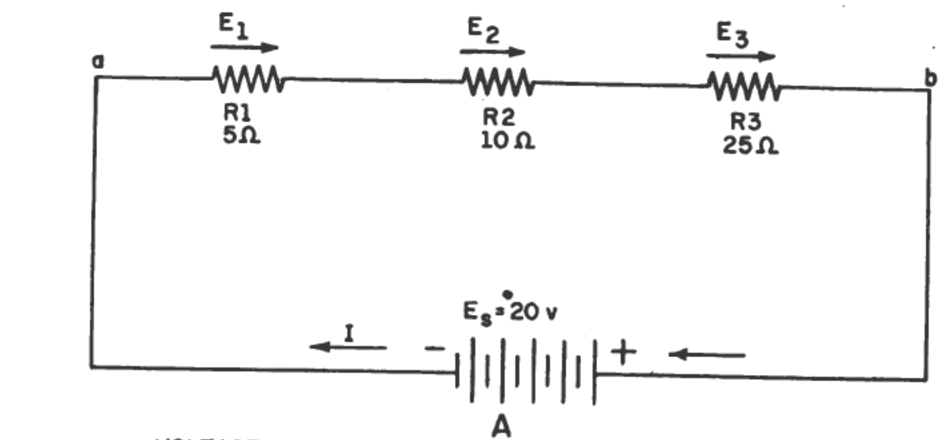
Typical Problems in Series Circuits

As seen by the preceding calculations, problems involving the determination of resistance, voltage, current, and power in a series circuit are relatively simple. It is necessary only to draw or visualize the circuit, to list the known values, and to determine by means of Ohm's law (or Kirchhoff's adaptation of Ohm's law) the unknown values.

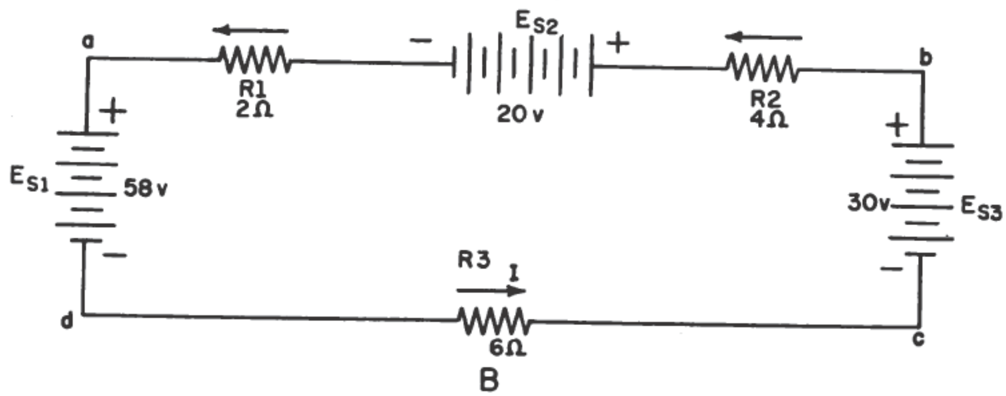
In figure 4–3, A, assume that the source voltage and the series resistors have values as indicated. It is desired to find the total resistance, R_t, the current, I, and the voltage drops across $R1$, $R2$, and $R3$.

The total resistance is equal to the sum of the individual resistors. Thus,

$$R_t=R_1+R_2+R_3=5+10+25=40.$$



VOLTAGE AROUND SERIES CIRCUIT HAVING ONE VOLTAGE SOURCE



VOLTAGES AROUND SERIES CIRCUIT HAVING VOLTAGE SOURCES IN OPPOSITION

Figure 4–3.—Solving for R_t, E, and I in series circuits.

The current is

$$I=\frac{E_s}{R_t}=\frac{20}{40}=0.5 \text{ ampere.}$$

The current is the same in all parts of the series circuit, and therefore the same current flows through $R1$, $R2$, and $R3$. The voltage drops across $R1$, $R2$, and $R3$ are indicated as

$$E_1=I\times R_1=0.5\times 5=2.5 \text{ volts,}$$

$$E_2=I\times R_2=0.5\times 10=5 \text{ volts,}$$

and

$$E_3=I\times R_3=0.5\times 25=12.5 \text{ volts.}$$

According to Kirchhoff's voltage law, the sum of the individual voltage drops is equal to the source voltage, or

$$E_s = E_1 + E_2 + E_3 = 2.5 + 5 + 12.5 = 20 \text{ volts.}$$

The power absorbed by $R1$ is $\frac{E_1^2}{R_1} = \frac{(2.5)^2}{5} = 1.25$ watts.

The power absorbed by $R2$ is $\frac{E_2^2}{R_2} = \frac{5^2}{10} = 2.50$ watts.

The power absorbed by $R3$ is $\frac{E_3^2}{R_3} = \frac{(12.5)^2}{25} = 6.25$ watts.

The total power absorbed by the 3 resistors is $1.25 + 2.5 + 6.25$, or 10 watts. This value may be checked by the equation,

$$P_t = E_t I_t = 20 \times 0.5 = 10 \text{ watts.}$$

In the example of figure 4–3, B, the circuit trace is arbitrarily started at point *a* and continued in a clockwise direction completely around the circuit and back to point *a*. The voltage equation is written in terms of both (1) known voltages and (2) current and resistance. The equation is solved for current in terms of voltage and resistance. The three voltage sources are arranged so that E_{s1} and E_{s2} are additive and both are opposed to E_{s3}. Since the sum of E_{s1} and E_{s2} is greater than E_{s3} the electron flow is determined by E_{s1} and E_{s2} and is seen to be in a counterclockwise direction around the circuit as indicated by the arrows above the resistors.

Representing the unknown current as I, the voltage equation is,

$$\begin{aligned} 2I - 20 + 4I + 30 + 6I - 58 &= 0 \\ 12I - 48 &= 0 \\ I = \frac{48}{12} &= 4 \text{ amperes.} \end{aligned}$$

The voltage drop across $R1$ is $IR_1 = 4 \times 2 = 8$ volts.
The voltage drop across $R2$ is $IR_2 = 4 \times 4 = 16$ volts.
The voltage drop across $R3$ is $IR_3 = 4 \times 6 = 24$ volts.
The total voltage acting in the direction of electron flow is equal to

$$8 + 16 + 24, \text{ or } 48 \text{ volts.}$$

The power absorbed in $R1$ is $I^2R_1=4^2\times2=32$ watts.
The power absorbed in $R2$ is $I^2R_2=4^2\times4=64$ watts.
The power absorbed in $R3$ is $I^2R_3=4^2\times6=96$ watts.
The total power absorbed by the three resistors is

$$32+64+96=192 \text{ watts.}$$

The calculation may be checked by the relation

$$P_t=E_tI=48\times4=192 \text{ watts.}$$

The filaments of light bulbs and electron tubes are made of materials that have appreciable resistance. Most incandescent lamps have tungsten filaments. The heat produced by the current flowing through the resistance of the filament is intense enough to make the tungsten wires white hot and thus produce artificial light. Radio receivers of the common a–c/d–c variety have their filaments connected in series, as shown in figure 4–4. The tube designations are shown directly above the filament symbols, $R1$, $R2$, and so forth. The first two numerals in the designation indicate the approximate voltage that must be applied to the filament to cause rated current to flow. For example, 12 volts must be applied to the filament of the 12SA7 tube to cause the rated current of 0.15 ampere to flow.

The first problem is to find the value of $R6$ that will cause 0.15 ampere to flow in the circuit when the applied voltage is 117 volts. By observation it is seen that the tube filaments will absorb

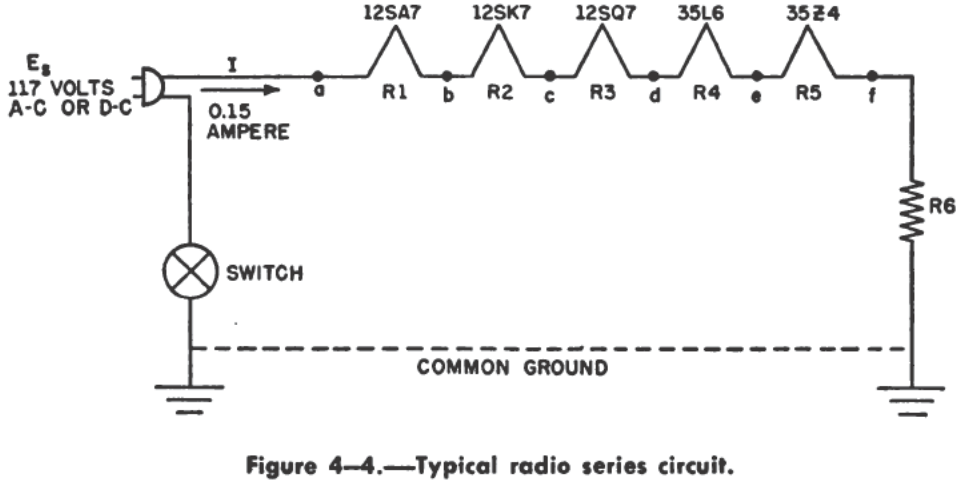


Figure 4–4.—Typical radio series circuit.

$$12+12+12+35+35=106 \text{ volts.}$$

Therefore, $R6$ must absorb the difference, or

$$117-106=11 \text{ volts.}$$

The resistance of $R6$ is,

$$R_6=\frac{E_6}{I}=\frac{11}{0.15}=73.3 \text{ ohms.}$$

The power, P_6, absorbed by $R6$ is

$$P_6=E_6\times I=11\times 0.15=1.65 \text{ watts.}$$

The power absorbed by the 12SA7 filament is

$$P_1=E_1I=12\times 0.15=1.8 \text{ watts.}$$

The power absorbed by the 12SK7 and the 12SQ7 filaments is also 1.8 watts each, since they both have equal voltages and the same value of current. The 35L6 filament absorbs an amount of power equal to $35\times 0.15=5.25$ watts and the 35Z4 filament absorbs an equal amount of power. The total power absorbed by the entire filament circuit is equal to the sum of the power absorbed by each separate unit. Thus,

$$P_t=1.8+1.8+1.8+5.25+5.25+1.65=17.55 \text{ watts.}$$

The total power may be checked as follows:

$$P_t=E_tI=117\times 0.15=17.55 \text{ watts.}$$

The voltages around the series circuit shown in figure 4–4 are distributed in proportion to the resistances around the circuit. Thus, the voltage between any point, for example, ground, and some other point such as *a, b,* or *c* may be determined. For example, the potential from *a* to ground is the full 117 volts because the entire circuit is included. From *b* to ground, the potential is 117 volts less the drop of 12 volts across $R1$, or 105 volts, because the circuit includes everything except $R1$. The potential between point *f* and ground is 11 volts because $R6$ alone is included in this part of the circuit.

In this type of circuit, when an open occurs in any of the filaments or in $R6$, the current ceases to flow. However, the input

current increases if any of the points, *a, b, c,* and so forth, should become grounded. For example, if point *f* should become grounded the resistance of *R*6 is no longer in the circuit, and 117 volts is applied between points *a* and *f* instead of 106 volts. The total filament current is increased to

$$\frac{117}{106}\times 0.15=0.165 \text{ ampere.}$$

The increase in current is proportioned to the increase in voltage. Overvoltage may cause the filament to overheat because of excessive current. Thus, the entire filament circuit may be deenergized when the circuit opens at same point due to filament burnout.

EFFECT OF SOURCE RESISTANCE ON VOLTAGE, POWER, AND EFFICIENCY

All sources of emf have some internal resistance that acts in series with the load resistance. The source resistance is generally indicated in circuit diagrams as a separate resistor connected in series with the source. Both the voltage and power made available to the load may be increased if the internal resistance of the source is reduced.

The effects of source resistance, R_s, on load voltage may be illustrated by the use of figure 4–5. In figure 4–5, A, the circuit is open, and therefore a voltmeter connected across the battery will read the open-circuit voltage. In the case of a dry cell, the open-circuit voltage is 1.5 volts. In figure 4–5, B, the cell is short-circuited through the ammeter, and a current of 30 amperes flows from the source. In this case the voltage of the cell is developed across the internal resistance of the cell. The internal resistance of the cell is therefore,

$$R_s=\frac{E_s}{I}=\frac{1.5}{30}=0.05 \text{ ohms.}$$

If a load, R_L, of 0.10 ohm is connected to the circuit, as shown in figure 4–5, C, the current, *I,* becomes

$$I=\frac{E_s}{R_t}=\frac{1.5}{0.15}=10 \text{ amperes.}$$

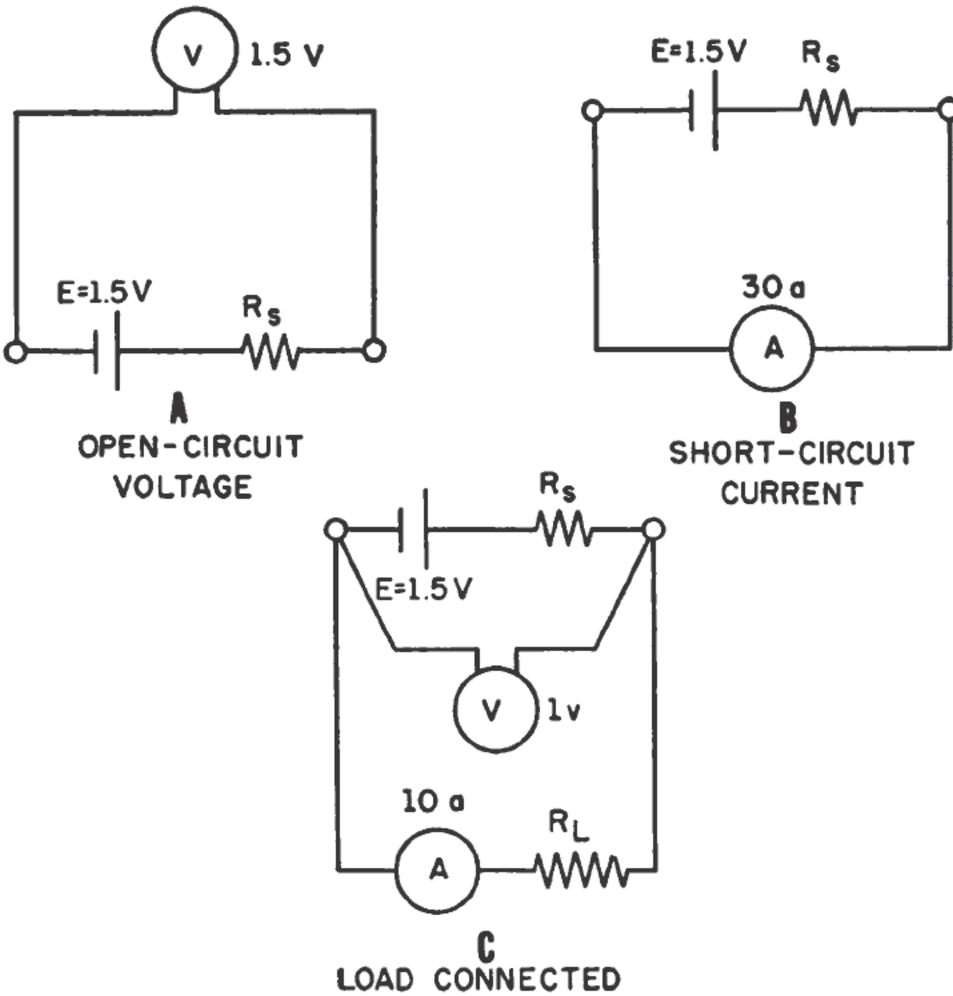


Figure 4–5.—Effect of source resistance on load voltage.

The voltage available at the load is

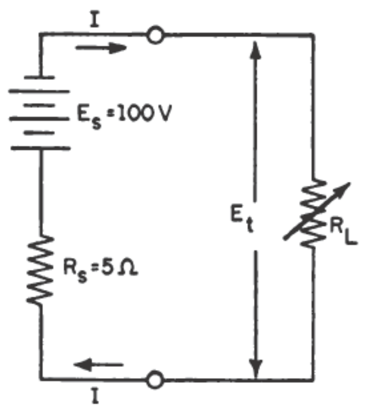
$$E_L = IR_L = 10 \times 0.1 = 1 \text{ volt.}$$

The voltage absorbed across the internal resistance of the cell is

$$IR_s = 10 \times 0.05 = 0.5 \text{ volt.}$$

Thus the effect of the internal resistance is to decrease the terminal voltage from 1.5 volts to 1 volt when the cell delivers 10 amperes to the load.

The effect of the source resistance on the power output of a d-c source may be shown by an analysis of the circuit in figure

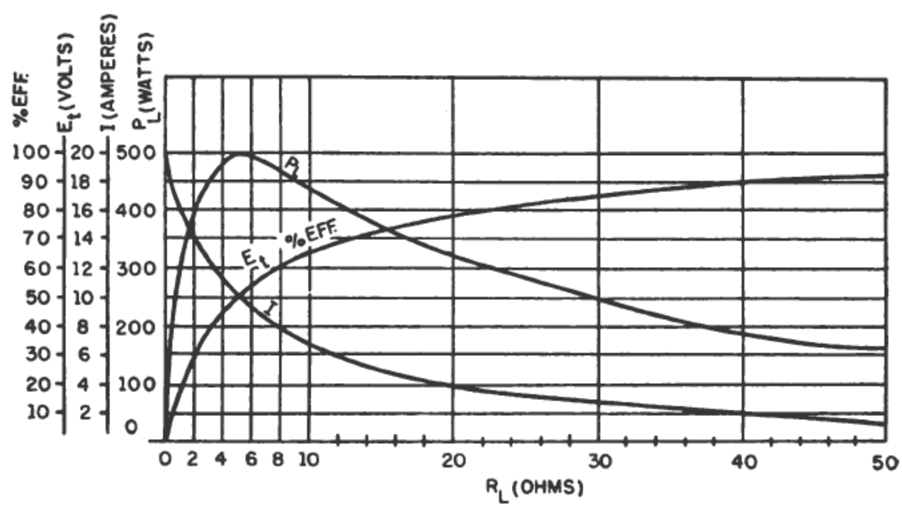


E_s=OPEN-CIRCUIT VOLTAGE OF SOURCE
R_s=INTERNAL RESISTANCE OF SOURCE
E_t=TERMINAL VOLTAGE
R_L=RESISTANCE OF LOAD
P_L=POWER USED IN LOAD
I =CURRENT FROM SOURCE
% EFF =PERCENTAGE OF EFFICIENCY

A
CIRCUIT AND SYMBOL DESIGNATIONS

R_L	E_t	I	P_L	% EFF.
0	0	20	0	0
1	16.6	16.6	267.6	16.6
2	28.6	14.3	409	28.6
3	37.5	12.5	468.8	37.5
4	44.4	11.1	492.8	44.4
5	**50**	**10**	**500**	**50**
6	54.5	9.1	495.4	54.5
7	58.1	8.3	482.2	58.1
8	61.6	7.7	474.3	61.6
9	63.9	7.1	453.7	63.9
10	66	6.6	435.6	66
20	80	4	320	80
30	87	2.9	252	87
40	88	2.2	193.6	88
50	95	1.9	180.5	95

B
CHART



C
GRAPH

Figure 4–6.—Effect of source resistance on power output.

4–6, A. When the load is zero ohms (equivalent to a short circuit) the current is limited only by the internal resistance, R_s, of the source and the inherent resistance of the connecting wires, which is neglected in this chapter. The short-circuit current, I, is determined as

$$I=\frac{E_s}{R_s}=\frac{100}{5}=20 \text{ amperes.}$$

This is the maximum current that may be drawn from the source. The terminal voltage across the short circuit is zero and all the voltage is absorbed within the internal resistance of the source.

If the load resistance, R_L, is increased (the internal resistance remaining the same), the current drawn from the source and the voltage drop across the internal resistance will decrease. At the same time, the terminal voltage applied across the load will increase and will approach a maximum as the current approaches zero.

The MAXIMUM POWER TRANSFER THEOREM says in effect that maximum power is transferred from the source to the load when the resistance of the load is equal to the internal resistance of the source. This theorem is illustrated in the tabular chart and the graph of figure 4–6, B and C. When the load resistance is 5 ohms, thus matching the source resistance, the maximum power of 500 watts is developed in the load.

The efficiency of power transfer (ratio of output to input power) from the source to the load increases as the load resistance is increased. The efficiency approaches 100 percent as the load resistance approaches a relatively large value compared with that of the source, since less power is lost in the source. The efficiency of power transfer is only 50 percent at the maximum power transfer resistance of 5 ohms and approaches zero efficiency at relatively low values of load resistance compared with that of the source.

Thus the problem of high efficiency and maximum power transfer is resolved as a compromise somewhere between the low efficiency of maximum power transfer and the high efficiency of the high-resistance load. Where the amounts of power involved are large and the efficiency is important, the load resistance is made large relative to the source resistance so that the losses are kept small. In this case the efficiency will be high. Where the problem of matching a source to a load is of paramount importance, as in communications circuits, a strong signal may be more important than the losses within the source, in which event the efficiency of transmission will be of the order of 50 percent and at the same time the power in the load will be the maximum which the source is capable of supplying.

Ohm's Law Applied to Parallel Circuits

The parallel circuit differs from the simple series circuit in that two or more resistors, or loads, are connected DIRECTLY to the same source of voltage. There is accordingly MORE THAN ONE PATH that the electrons can take. The more paths (or resistors) that are added in parallel the less opposition there is to the flow of electrons from the source. This condition is opposite to the effect that is produced in the series circuit where added resistors increase the opposition to the electron flow.

As may be seen from figure 4–7, the same voltage is applied across each of the parallel resistors. In this case the voltage applied across the resistors is the same as the source voltage, E_s.

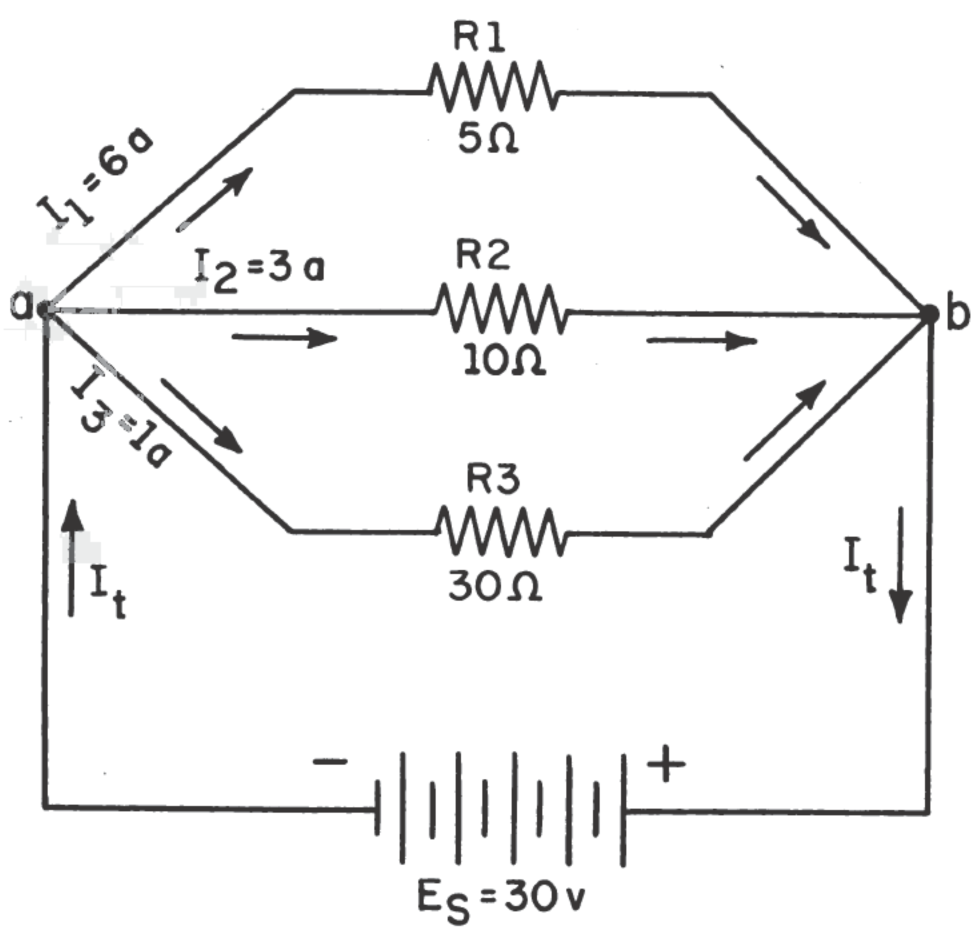


Figure 4–7.—Resistors in parallel.

Current flows from the negative terminal of the source to point *a* where it divides and passes through the three resistors to point *b* and back to the positive terminal of the voltage source. The amount of current flowing through each individual branch depends on the source voltage and on the resistance of that branch—that is, the lower the resistance of the branch the higher will be the current through that branch. The individual currents can be found by the application of Ohm's law to the individual resistors. Thus,

$$I_1=\frac{E_s}{R_1}=\frac{30}{5}=6 \text{ amperes,}$$

$$I_2=\frac{E_s}{R_2}=\frac{30}{10}=3 \text{ amperes,}$$

and

$$I_3=\frac{E_s}{R_3}=\frac{30}{30}=1 \text{ ampere.}$$

The total current, I_t, of the parallel circuit is equal to the sum of the currents through the individual branches. This, in slightly different words, is Kirchhoff's current law, which is treated later in this chapter. In this case, the total current is

$$I_t=I_1+I_2+I_3=6+3+1=10 \text{ amperes.}$$

In order to find the equivalent, or total, resistance, R_t (for simplicity, R_t is used instead of R_{eq}), of the combination shown in figure 4–7, Ohm's law is used to find each of the currents (I_t, I_1, I_2, and I_3) in the preceding formula. The total current is equal to the sum of the branch currents. Thus,

$$\frac{E_s}{R_t}=\frac{E_s}{R_1}+\frac{E_s}{R_2}+\frac{E_s}{R_3}$$

$$\frac{E_s}{R_t}=E_s\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right).$$

Both sides of this equation may be divided by E_s without changing the value of the equation; and therefore

$$\frac{1}{R_t}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}.$$

The fraction, $\frac{1}{R_t}$, is the reciprocal (the reciprocal of a value is that value divided into one) of the total, or equivalent, resistance of the parallel circuit. THE RECIPROCAL OF THE EQUIVALENT RESISTANCE OF A PARALLEL CIRCUIT IS EQUAL TO THE SUM OF THE RECIPROCALS OF ALL THE BRANCHES.

The quantity $\frac{1}{R}$ is an expression called CONDUCTANCE, G, which is the reciprocal of resistance. It is equal to the amperes per volt of applied pressure. When voltage is multiplied by conductance the value of the current is obtained. Conductance is expressed in mhos (ohm spelled backward) and is defined AS THE ABILITY OF A CIRCUIT TO CONDUCT CURRENT.

By means of the preceding equation the total resistance of the circuit shown in figure 4–7 may be determined. Thus,

$$\frac{1}{R_t}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3},$$

$$\frac{1}{R_t}=\frac{1}{5}+\frac{1}{10}+\frac{1}{30},$$

and

$$\frac{1}{R_t}=\frac{10}{30}.$$

Taking the reciprocals of both sides (dividing them into 1),

$$R_t=\frac{30}{10}=3 \text{ ohms.}$$

A useful rule to remember in computing the equivalent resistance of a d-c parallel circuit is that THE TOTAL RESISTANCE IS ALWAYS LESS THAN THE SMALLEST RESISTANCE IN ANY OF THE BRANCHES.

There are two shortcuts that may be used in solving for the equivalent resistance of a parallel circuit.

The first applies to any number of parallel resistors all of which have the same value of resistance. The equivalent resistance is determined by dividing the resistance of one by the number in

parallel. Thus, the equivalent resistance of five 10-ohm resistors connected in parallel is

$$R_t=\frac{10}{5}=2 \text{ ohms.}$$

The second shortcut applies to two resistors of different values in parallel. The equivalent resistance is equal to their product divided by their sum. For example, if two resistors having resistances of 3 ohms and 6 ohms are connected in parallel, the equivalent resistance is

$$R_t=\frac{R_1R_2}{R_1+R_2}=\frac{3\times 6}{3+6}=\frac{18}{9}=2 \text{ ohms.}$$

In addition to adding the individual branch currents to obtain the total current in a parallel circuit, the total current may be found directly by dividing the applied voltage by the equivalent resistance, R_t. For example, in figure 4–7

$$I_t=\frac{E_s}{R_t}=\frac{30}{3}=10 \text{ amperes.}$$

Kirchhoff's Current Law Applied to Parallel Circuits

Kirchhoff's current law states that AT ANY JUNCTION OF CONDUCTORS THE ALGEBRAIC SUM OF THE CURRENTS IS ZERO. This is another way of saying that as many electrons leave a junction as enter it. Consider junction *a* of figure 4–7. Assume that the current flowing toward junction *a* is positive and that the currents flowing away from the junction—that is, currents I_1, I_2, and I_3—are negative. Kirchhoff's current law is then expressed mathematically as

$$+I_t-I_1-I_2-I_3=0,$$

or

$$10-6-3-1=0.$$

As in the series circuit, the total power consumed in a parallel circuit is equal to the sum of the power consumed in the individual resistors. The power, P_1, consumed in $R1$ of figure 4–7 is

$$P_1=EI_1=30\times 6=180 \text{ watts;}$$

the power, P_2, consumed in $R2$ is

$$P_2 = EI_2 = 30 \times 3 = 90 \text{ watts;}$$

and the power, P_3, consumed in $R3$ is

$$P_3 = EI_3 = 30 \times 1 = 30 \text{ watts.}$$

The total power, P_t, consumed is the sum of the power consumed in the individual branch units—that is,

$$P_t = P_1 + P_2 + P_3 = 180 + 90 + 30 = 300 \text{ watts.}$$

The total power may be checked as

$$P_t = EI_t = 30 \times 10 = 300 \text{ watts.}$$

Typical Problems in Parallel Circuits

Problems involving the determination of resistance, voltage, current, and power in a parallel circuit are solved as simply as in a series circuit. The procedure is the same—(1) draw a circuit diagram, (2) state the values given and the values to be found, (3) state the applicable equations, and (4) substitute the given values and solve for the unknown.

For example, the parallel circuit of figure 4–8 consists of 2

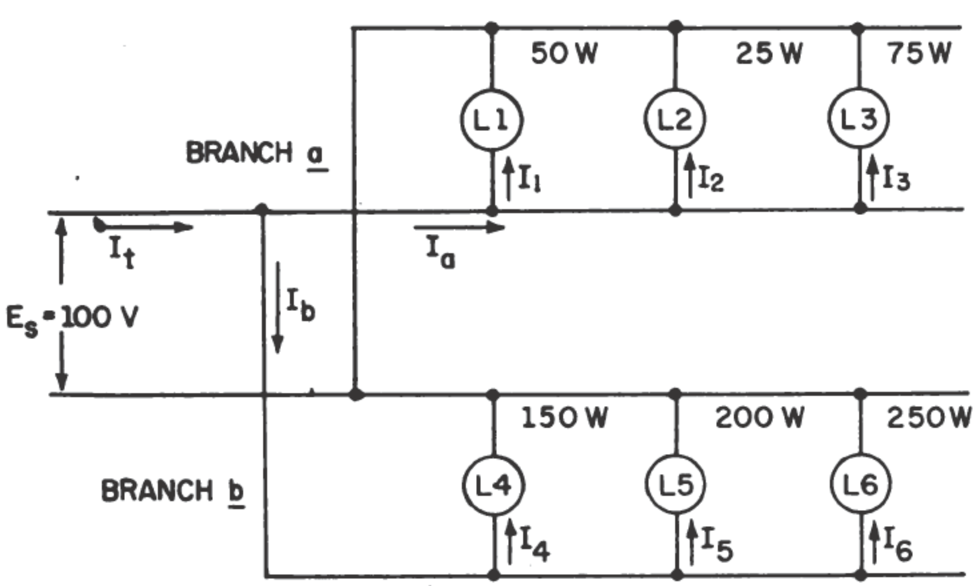


Figure 4–8.—Typical parallel circuit.

branches (a and b). Branch a consists of 3 lamps in parallel. Their ratings are $L_1=50$ watts, $L_2=$ 25 watts, and $L_3=$ 75 watts. Branch b also has 3 lamps in parallel with ratings of $L_4=150$ watts, $L_5=200$ watts, and $L_6=250$ watts. The source voltage is 100 volts.

1. Find the current in each lamp.
2. Find the resistance of each lamp.
3. Find the current in branch a.
4. Find the current in branch b.
5. Find the total circuit current.
6. Find the total circuit resistance.
7. Find the total power supplied to the circuit.
8. Check 7 by a separate calculation.

1. The current in L_1 is $I=\frac{P}{E}=\frac{50}{100}=0.50$ ampere.

   The current in L_2 is $\frac{25}{100}=0.25$ ampere.

   The current in L_3 is $\frac{75}{100}=0.75$ ampere.

   The current in L_4 is $\frac{150}{100}=1.50$ amperes.

   The current in L_5 is $\frac{200}{100}=2.00$ amperes.

   The current in L_6 is $\frac{250}{100}=2.5$ amperes.

2. The resistance of L_1 is $R=\frac{E}{I}=\frac{100}{0.5}=200$ ohms.

   The resistance of L_2 is $\frac{100}{0.25}=400$ ohms.

   The resistance of L_3 is $\frac{100}{0.75}=133$ ohms.

   The resistance of L_4 is $\frac{100}{1.5}=66.7$ ohms.

   The resistance of L_5 is $\frac{100}{2.0}=50$ ohms.

   The resistance of L_6 is $\frac{100}{2.5}=40$ ohms.

3. The current in branch *a* is

$$I_1+I_2+I_3=0.5+0.25+0.75=1.5 \text{ amperes.}$$

4. The current in branch *b* is

$$I_4+I_5+I_6=1.5+2.0+2.5=6.0 \text{ amperes.}$$

5. The total circuit current is

$$I_a+I_b=1.5-6.0=7.5 \text{ amperes.}$$

6. The total circuit resistance is

$$R_t=\frac{E}{I_t}=\frac{100}{7.5}=13.3 \text{ ohms.}$$

7. The total power supplied to the circuit is

$$50 \text{ w}+25 \text{ w}+75 \text{ w}+150 \text{ w}+200 \text{ w}+250 \text{ w}=750 \text{ watts.}$$

8. The total power is also equal to

$$P_t=EI_t=100\times 7.5=750 \text{ watts.}$$

PRACTICAL APPLICATIONS OF SERIES AND PARALLEL CIRCUITS

One of the most common series circuits is the familiar string of Christmas tree lamps. When one of the lamps in the string burns out all of the lamps go out because there is no longer a complete pathway through which the electrons can flow.

If the string is composed of 10 lamps of the same power rating and it is connected to a 120-volt source, 12 volts will be applied across each lamp. If one of the lamps burns out and the line is then shorted across the socket of the burned-out lamp, current will continue to flow through the remaining 9 lamps. However, the source voltage of 120 volts will be applied to the remaining lamps, and 13.3 volts will be developed across each lamp. The resultant increase in current flow may quickly burn out another lamp in the string.

Airport runway lamps and street lamps are other applications of series circuits. For these applications, constant-current variable-voltage sources are employed. Where long circuits like these are used, line losses are kept to a minimum by the use of high voltage

and low current. When one of the lamps burns out, a device at the lamp automatically shorts out the defective lamp, thus allowing the others to continue to burn. At the variable-voltage source there is an automatic reduction in voltage when the current tends to increase so that the rise in current through the remaining lamps is checked and the circuit current remains approximately constant at the rated value.

Most power and light distribution circuits are basically parallel circuits supplied by constant-potential variable-current sources having low impedance.

Ship's service distribution systems are typical parallel circuits where many branch circuits are connected in parallel across the bus bars (the massive conductors that tie to the source).

Distribution systems used in homes and factories are essentially networks of parallel circuits. The service conductors (the wires that bring the current to the home) are connected to the branch parallel circuits via a service switch and fuses. Each parallel circuit that is connected to the main circuit is likewise supplied with a fuse or circuit breaker of a lower current rating. On each individual branch circuit lamps or other loads are connected in parallel. Thus, in the home there are branch parallel circuits in parallel with other branch parallel circuits. Actually, nearly all the commonly encountered distribution electrical circuits are parallel circuits if the line resistance is disregarded.

QUIZ

1. What feature distinguishes a series circuit from a parallel circuit?
2. What is Kirchhoff's law of voltages as applied to the simple series circuit?
3. In applying Kirchhoff's voltage law to a series circuit, how are the algebraic signs of the source emf and voltage drops across loads determined?
4. If the assumed direction of electron flow is incorrect, how will the result be affected when Kirchhoff's voltage law is applied to determine the magnitude of current flow in a series circuit?
5. If in the circuit of figure 4–1, resistor $R2$ is replaced by one having a value of 40 ohms, what will be the resultant voltage drop across $R1$, $R2$, and $R3$?
6. If resistor $R2$ is 40 ohms (fig. 4–1), how much power is absorbed in $R1$, $R2$, and $R3$?

7. What current will flow through the circuit of figure 4–1, if a 15-volt battery is inserted in series opposition to the 30-volt battery?
8. If in the circuit of figure 4–2, A, the terminals of E_{s2} are reversed, what will be the resultant line current, I?
9. If in the circuit of figure 4–2, B, the terminals of E_{s1} are reversed, what will be the resultant current in $R4$?
10. If the voltage in the circuit of figure 4–3, A, is reduced to 10 volts, how much current will flow through the circuit?
11. In the circuit of figure 4–3, A, what value of source voltage is necessary to establish a power loss of 100 watts in $R3$?
12. If in figure 4–3, B, the voltage of E_{s3} is increased from 30 volts to 78 volts, what will be the power dissipated in $R1$, $R2$, and $R3$?
13. In the circuit of figure 4–4, if the 35*L*6 is replaced by a 50*L*6, will it then be necessary to employ $R6$?
14. If 6-volt tubes of the same current rating are used throughout the circuit shown in figure 4–4, (a) what value of resistance would be necessary for $R6$ and (b) what wattage would be dissipated in it?
15. How does a high internal source resistance affect the voltage and power made available to a load?
16. If the terminals of a voltage source are short-circuited, where does the voltage drop occur?
17. Under what circumstances is there a maximum transfer of energy from the source to the load?
18. Under what circumstances does the efficiency of power transfer from the source to the load approach 100 percent?
19. How is the combined resistance of a parallel circuit affected when more resistors are added in parallel?
20. If in figure 4–7, resistor $R2$ is replaced by one having a value of 60 ohms, what will be the value of I_t?
21. If in figure 4–7, resistor $R2$ is replaced by one having a value of 60 ohms, what will be the total wattage consumed?
22. What is the equivalent resistance of three 3-ohm resistors connected in parallel?
23. What is the equivalent resistance of a 6-ohm resistor and a 12-ohm resistor connected in parallel?
24. State Kirchhoff's current law.
25. Assume that a string of ten 12-volt Christmas-tree lamps, are connected in series across a 120-volt source. If one additional 12-volt lamp is added to the string, what will be the voltage drop across each lamp?

26. If each lamp in the 10-lamp string dissipates 10 watts, what will each lamp dissipate if eleven lamps are used?
27. Why are airport lamps and street lamps connected in series?
28. What type of circuit (series or parallel) is commonly used to distribute power within the home?

CHAPTER 5

DIRECT-CURRENT COMPOUND AND BRIDGE CIRCUITS

Direct-current compound circuits contain resistors that are connected in series AND in parallel. Bridge circuits contain series-parallel groups that are connected by a common bridge. The bridge frequently takes the form of a meter in certain test circuits, as in the Wheatstone bridge.

Frequently the technician will encounter combinations of series and parallel arrangements. He will need to know how to recognize and to segregate these combinations. Then by the application of Ohm's and Kirchhoff's laws and the power formulas, circuit problems may be solved with ease.

SERIES-PARALLEL COMBINATIONS

Three or more resistors may be connected in series and parallel combinations to form a compound circuit. Two basic series-parallel circuits composed of three resistors are shown in figure 5–1. In figure 5–1, A, $R1$ is connected in series with the parallel combination made up of $R2$ and $R3$.

The total resistance, R_t, of figure 5–1, A, is determined in two steps. First, the resistance, $R_{2,\,3}$, of the parallel combination of $R2$ and $R3$ is determined as

$$R_{2,3}=\frac{R_2R_3}{R_2+R_3}=\frac{3\times 6}{3+6}=\frac{18}{9}=2 \text{ ohms.}$$

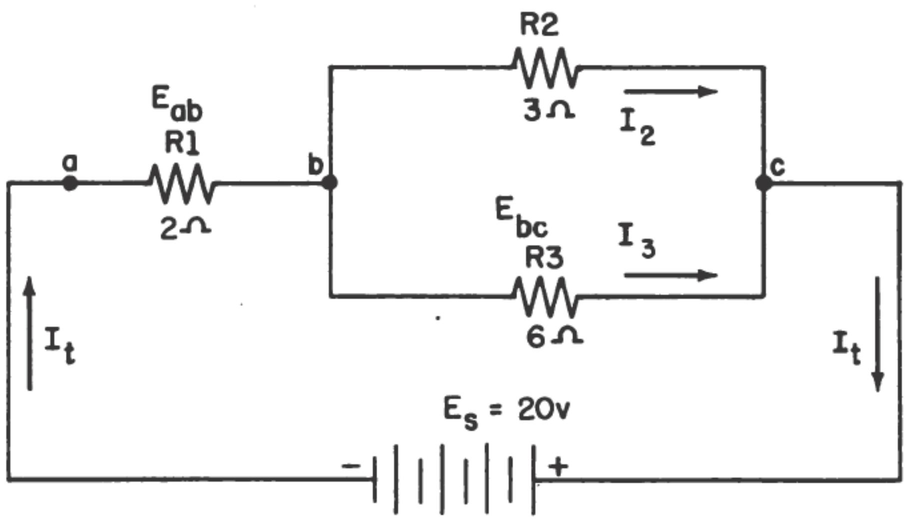
The sum of $R_{2,\,3}$ and R_1—that is, R_t—is

$$R_t=R_{2,3}+R_1=2+2=4 \text{ ohms.}$$

If the total resistance, R_t, and the source voltage, E_s, are known, the total current, I_t, may be determined by Ohm's law. Thus, in figure 5–1, A,

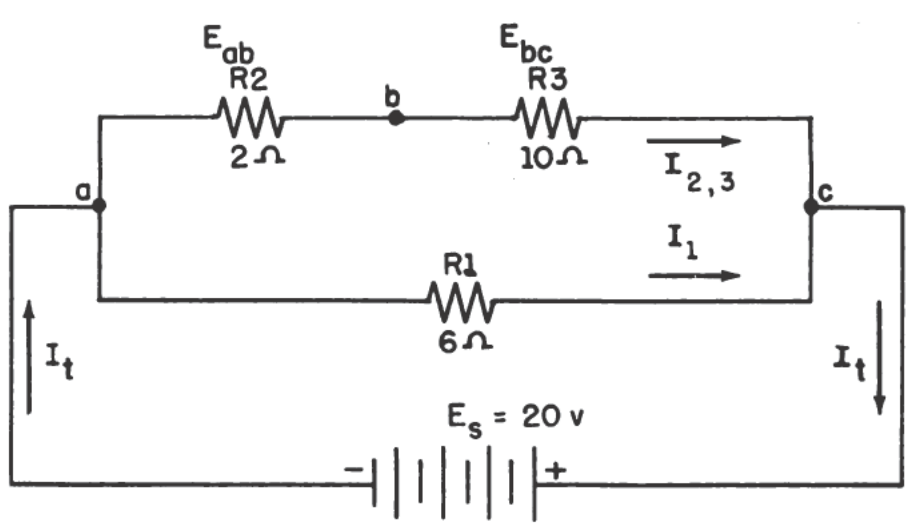
$$I_t=\frac{E_s}{R_s}=\frac{20}{4}=5 \text{ amperes.}$$

If the values of the various resistors and the current through them are known, the voltage drops across the resistors may be determined by Ohm's law. Thus, in figure 5–1, A,



R1 IN SERIES WITH PARALLEL COMBINATION OF R2 AND R3

A



R1 IN PARALLEL WITH THE SERIES COMBINATION OF R2 & R3

B

Figure 5–1.—Compound circuits—series-parallel connections.

$$E_{ab}=I_tR_1=5\times2=10 \text{ volts},$$

and

$$E_{bc}=I_tR_{2,3}=5\times2=10 \text{ volts}.$$

According to Kirchhoff's voltage law, the sum of the voltage drops around the closed circuit is equal to the source voltage. Thus,

$$E_{ab}+E_{bc}=E_s,$$

or

$$10+10=20 \text{ volts}.$$

If the voltage drop, E_{bc}, across $R_{2,3}$—that is, the drop between points b and c—is known, the current through the individual branches may be determined as

$$I_2=\frac{E_{bc}}{R_2}=\frac{10}{3}=3.333 \text{ amperes},$$

and

$$I_3=\frac{E_{bc}}{R_3}=\frac{10}{6}=1.666 \text{ amperes}.$$

According to Kirchhoff's current law, the sum of the currents flowing in the individual parallel branches is equal to the total current. Thus,

$$I_2+I_3=I_t,$$

or

$$3.333+1.666=5 \text{ amperes (approx.)}.$$

The total current flows through $R1$; and at point b it divides between the two branches in inverse proportion to the resistance of the branches. Twice as much goes through $R2$ as through $R3$ because $R2$ has one-half the resistance of $R3$. Thus, 3.333, or two-thirds of 5, amperes flow through $R2$; and 1.666, or one-third of 5, amperes flow through $R3$.

In figure 5–1, B, $R1$ is in parallel with the series combination of $R2$ and $R3$. The total resistance, R_t, is determined in two steps. First, the sum of the resistance of $R2$ and $R3$—that is, $R_{2,3}$—is determined as

$$R_{2,3}=R_2+R_3=2+10=12 \text{ ohms}.$$

Second, the total resistance, R_t, is the result of combining $R_{2,3}$ in parallel with $R1$, or

$$R_t = \frac{R_{2,3}R_1}{R_{2,3}+R_1} = \frac{12\times 6}{12+6} = 4 \text{ ohms.}$$

If the total resistance, R_t, and the source voltage, E_s, are known, the total current, I_t may be determined by Ohm's law. Thus, in figure 5–1, B,

$$I_t = \frac{E_s}{R_t} = \frac{20}{4} = 5 \text{ amperes.}$$

A portion of the total current flows through the series combination of $R2$ and $R3$ and the remainder flows through $R1$. Because current varies inversely with the resistance, two-thirds of the total current flows through $R1$ and one-third flows through the series combination of $R2$ and $R3$, since R_1 is one-half of R_2+R_3.

The source voltage, E_s, is applied between points a and c, and therefore the current I_1 through $R1$ is

$$I_1 = \frac{E_s}{R_1} = \frac{20}{6} = 3.333 \text{ amperes;}$$

and the current, $I_{2,3}$, through $R_{2,3}$ is

$$I_{2,3} = \frac{E_s}{R_{2,3}} = \frac{20}{12} = 1.666 \text{ amperes.}$$

According to Kirchhoff's current law the sum of the individual branch currents is equal to the total current, or

$$I_t = I_1 + I_{2,3},$$

$$5 = 3.333 + 1.667.$$

Compound circuits may be made up of a number of resistors arranged in numerous series and parallel combinations. In more complicated circuits, special theorems, rules, and formulas are used. These are based on Ohm's law and provide faster solutions for particular applications. Series formulas are applied to the

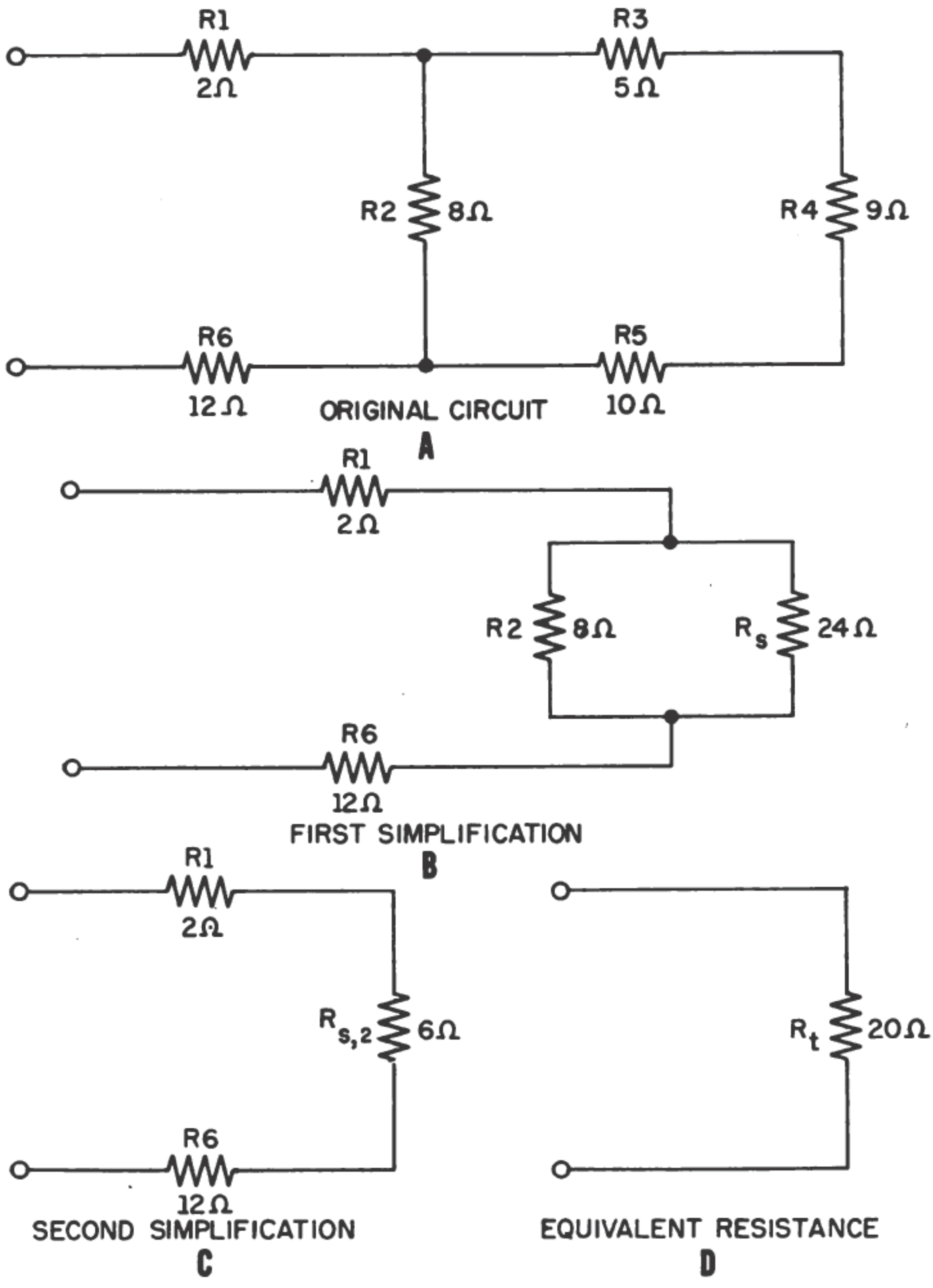


Figure 5–2.—Method of solving for the total resistance of a compound circuit.

series parts of the circuit, and parallel formulas are applied to the parallel parts. For example, in figure 5–2, the total resistance, R_t, may be obtained in three logical steps.

First, $R3$, $R4$, and $R5$ in figure 5–2, A, are in series (there is

only one path for current) and may be combined in figure 5–2, **B**, to give the resistance, R_s, of the three resistors. Thus,

$$R_s = R_3 + R_4 + R_5 = 5 + 9 + 10 = 24 \text{ ohms},$$

and it is now in parallel with $R2$ (because they both receive the same voltage).

The combined resistance of R_s in parallel with $R2$ is

$$R_{s,2} = \frac{R_2 R_s}{R_2 + R_s} = \frac{8 \times 24}{8 + 24} = 6 \text{ ohms},$$

as in figure 5–2, **C**.

Third, the total resistance, R_t, is determined by combining resistors $R1$ and $R6$ with $R_{s,2}$, as

$$R_t = R_1 + R_6 + R_{s,2} = 2 + 12 + 6 = 20 \text{ ohms}.$$

Other compound circuits may be solved in a similar manner. For example, in figure 5–3, the equivalent resistance, R_t, may be found by simplifying the circuit in successive steps beginning with the resistances, R_1 and R_2. Thus,

$$R_{1,2} = \frac{R_1 R_2}{R_1 + R_2} = \frac{3 \times 6}{3 + 6} = \frac{18}{9} = 2 \text{ ohms},$$

and it is in series with $R3$.

The resistances, $R_{1,2}$ and R_3, are added to give the resultant resistance, $R_{1,2,3}$. Thus,

$$R_{1,2,3} = R_{1,2} + R_3 = 2 + 4 = 6 \text{ ohms}.$$

$R_{1,2,3}$ is in parallel with $R4$. The combined resistance, $R_{1,2,3,4}$, is determined as

$$R_{1,2,3,4} = \frac{R_{1,2,3} R_4}{R_{1,2,3} + R_4} = \frac{6 \times 12}{6 + 12} = \frac{72}{18} = 4 \text{ ohms}.$$

This equivalent resistance is in series with $R5$. Thus, the total resistance, R_t, of the circuit is

$$R_t = R_{1,2,3,4} + R_5 = 4 + 8 = 12 \text{ ohms}.$$

By Ohm's law, the line current, I_t, is

$$I_t = \frac{E_s}{R_t} = \frac{54}{12} = 4.5 \text{ amperes.}$$

The line current flows through $R5$ and therefore the voltage drop, E_5, across $R5$ is

$$E_5 = I_t R_5 = 4.5 \times 8 = 36 \text{ volts.}$$

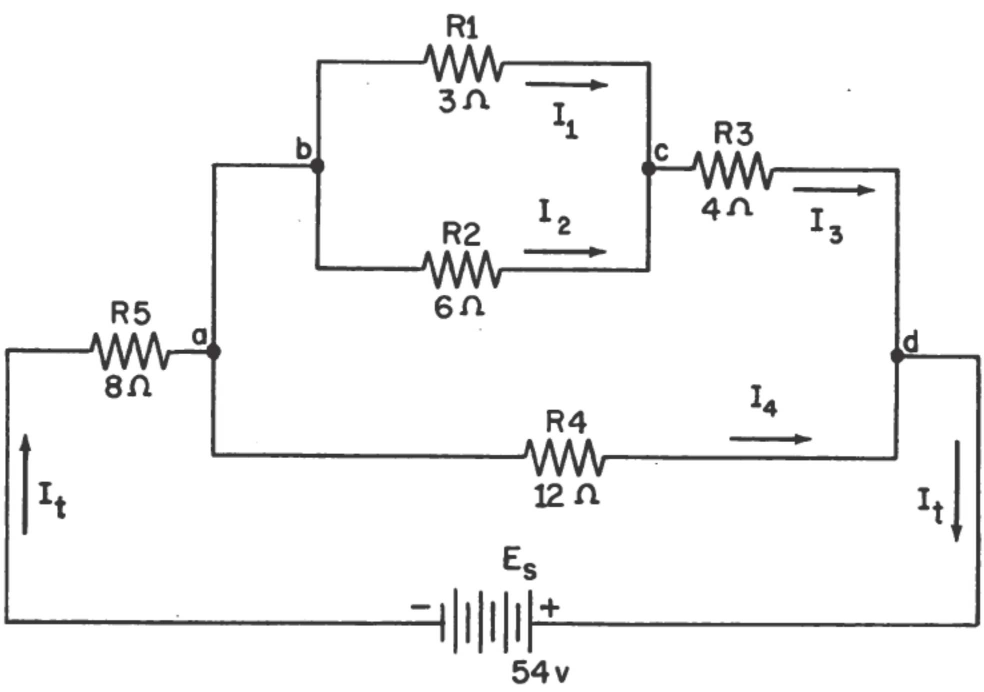


Figure 5–3.—Solving for resistance, voltage, current, and power in a compound circuit.

According to Kirchhoff's voltage law, the sum of the voltage drops around the circuit is equal to the source voltage; accordingly, the voltage between points *a* and *d* is

$$E_{ad} = E_s - E_5 = 54 - 36 = 18 \text{ volts.}$$

The current through $R4$ is

$$I_4 = \frac{E_4}{R_4} = \frac{18}{12} = 1.5 \text{ amperes.}$$

The resistance, $R_{1,2,3}$, of parallel resistors $R1$ and $R2$ in series with resistor $R3$ is 6 ohms. E_{ad} is applied across 6 ohms; therefore the current, I_3, through $R3$ is

$$I_3=\frac{E_{ad}}{R_{1,2,3}}=\frac{18}{6}=3 \text{ amperes.}$$

The voltage drop, E_3, across $R3$ is

$$E_3=I_3R_3=3\times4=12 \text{ volts,}$$

and the voltage across the parallel combination of $R1$ and $R2$—that is, E_{bc}—is

$$E_{bc}=I_{1,2}R_{1,2}=3\times2=6 \text{ volts,}$$

where $I_{1,2}$ is the current through the parallel combinations of R1 and $R2$. By Kirchhoff's current law, $I_{1,2}$ is equal to I_3. The current, I_1 through $R1$ is

$$I_1=\frac{E_{bc}}{R_1}=\frac{6}{3}=2 \text{ amperes,}$$

and the current, I_2, through $R2$ is

$$I_2=\frac{E_{bc}}{R_2}=\frac{6}{6}=1 \text{ ampere.}$$

The preceding computations may be checked by the application of Kirchhoff's voltage and current law to the entire circuit. Briefly, the sum of the voltage drops around the circuit is equal to the source voltage. Voltage E_5 across $R5$ is 36 volts and voltage E_{ad} across $R4$ is 18 volts—that is,

$$E_s=E_5+E_{ad},$$

or

$$54=36+18 \text{ volts.}$$

Likewise, the voltage drop, E_{bc}, across the parallel combination of $R1$ and $R2$ plus the voltage drop, E_3, across $R3$ should be equal to the voltage across points a and d. E_{bc} is 6 volts and E_3 is 12 volts. Therefore,

$$E_{ad}=E_{bc}+E_3=6+12=18 \text{ volts.}$$

Kirchhoff's current law says in effect that the sum of the branch currents is equal to the line current, I_t. The line current is 4.5

amperes, and therefore the sum of I_4 and I_3 should be 4.5 amperes, or

$$I_t = I_4 + I_3 = 1.5 + 3 = 4.5 \text{ amperes.}$$

The power consumed in a circuit element is determined by one of the three power formulas. For example, in figure 5–3 the power, P_1, consumed in $R1$ is

$$P_1 = I_1 E_{bc} = 2 \times 6 = 12 \text{ watts;}$$

the power P_2 consumed in $R2$ is

$$P_2 = I_2 E_{bc} = 1 \times 6 = 6 \text{ watts;}$$

the power P_3 consumed in $R3$ is

$$P_3 = I_3 E_3 = 3 \times 12 = 36 \text{ watts;}$$

the power P_4 consumed in $R4$ is

$$P_4 = I_4 E_4 = 1.5 \times 18 = 27 \text{ watts;}$$

and the power P_5 consumed in $R5$ is

$$P_5 = I_5 E_5 = 4.5 \times 36 = 162 \text{ watts.}$$

The total power, P_t, consumed is

$$\begin{aligned} P_t &= P_1 + P_2 + P_3 + P_4 + P_5 \\ &= 12 + 6 + 36 + 27 + 162 \\ &= 243 \text{ watts.} \end{aligned}$$

The total power is also equal to the total current multiplied by the source voltage, or

$$P_t = I_t E_s = 4.5 \times 54 = 243 \text{ watts.}$$

VOLTAGE DIVIDERS

A basic voltage divider may consist of a series of resistors having two input terminals, across which the source voltage is applied, and two or more output terminals across which the desired fraction of the total voltage is obtained. Voltage dividers are frequently used across the output of power supplies to provide a variety of voltages for electron tubes. The values of resistance to

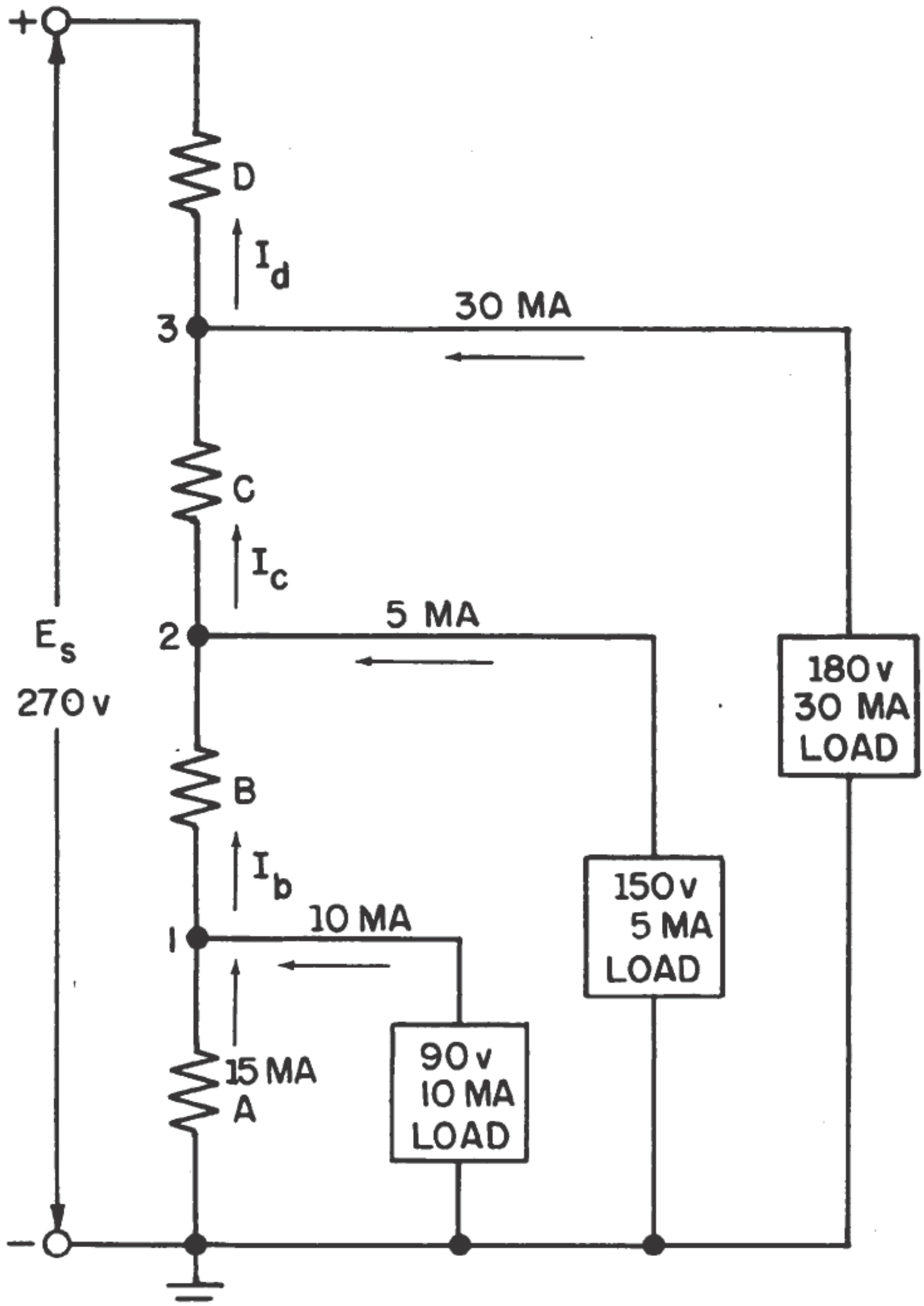


Figure 5–4.—Voltage divider—determination of *R* and *P*.

be used are determined by applying Ohm's law and Kirchhoff's laws.

A voltage divider circuit is shown in figure 5–4. The divider is connected across a 270-volt source and supplies three loads simultaneously—10 ma (one MILLIAMPERE is 0.001 ampere) at 90 volts, between terminal 1 and ground; 5 ma at 150 volts, between terminal 2 and ground; and 30 ma at 180 volts, between terminal 3 and ground. The current in resistor A is 15 ma. The current, voltage, resistance, and power of the 4 resistors are to be determined.

Kirchhoff's law of currents applied to terminal 1 indicates that the current in resistor B is equal to the sum of 15 ma from resistor A and 10 ma from the 90-volt load. Thus,

$$I_b = 15 + 10, \text{ or } 25 \text{ ma.}$$

Similarly,

$$I_c = 25 + 5 = 30 \text{ ma,}$$

and

$$I_d = 30 + 30 = 60 \text{ ma.}$$

Kirchhoff's voltage law indicates that the voltage across resistor A is 90 volts; the voltage across B is

$$E_b = 150 - 90, \text{ or } 60 \text{ volts;}$$

the voltage across C is

$$E_c = 180 - 150, \text{ or } 30 \text{ volts;}$$

and the voltage across D is

$$E_d = 270 - 180, \text{ or } 90 \text{ volts.}$$

Before solving for the various resistances it should be recalled that in the formula, $R = \frac{E}{I}$, R will be in ohms if E is in volts and I is in amperes. In many electronic circuits, particularly those being considered, it is just as valid and considerably simpler to let R be in thousands of ohms (k-ohms), E in volts, and I in milliamperes. In the following formulas this convention will be followed.

373023°—56——8

Applying Ohm's law to determine the resistances—

$$\text{resistance of } A \text{ is } R_a=\frac{E_a}{I_a}=\frac{90}{15}=6 \text{ k-ohms},$$

$$\text{resistance of } B \text{ is } R_b=\frac{E_b}{I_b}=\frac{60}{25}=2.4 \text{ k-ohms},$$

$$\text{resistance of } C \text{ is } R_c=\frac{E_c}{I_c}=\frac{30}{30}=1 \text{ k-ohm},$$

$$\text{resistance of } D \text{ is } R_d=\frac{E_d}{I_d}=\frac{90}{60}=1.5 \text{ k-ohms}.$$

The power absorbed by

$$\text{resistor } A \text{ is } P_a=E_aI_a=90\times0.015=1.35 \text{ watts},$$

$$\text{resistor } B \text{ is } P_b=E_bI_b=60\times0.025=1.50 \text{ watts},$$

$$\text{resistor } C \text{ is } P_c=E_cI_c=30\times0.030=0.90 \text{ watts},$$

$$\text{resistor } D \text{ is } P_d=E_dI_d=90\times0.060=5.40 \text{ watts}.$$

The total power absorbed by the 4 resistors is

$$1.35+1.50+0.90+5.40=9.15 \text{ watts}.$$

The power absorbed by the load connected to

$$\text{terminal 1 is } P_1=E_1I_1=90\times0.010=0.90 \text{ watt},$$

$$\text{terminal 2 is } P_2=E_2I_2=150\times0.005=0.75 \text{ watt},$$

$$\text{terminal 3 is } P_3=E_3I_3=180\times0.030=5.4 \text{ watts}.$$

The total power supplied to the 3 loads is

$$0.90+0.75+5.4=7.05 \text{ watts}.$$

The total power supplied to the entire circuit including the voltage divider and the 3 loads is

$$9.15+7.05=16.2 \text{ watts}.$$

This value is checked as

$$P_t=E\times I_t=270\times0.060=16.2 \text{ watts}.$$

In figure 5–5 the voltage divider resistances are given and the current in $R5$ is to be found. The load current in $R1$ is 6 ma; the current in $R2$ is 4 ma; and the current in $R3$ is 10 ma. The source voltage is 510 volts. Kirchhoff's current law may be applied at the junctions a, b, c, and d to determine expressions for

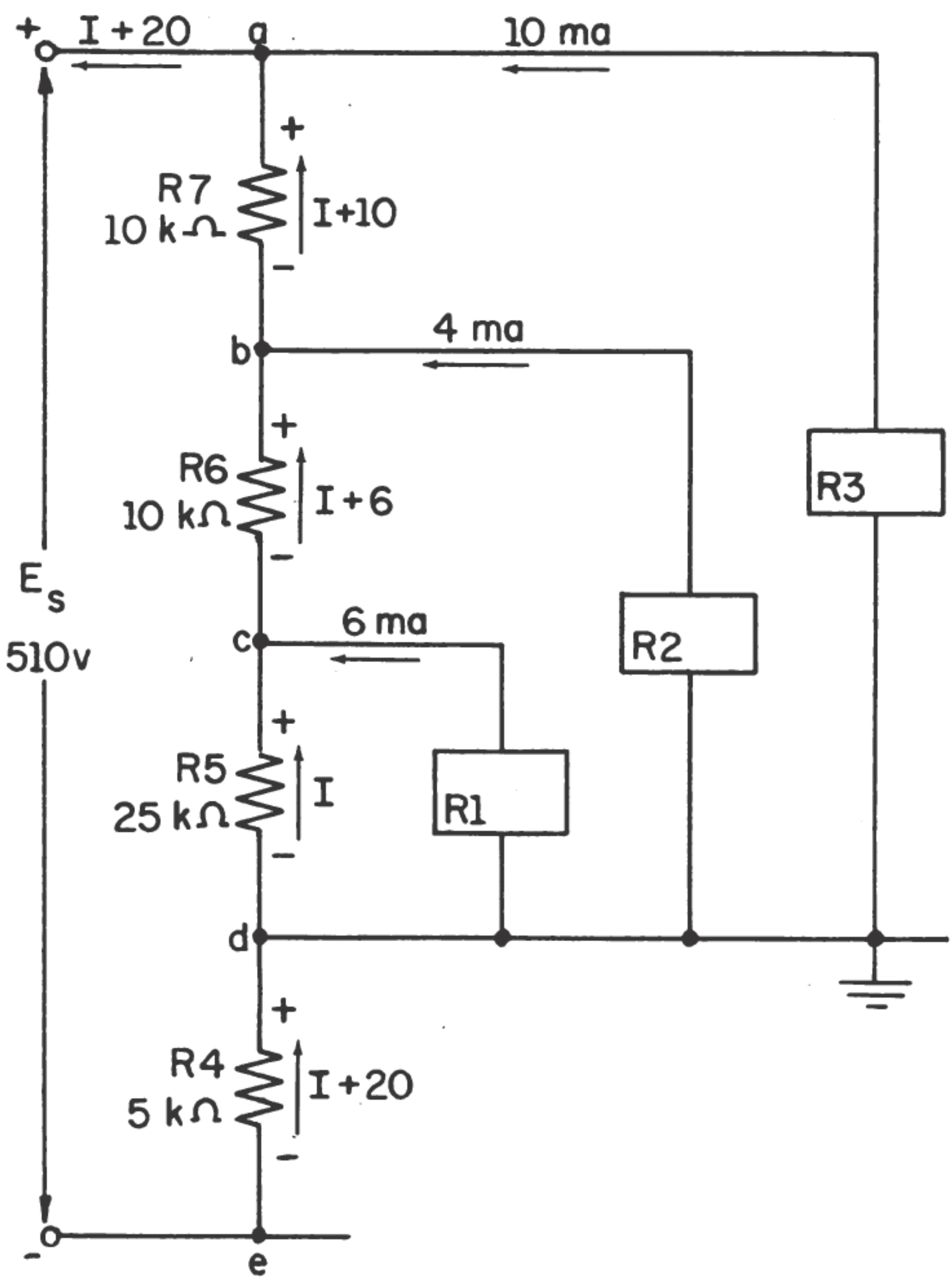


Figure 5–5.—Voltage divider—determination of *E* and *R*.

the current in resistors $R4$, $R5$, $R6$, and $R7$. Accordingly, the current in $R4$ is $I+6+4+10$, or $I+20$; the current in $R5$ is I; the current in $R6$ is $I+6$; the current in $R7$ is $I+6+4$, or $I+10$.

The voltage across $R4$ may be expressed in terms of the resistance in k-ohms and the current in milliamperes as $5(I+20)$ volts. Similarly, the voltage across $R5$ is equal to $25I$; the voltage across $R6$ is $10(I+6)$ and the voltage across $R7$ is $10(I+10)$. Kirchhoff's law of voltages may be applied to the voltage divider to solve for the unknown current, I, by expressing the source voltage in terms of the given values of voltage resistance and current (both known and unknown values). The sum of the voltages across $R4$, $R5$, $R6$, and $R7$ is equal to the source voltage as follows:

$$\begin{aligned} E_4+E_5+E_6+E_7&=E_s \\ 5(I+20)+25I+10(I+6)+10(I+10)&=510 \\ 50I&=250 \\ I&=5 \text{ ma.} \end{aligned}$$

The current of 5 ma through $R5$ produces a voltage drop across $R5$ of 5×25, or 125 volts. Since $R1$ is in parallel with $R5$, the voltage across load $R1$ is 125 volts. The current through $R4$ is $5+20$, or 25 ma and the corresponding voltage is 5×25, or 125 volts. Since point *d* is at ground potential, point *c* is 125 volts positive with respect to ground, whereas point *e* is 125 volts negative with respect to ground. The current in $R6$ is $5+6$ or 11 ma, and the voltage drop across $R6$ is 11×10, or 110 volts. The current in $R7$ is $5+10$, or 15 ma, and the voltage drop is 15×10, or 150 volts. The total voltage is the sum of the voltages across the divider. Thus,

$$125+125+110+150=510.$$

The power absorbed by each resistor in the voltage divider may be found by multiplying the voltage across the resistor by the current in the resistor. If the current is expressed in amperes and the emf in volts the power will be expressed in watts. Thus the power in $R4$ is

$$P_4=E_4I_4=125\times 0.025=3.125 \text{ watts.}$$

Similarly the power in $R5$ is $125\times 0.005=0.625$ watts; the power

in $R6$ is $110 \times 0.011 = 1.21$ watts; and in $R7$ is $150 \times 0.015 = 2.25$ watts. The total power in the divider is

$$3.125 + 0.625 + 1.21 + 2.25 = 7.21 \text{ watts.}$$

The voltage across load $R1$ is the voltage across $R5$, or 125 volts. The power in $R1$ is

$$P_1 = E_1 I_1 = 125 \times 0.006 = 0.750 \text{ watts.}$$

The voltage across load $R2$ is equal to the sum of the voltages across $R5$ and $R6$. Thus,

$$E_2 = E_5 + E_6 = 125 + 110 = 235 \text{ volts.}$$

The power in load $R2$ is

$$P_2 = E_2 I_2 = 235 \times 0.004 = 0.940 \text{ watts.}$$

The voltage across load $R3$ is equal to the sum of the voltages across $R5$, $R6$, and $R7$. Thus,

$$E_3 = E_5 + E_6 + E_7 = 125 + 110 + 150 = 385 \text{ volts.}$$

The power in load $R3$ is

$$P_3 = E_3 I_3 = 385 \times 0.010 = 3.85 \text{ watts.}$$

The total power in the three loads is

$$0.75 + 0.94 + 3.85 = 5.54 \text{ watts,}$$

and the total power supplied by the source is equal to the sum of the power absorbed by the voltage divider and the three loads, or

$$7.21 + 5.54 = 12.75 \text{ watts.}$$

The total power may be checked by

$$P_t = E_t I_t = 510 \times 0.025 = 12.75 \text{ watts.}$$

The resistances of load resistors $R1$, $R2$, and $R3$ are determined by means of Ohm's law as follows:

$$R_1 = \frac{E_1}{I_1} = \frac{125}{6} = 20.83 \text{ k-ohms,}$$

$$R_2=\frac{E_2}{I_2}=\frac{235}{4}=58.75 \text{ k-ohms},$$

and

$$R_3=\frac{E_3}{I_3}=\frac{385}{10}=38.5 \text{ k-ohms.}$$

ATTENUATORS

Attenuators are networks of resistors that are used to reduce (or attenuate) voltage, current, or power delivered to a load. Two of the simpler types of attenuators are the **L** and **T** types shown in figure 5–6.

If the device is adjustable, as shown, it is called an "attenuator;" if it is nonadjustable it is commonly called a "pad."

The **L**-type attenuator maintains constant resistance at ONE pair of terminals for any setting of the variable resistors. For example, in figure 5–6, A, the resistance presented by the load and the attenuator to terminals *ab* will be constant for any setting of ganged resistors $R1$ and $R2$.

In figure 5–6, A, the resistance offered to the flow of current through the terminals *a* and *b* includes (1) R_s and (2) R_1 acting in series with the parallel combination of R_2 and R_L. In this circuit $R_s=R_L$, and

$$R_s=R_1+\frac{R_2R_L}{R_2+R_L}.$$

The resistance offered to the flow of current through terminals *c* and *d* includes (1) R_2 acting in parallel with the series combination of R_1 and R_s, and (2) R_L. The **L** attenuator operates properly only if the load is connected to terminals *cd* and the source to terminals *ab*. The load and the source are not interchangeable, because R_L is not equal to

$$\frac{R_2\ (R_1+R_s)}{R_2+R_1+R_s}.$$

The **T**-type attenuator, shown in figure 5–6, B, maintains at BOTH pairs of terminals a constant resistance for any setting of the ganged variable resistors. As may be seen in the figure, an extra variable resistor is needed in the **T**-type attenuator, and the circuit is therefore more complicated than the **L**-type attenuator circuit.

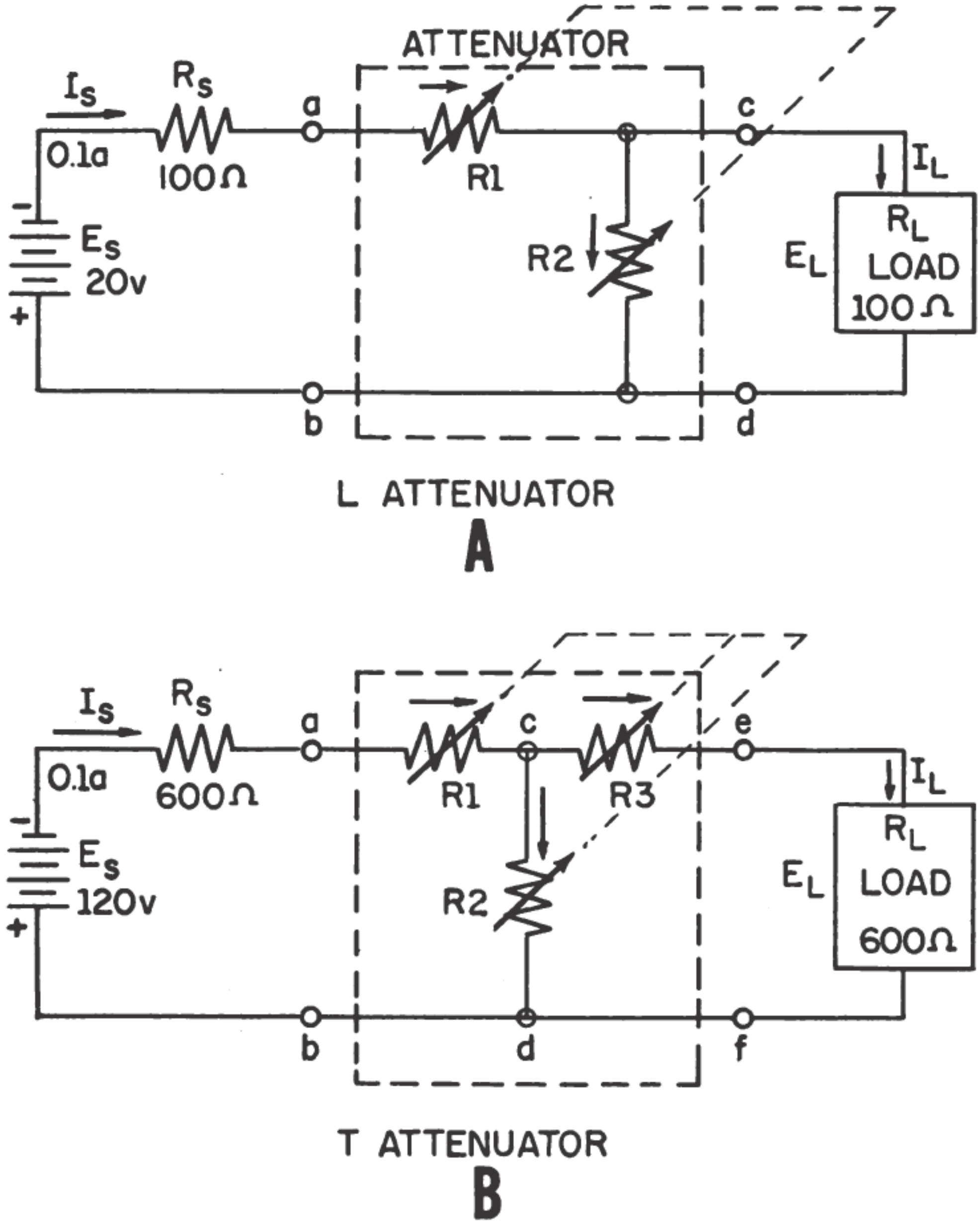


Figure 5–6.—L and T attenuators.

In the T attenuator the resistance offered to the flow of current through terminals *a* and *b* includes (1) R_s and (2) R_1 acting in series with the parallel combination, one branch of which is R_2 and the other branch of which is R_3 acting in series with R_L. In this circuit,

$$R_s = R_1 + \frac{R_2\,(R_3 + R_L)}{R_2 + R_3 + R_L}.$$

The resistance offered to the flow of current through terminals *e* and *f* includes (1) R_3 in series with the parallel combination, one branch of which is R_2 and the other branch of which is R_1 acting in series with R_s and (2) R_L. In this circuit,

$$R_L = R_3 + \frac{R_2(R_1 + R_s)}{R_2 + R_1 + R_3}.$$

Because $R_s = R_L$ and $R_1 = R_3$, the load and the source are interchangeable without affecting the operation of the **T** attenuator. Thus, the resistance offered to the flow of current through terminals *a* and *b* is equal to the resistance offered to the flow of current through terminals *e* and *f*.

There are other types of attenuators that are even more complicated—for example, ladder attenuators, so called because the circuit looks like a ladder; bridge-**T** attenuators, in which the two resistors forming the top member of the **T** are paralleled, or bridged, by another resistor; and decimal attenuators, in which the resistors are arranged so that the current or voltage may be reduced in steps equal to decimal fractions of the full-load values.

It is often desirable, especially in communication circuits, to lower the voltage applied to a load in order to attenuate the signal, and yet to permit the same amount of current to be drawn from the source as was drawn before the voltage across the load was lowered. Thus the load on the source remains constant and its characteristics are not altered when the load signal is weakened. The **L**-type of attenuator is one of the simplest circuits that may be used to accomplish this result.

L Attenuator

In the **L**-type attenuator shown in figure 5–6, A, the two objectives are: (1) To vary the load voltage, and (2) to maintain constant current through the source when the load voltage is varied. The source resistance is 100 ohms, the load resistance is 100 ohms, and the source voltage is 20 volts. Before the attenuator is inserted, the current delivered to the load is

$$I_s = \frac{E_s}{R_L + R_s} = \frac{20}{100 + 100} = 0.1 \text{ ampere.}$$

The load voltage in this case is $E_L = I_s R_L = 0.1 \times 100 = 10$ volts.

The voltage lost in the source is $20-10=10$ volts, which is the equivalent of 0.1 ampere flowing through the source resistance of 100 ohms.

To reduce the voltage across R_L in figure 5–6, A, to 5 volts and maintain the source current at 0.1 ampere, $R1$ and $R2$ are inserted. The current, I_L, through R_L is

$$I_L=\frac{E_L}{R_L}=\frac{5}{100}=0.05 \text{ ampere.}$$

The current through $R1$ is 0.1 ampere, since it is in series with the source, and the voltage drop across $R1$ is the difference between the source voltage and the sum of the voltage drops in the source and across the load, or $20-(10+5)=5$ volts. The resistance of $R1$ is $\frac{5}{0.1}=50$ ohms.

The current through $R2$ is the difference between the source current of 0.1 ampere and the load current of 0.05 ampere, or 0.05 ampere. The voltage across $R2$ is 5 volts because it is in parallel with the load. The resistance of $R2$ is $\frac{5}{0.05}=100$ ohms. Thus, by inserting 50 ohms in series with the source and 100 ohms in shunt with the load, the load voltage is reduced to 5 volts and the source current is maintained at 0.1 ampere. In this way the load on the source is maintained constant while the signal voltage at the load is reduced to one-half its original value.

The parallel resistance of $R2$ and R_L is 50 ohms and the total resistance of $R1$ in series with the parallel combination is $50+50$, or 100 ohms. Thus, the same resistance is presented to the source after the insertion of $R1$ and $R2$ as before, and the load on the source remains the same.

T Attenuator

In the **T** attenuator of figure 5–6, B, $R_s=600$ ohms, $R_L=600$ ohms, and the source voltage is 120 volts. In this example, the attenuator is adjusted to reduce the load voltage to one-half its rated value. The problem is to find the necessary resistance values of $R1$, $R2$, and $R3$. Before the attenuator is inserted, the load current is equal to $\frac{120}{600+600}=0.1$ ampere and the rated load

voltage, E_L, is $0.1 \times 600 = 60$ volts. The attenuator reduces the load voltage to ½ of 60, or 30 volts.

The load current with 30 volts applied to the load will be $\frac{30}{600} = 0.05$ ampere. Since the total current must remain 0.1 ampere, the current through $R2$ will be $0.1 - 0.05 = 0.05$ ampere.

The algebraic sum of the voltages around circuit *dcefd* is equal to zero, and these may be expressed in terms of current, resistance, and voltage as,

$$+0.05R_2 - 0.05R_3 - 30 = 0,$$

and since $R_3 = R_1$,

$$0.05R_2 - 0.05R_1 = 30. \tag{5–1}$$

The algebraic sum of the voltages around circuit *bacdb* is equal to zero. Thus,

$$\begin{aligned} 120 - 0.1 \times 600 - 0.1R_1 - 0.05R_2 &= 0 \\ 0.05R_2 + 0.1R_1 &= 60. \end{aligned} \tag{5–2}$$

Subtracting equation (5–1) from equation (5–2) and solving for R_1,

$$\begin{array}{r} 0.05R_2 + 0.1R_1 = 60 \\ 0.05R_2 - 0.05R_1 = 30 \\ \hline 0 \quad + 0.15R_1 = 30 \end{array}$$

$$R_1 = 200 \text{ ohms} = R_3.$$

Substituting the value of R_1 in equation (5–1),

$$\begin{aligned} 0.05R_2 - 0.05(200) &= 30 \\ 0.05R_2 &= 40 \\ R_2 &= 800 \text{ ohms.} \end{aligned}$$

The resistance to the flow of current through terminals *a* and *b* includes (1) the 600-ohm source and (2) the attenuator resistance, R_{ab}.

$$\begin{aligned} R_{ab} &= R_1 + \frac{R_2(R_3 + R_L)}{R_2 + R_3 + R_L} \\ &= 200 + \frac{800(200 + 600)}{800 + 200 + 600} \\ &= 200 + 400 \\ &= 600 \text{ ohms.} \end{aligned}$$

The resistance to the flow of current through terminals *e* and *f* includes (1) the 600-ohm load and (2) attenuator resistance, R_{ef}.

$$R_{ef}=R_3+\frac{R_3(R_1+R_s)}{R_2+R_1+R_s}$$
$$=200+\frac{800(200+600)}{800+200+600}$$
$$=200+400$$
$$=600 \text{ ohms.}$$

Thus, $R_{ef}=R_{ab}$ and both the load and the source see the same impedance "looking" into the **T** attenuator.
The source current I_s is

$$\frac{E_s}{R_s+R_{ab}}=\frac{120}{600+600}=0.1 \text{ ampere.}$$

The input voltage E_{ab} to the attenuator is

$$I_sR_{ab}=0.1\times600=60 \text{ volts.}$$

The voltage drop across R_1 is

$$I_sR_1=0.1\times200=20 \text{ volts.}$$

The voltage drop across R_2 is

$$I_2R_2=0.05\times800=40 \text{ volts.}$$

The voltage drop across R_3 is

$$I_3R_3=0.05\times200=10 \text{ volts.}$$

Thus, the voltage across the load is

$$40-10=30 \text{ volts.}$$

BRIDGE CIRCUITS

Unbalanced Resistance Bridge

A resistance bridge circuit is shown in figure 5–7. Resistors *R*1, *R*3, *R*4, and *R*5, form the 4 arms of the bridge. The source voltage E_s, is connected between junctions *a* and *c* and resistor

*R*2 is connected between junctions *b* and *d*. The resistance bridge is frequently used to determine the value of an unknown resistance, for example that of *R*5, by balancing (equalizing) the voltage across *R*1 and *R*3, or *R*4 and *R*5. In this case *R*2 may represent the resistance of a galvanometer used to detect the balance. Other forms of bridge circuits are widely used in a-c circuits for the purpose of measuring inductance, capacitance, and impedance.

In the example of figure 5–7 the bridge is not balanced—that is, current flows through *R*2. Resistance $R_1=50\Omega$, $R_2=100\Omega$, $R_3=70\Omega$, $R_4=250\Omega$, and $R_5=150\Omega$. The total current delivered

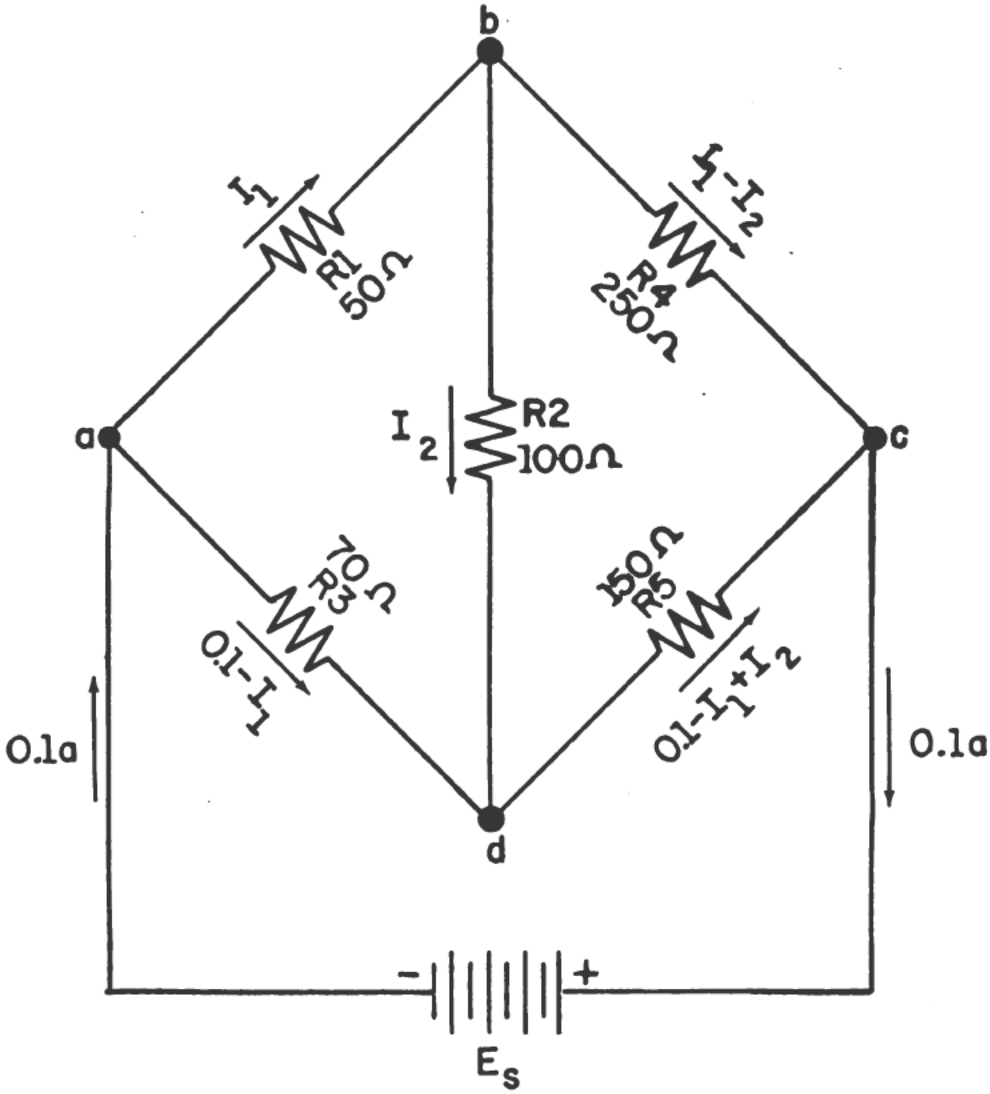


Figure 5–7.—Resistance bridge circuit.

to the bridge is 0.1 ampere. It is desired to find the current flow in each resistor, the voltage drop across each resistor, and the source voltage.

The current of 0.1 ampere flowing into junction *a* divides into two parts. The part flowing through $R1$ is indicated as I_1 and the part through $R3$ is $0.1-I_1$. Similarly, at junction *b*, I_1 divides, part flowing through $R2$ and the remainder through $R4$. The part through $R2$ is designated I_2 and the part through $R4$ is I_1-I_2. The direction of current through $R2$ may be assumed arbitrarily.

If the solution indicates a positive value for I_2 the assumed direction is proved to be correct. The current through $R4$ is I_1-I_2. At junction *d* the currents may be analyzed in a similar manner. Current I_2 through $R2$ joins current $0.1-I_1$, from $R3$; and the current through $R5$ is $0.1-I_1+I_2$.

The unknown currents, I_1 and I_2, may be determined by establishing two voltage equations in which they both appear. These equations are solved for I_1 and I_2 in terms of the given values of current and resistance. The first voltage equation is developed by tracing clockwise around the closed circuit containing resistors $R1$, $R2$, and $R3$. The trace starts at junction *a*, and proceeds to *b*, to *d*, and back to *a*. The algebraic sum of the voltages around this circuit is zero. These voltages are expressed in terms of resistance and current. Going from *a* to *b*, the voltage drop is in the direction of the arrow and is equal to $-50I_1$; the drop across $R2$, going from *b* to *d*, is $-100I_2$; and the voltage from *d* to *a* (in the opposite direction to the arrow) is $+70(0.1-I_1)$. Thus,

$$-50I_1-100I_2+70(0.1-I_1)=0.$$

Multiplying both sides by -1,

$$50I_1+100I_2-70(0.1-I_1)=0,$$

and transposing and simplifying,

$$120I_1+100I_2=7. \qquad (5\text{–}3)$$

The second voltage equation is established by tracing clockwise around the circuit, which includes resistors $R4$, $R5$, and $R2$. Starting at junction *b*, the trace proceeds to *c*, to *d*, and returns to *b*. The voltage across $R4$, from *b* to *c*, is $-250(I_1-I_2)$; the

voltage across $R5$, from c to d, is $+150(0.1-I_1+I_2)$; and the voltage across $R2$, from d to b, is $+100I_2$. Thus,

$$-250(I_1-I_2)+150(0.1-I_1+I_2)+100I_2=0$$

from which,

$$400I_1-500I_2=15. \tag{5-4}$$

Equations (5–3) and (5–4) may be solved simultaneously by multiplying equation (5–3) by the factor 5 and then adding the equations to eliminate I_2 as follows:

$$\begin{array}{r} 400I_1-500I_2=15 \\ 600I_1+500I_2=35 \\ \hline 1{,}000I_1 \qquad\quad =50 \\ I_1=0.05 \text{ ampere.} \end{array}$$

Substituting the value of 0.05 for I_1 in equation (5–3) and solving for I_2,

$$\begin{aligned} 120(0.05)+100I_2&=7 \\ 100I_2&=1 \\ I_2&=0.01 \text{ ampere.} \end{aligned}$$

Thus the current in $R1$ is $I_1=0.05$ ampere. The current in $R2$ is $I_2=0.01$ ampere. The current in $R3$ is $0.1-I_1=0.1-0.05$, or 0.05 ampere. The current in $R4$ is $I_1-I_2=0.05-0.01$, or 0.04 ampere. The current in $R5$ is $0.1-I_1+I_2=0.1-0.05+0.01=0.06$ ampere. The voltages E_1, E_2, E_3, E_4 and E_5 are as follows:

E_1 across $R1$ is $I_1R_1=0.05\times 50=2.5$ volts.
E_2 across $R2$ is $I_2R_2=0.01\times 100=1.0$ volt.
E_3 across $R3$ is $(0.1-I_1)R_3=0.05\times 70=3.5$ volts.
E_4 across $R4$ is $(I_1-I_2)R_4=0.04\times 250=10$ volts.
E_5 across $R5$ is $(0.1-I_1+I_2)R_5=0.06\times 150=9.0$ volts

The source voltage E_s is equal to the sum of the voltages across $R3$ and $R5$ or $R1$ and $R4$.
Thus,

$$\begin{aligned} E_s&=E_1+E_4 \\ &=2.5+10=12.5 \text{ volts} \end{aligned}$$

and

$$E_s = E_3 + E_5$$
$$= 3.5 + 9 = 12.5 \text{ volts}$$

The voltage across $R2$ is the difference in the voltages across $R3$ and $R1$. It is also the difference in the voltages across $R4$ and $R5$.

Wheatstone Bridge

A type of circuit that is widely used for precision measurements of resistance is the Wheatstone bridge. The circuit diagram of a Wheatstone bridge is shown in figure 5–8, A. $R1$, $R2$, and $R3$ are precision variable resistors, and R_x is the resistor whose value of resistance is to be determined. After the bridge has been properly balanced, the unknown resistance may be determined by means of a simple formula. The galvanometer, G, is inserted across terminals b and d to indicate the condition of balance. When the bridge is properly balanced there is no difference in potential across terminals bd, and the galvanometer deflection, when the key is closed, will be zero.

The operation of the bridge is explained in a few logical steps. When the switch to the battery is closed, electrons flow from the negative terminal of the battery to point a. Here the current divides, as it would in any parallel circuit, a part of it passing through $R1$ and $R2$ and the remainder passing through $R3$ and R_x. The two currents, labeled I_1 and I_2, unite at point c and return to the positive terminal of the battery. The value of I_1 depends on the sum of resistances R_1 and R_2, and the value of I_2 depends on the sum of resistances R_3 and R_x. In each case, according to Ohm's law, the current is inversely proportional to the resistance.

$R1$, $R2$, and $R3$ are adjusted so that when the galvanometer switch is closed there will be no deflection of the needle. When the galvanometer shows no deflection there is no difference of potential between points b and d. This means that the voltage drop (E_1) across $R1$, between points a and b, is the same as the voltage drop (E_3) across $R3$, between points a and d. By similar reasoning, the voltage drops across $R2$ and R_x—that is, E_2 and E_x—are also equal. Expressed algebraically,

$$E_1 = E_3,$$

or

$$I_1R_1 = I_2R_3;$$

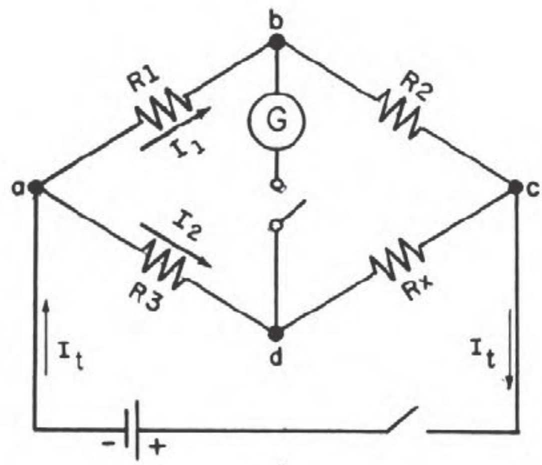
and

$$E_2 = E_x,$$

or

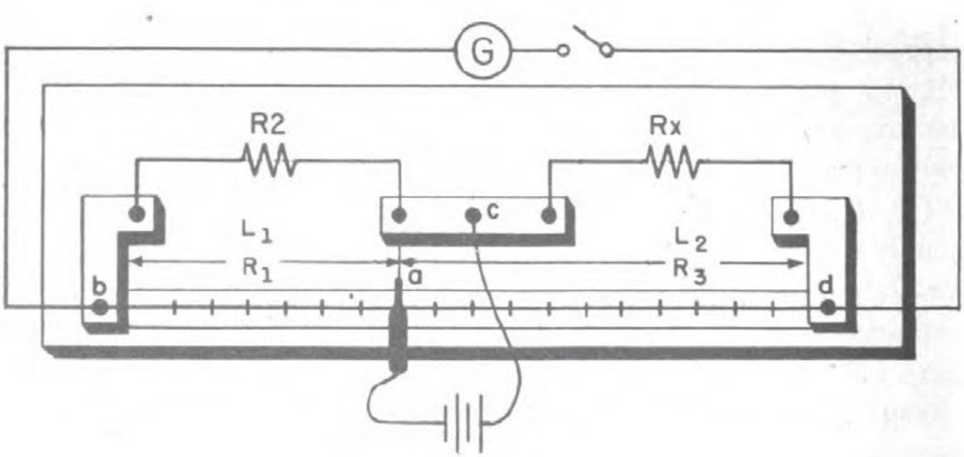
$$I_1R_2 = I_2R_x.$$

Dividing the voltage drops across $R1$ and $R3$ by the respective voltage drops across $R2$ and R_x,



A

SCHEMATIC WHEATSTONE-BRIDGE CIRCUIT



B

SLIDE-WIRE BRIDGE

Figure 5–8.—Circuits of Wheatstone bridges.

$$\frac{I_1R_1}{I_1R_2}=\frac{I_2R_3}{I_2R_x}.$$

Simplifying,

$$\frac{R_1}{R_2}=\frac{R_3}{R_x}.$$

Therefore,

$$R_x=\frac{R_2R_3}{R_1}.$$

The resistance values of *R*1, *R*2, and *R*3 are readily determined from the markings on the standard resistors, or from the calibrated dials if a dial-type bridge is used.

The Wheatstone bridge may be of the slide-wire type, as shown in figure 5–8, B. In this circuit, *R*1 and *R*2 (fig. 5–8, A) are replaced by a slide-wire of uniform cross-section. The wire may be an alloy, for example German silver or nichrome, having a resistance of about 100 ohms. Point *a* is established by moving the pointer along the wire until the bridge is balanced, as indicated by a zero reading on galvanometer *G* when the battery circuit is energized and the galvanometer switch is closed.

The equation for solving for R_x in the circuit of figure 5–8, B, is similar to the one used for solving for R_x in the circuit of figure 5–8, A. However, in the slide-wire bridge the length L_1 corresponding to the resistance R_1, and the lenght L_2 corresponding to the resistance R_2 may be substituted for R_1 and R_2. This is possible because for a wire of uniform cross section the resistance varies directly with the length and therefore the ratio of the resistances is equal to the corresponding ratio of the lengths. Thus,

$$R_x=\frac{L_2}{L_1}R_3.$$

A meter stick is mounted underneath the slide wire and L_1 and L_2 are easily read in centimeters. For example, if a balance is obtained when $R_3=150$ ohms, $L_1=25$ cm, and $L_2=75$ cm, the unknown resistance is,

$$R_x=\frac{75}{25}\times 150=450 \text{ ohms}.$$

PARALLEL SOURCES SUPPLYING A COMMON LOAD

The circuit shown in figure 5–9 illustrates two sources of emf, E_{s1} and E_{s2}, having internal resistances of 2 and 2.5 ohms respectively, connected in parallel, and supplying a 5-ohm load. Neglecting the resistance of the lead wires, it is desired to determine the current delivered by each source to the load, the load current, and the load voltage.

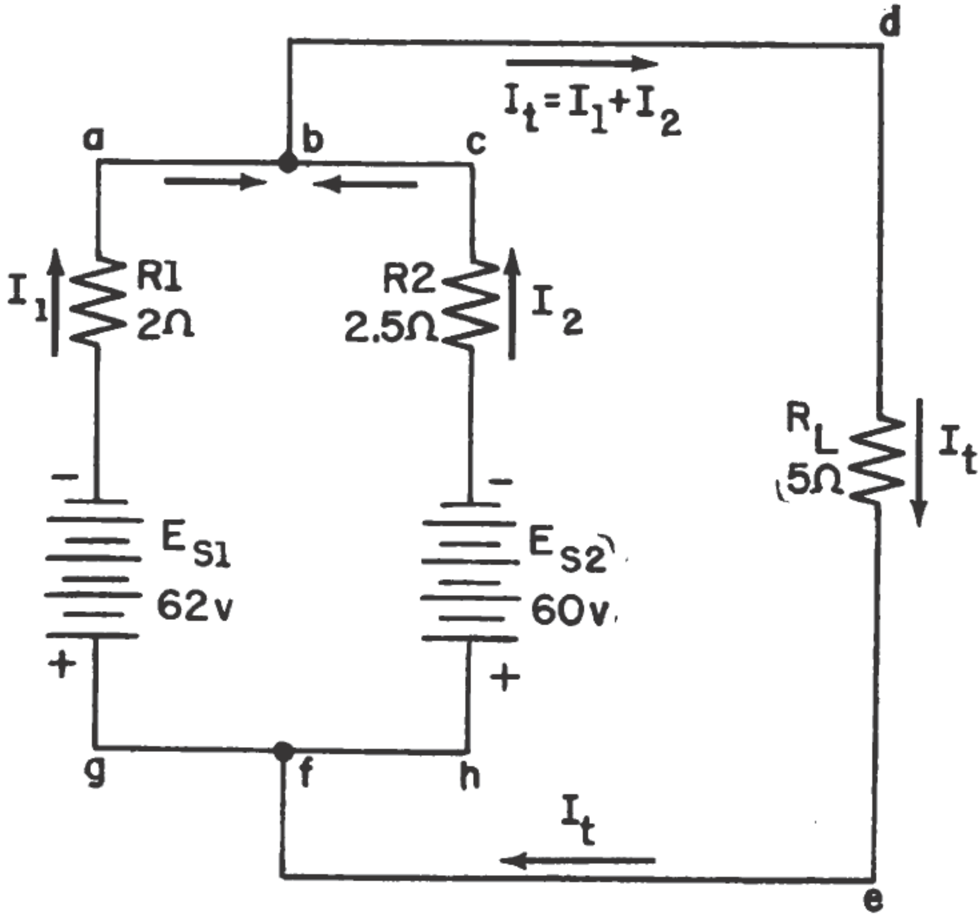


Figure 5–9.—Parallel sources supplying a common load.

The problem may be solved by establishing two voltage equations in which the voltages are expressed in terms of the unknown currents I_1 and I_2, the known resistances, and the known voltages. The equations are then solved simultaneously as in previous examples, to eliminate one of the unknown currents. The other unknown current is solved by substitution.

The first voltage equation is established by starting at point g and tracing clockwise around circuit $gabdefg$. The total load

current is equal to the sum of the source currents, I_1+I_2. The first voltage equation is,

$$62-2I_1-5(I_1+I_2)=0,$$

from which,

$$7I_1+5I_2=62. \tag{5–5}$$

The second voltage equation is established by starting at point h and tracing around circuit $hcbdefh$. Thus,

$$60-2.5I_2-5(I_1+I_2)=0,$$

from which,

$$5I_1+7.5I_2=60. \tag{5–6}$$

I_2 is eliminated by multiplying equation (5–5) by 1.5 and subtracting equation (5–6) from the result, as follows:

$$\begin{array}{r}
10.5I_1+7.5I_2=93 \\
5.0I_1+7.5I_2=60 \\
\hline
5.5I_1 \qquad\quad =33 \\
I_1=6 \text{ amperes.}
\end{array}$$

Substituting this value in equation (5–5),

$$7\times 6+5I_2=62,$$

from which

$$I_2=4 \text{ amperes.}$$

The load current is $I_1+I_2=6+4=10$ amperes. Thus source E_{s1} supplies 6 amperes and source E_{s2} supplies 4 amperes.

The load voltage is equal to the voltage developed across terminals f and b and is equal to the difference in a given source voltage and the internal voltage absorbed across the corresponding source resistance.

The statement applies equally to either source since both are in parallel with the load. In terms of source E_{s1}

$$\begin{aligned}
E_{fb}&=E_{s1}-I_1R_1 \\
&=62-6\times 2 \\
&=50 \text{ volts,}
\end{aligned}$$

and in terms of source E_{s2}

$$\begin{aligned} E_{fb} &= E_{s2} - I_2 R_2 \\ &= 60 - 4 \times 2.5 \\ &= 50 \text{ volts.} \end{aligned}$$

A further check on the load voltage is to express this value in terms of the load current and the load resistance as follows:

$$\begin{aligned} E_{fb} &= (I_1 + I_2) R_L \\ &= (6+4)(5) \\ &= 50 \text{ volts.} \end{aligned}$$

DISTRIBUTION CIRCUITS

Two-Wire Distribution Circuits

Up to this point the voltage drop and the power lost in the line wires connecting the load and the source have been neglected. When the load is located at some distance from the source, the line resistance becomes an appreciable part of the total circuit resistance and the voltage and power lost in the line become significant even with moderate loads.

In figure 5–10, load *M* draws 7 amperes through terminals *b* and *e*, and the parallel group of lamps draws 5 amperes through terminals *c* and *d*. The current in line wires *bc* and *de* is 5 amperes. The line current in wires *ab* and *ef* is 5+7=12 amperes.

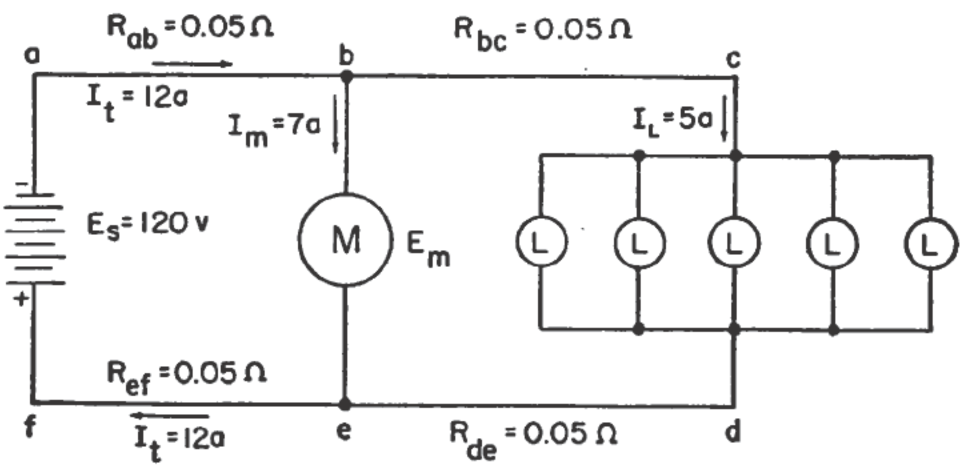


Figure 5–10.—Simple two-wire distribution circuit.

The voltage source supplies a constant potential of 120 volts between points a and f. The resistance of line wires ab and ef is $2\times0.05=0.1$ ohm. The voltage drop across line wires ab and ef is $12\times0.1=1.2$ volts. The voltage drop across line wires bc and de is $5\times0.1=0.5$ volt. The voltage across M is $120-1.2=118.8$ volts and the voltage across the five lamps is $118.8-0.5=118.3$ volts.

The power dissipated in line wires ab and ef is equal to $(12)^2\times0.1=14.4$ watts. The power absorbed by line wires bc and de is $(5)^2\times0.1=2.5$ watts. The total power absorbed by the line wires is $14.4+2.5=16.9$ watts.

The power delivered to load M is $118.8\times7=831.6$ watts, and to the 5 lamps is $118.3\times5=591.5$ watts. The total power supplied to the entire circuit is equal to $16.9+831.6+591.5=1{,}440$ watts and is equal to the product of the total applied voltage and the total current. Thus,

$$P_t=E_tI_t=120\times12=1{,}440 \text{ watts.}$$

Three-Wire Distribution Circuits

Three-wire distribution circuits transmit power at 240 volts and utilize it at 120 volts. The direct-current 3-wire system includes a positive feeder, a negative feeder, and a neutral wire, as shown in figure 5–11, A. The loads are connected between the negative feeder and the neutral, and between the positive feeder and the neutral. When the loads are unbalanced (unequal) the neutral wire carries a current equal to the difference in the currents in the negative and positive feeders.

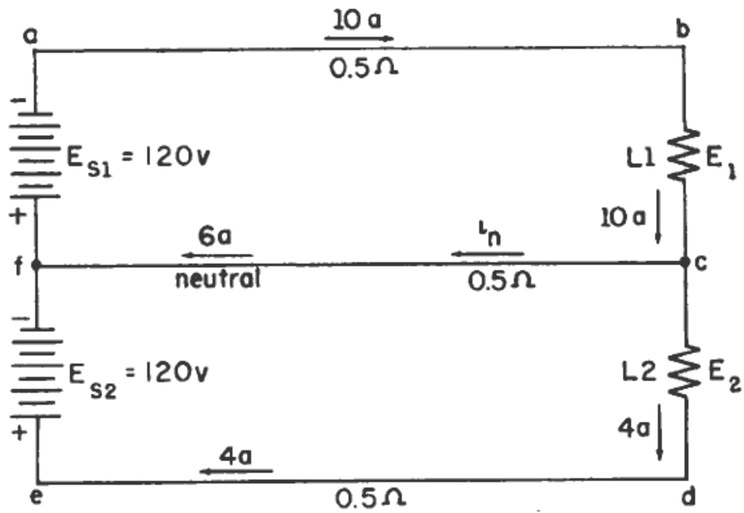
In the example of figure 5–11, A, load $L1$ draws 10 amperes, load $L2$ draws 4 amperes, and the neutral wire carries a current of $10-4=6$ amperes. The direction of flow of the current in the neutral wire is always the same as that of the smaller of the currents in the positive and negative feeders. Thus the flow is to the left in the lower (positive) wire and also in the neutral. The current in the upper (negative) wire is 10 amperes and in the lower wire is 4 amperes. The algebraic sum of the currents entering and leaving junction c is equal to zero. Thus,

$$+10-6-4=0.$$

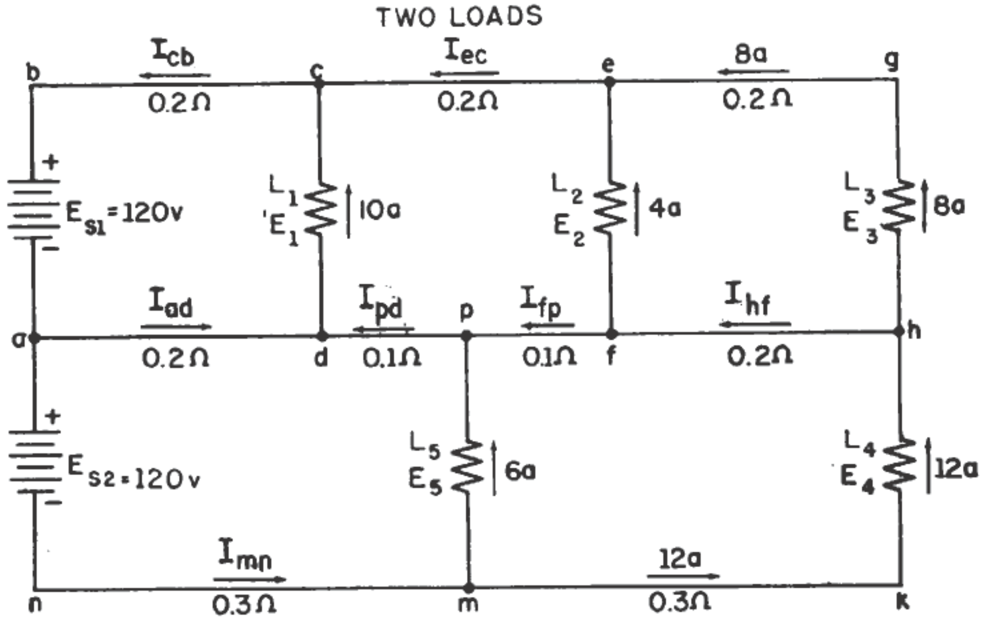
To find load voltage, E_1, a voltage equation is established in which E_1 is expressed in terms of the source voltage, E_{s1}, and the IR drops in the negative feeder and neutral wire. The algebraic sum of the voltages around the circuit *fabcf*, is equal to zero. Starting at *f* and proceeding clockwise,

$$+120 - 10 \times 0.5 - E_1 - 6 \times 0.5 = 0$$

$$E_1 = 112 \text{ volts.}$$



A
TWO LOADS



B
FIVE LOADS

Figure 5–11.—Three-wire distribution circuits.

Thus the voltage across load $L1$ is 112 volts. This voltage is less than the source voltage by an amount equal to the sum of the voltage drops in the negative (5 volts) and the neutral (3 volts) wires.

To find load voltage E_2 a voltage equation is established in which E_2 is expressed in terms of the source voltage, E_{s2}, and the IR drops in the positive feeder and the neutral wire. The algebraic sum of the voltages around the circuit *efcde* is zero. Starting at *e* and proceeding clockwise,

$$+120+(6\times 0.5)-E_2-(4\times 0.5)=0$$
$$E_2=121 \text{ volts.}$$

In tracing the circuit from *f* to *c* note that the direction is against the arrow representing current flow, and therefore that the IR drop of (6×0.5) volts is preceded by a plus sign. The load voltage, E_2, is 121 volts, which is 1 volt higher than the source voltage, E_{s2}. The total source voltage $(E_{s1}+E_{s2})$ is 240 volts and the total load voltage (E_1+E_2) is $112+121=233$ volts. This value is also equal to the difference between the total source voltage and the sum of the voltage drops in the positive and negative feeders, or $240-(2+5)=233$ volts.

When the loads are balanced on the positive and negative sides of the 3-wire system, the neutral current is zero and the currents in the outside wires (positive and negative) are equal. When the loads are unbalanced the neutral wire carries the unbalanced current. The voltage on the heavily loaded side falls while the voltage on the lightly loaded side rises. The lower the resistance of the neutral wire, the less unbalance in voltage there will be for a given unbalanced load.

A more complicated 3-wire circuit is shown in figure 5–11, B. The source voltage is 120 volts between each outside wire and the neutral, or center, wire. Load currents in the upper side of the system are indicated as 10, 4, and 8 amperes respectively for loads 1, 2, and 3. In the lower side of the system, the load currents are 12 and 6 amperes respectively for loads 4 and 5. In order to determine the various load voltages it is necessary to find the currents in each outside wire and in the neutral wire. The resistances of these wires are indicated, and therefore the voltage drops and the load voltages may be calculated after the currents are determined.

To find the currents in the various sections of the wires, it is best to start at the load farthest removed from the source. The polarities of the sources are such that electrons flow out of the negative terminal at n and return to the positive terminal at b.

Currents flowing toward a junction are assumed to be positive, and those flowing away from a junction are assumed to be negative. Applying Kirchhoff's current law at junction h, the neutral current, I_n (flowing from h to f) is determined as

$$12-8-I_{hf}=0$$
$$I_{hf}=4 \text{ amperes.}$$

Applying the same rule successively to junctions f, e, p, m, d, and c, it follows that:

at junction f,

$$4-4-I_{fp}=0$$
$$I_{fp}=0 \text{ ampere}$$

at junction e,

$$4+8-I_{ec}=0$$
$$I_{ec}=12 \text{ amperes}$$

at junction p,

$$6+0-I_{pd}=0$$
$$I_{pd}=6 \text{ amperes}$$

at junction m,

$$+I_{mn}-6-12=0$$
$$I_{mn}=18 \text{ amperes}$$

at junction d,

$$I_{ad}+6-10=0$$
$$I_{ad}=4 \text{ amperes}$$

at junction c,

$$-I_{cb}+10+12=0$$
$$I_{cb}=22 \text{ amperes.}$$

Thus, E_{s1} supplies 22 amperes and E_{s2} supplies 18 amperes. The electron flow in all parts of the lower wire is outward from the source, and the electron flow in all parts of the upper wire is back toward the source. The current in the neutral wire is always equal to the difference in the currents in the two outside wires, and the electron flow is in the direction of the smaller of these two currents. Thus in figure 5–11, B, the neutral current

in section *ad* is 4 amperes, which is the difference between 18 amperes and 22 amperes; and it is in the direction of the smaller current in section *nm*. The neutral current in section *pd* is 6 amperes, which is the difference between 18 amperes and 12 amperes; and it is in the same direction as the 12 amperes in section *ec*. The neutral current in section *fp* is zero because the current in each outside wire in that section is 12 amperes. The neutral current in section *hf* is 4 amperes, which is the difference between 12 amperes and 8 amperes, and it is in the direction of the smaller outside current in section *ge*.

In order to find the voltages across the loads in figure 5–11, B, Kirchhoff's voltage law is applied to the various individual circuits. Thus to find the voltage, E_1, across load L_1, the algebraic sum of the voltages around the circuit *abcda* is equated to zero, and E_1 is then readily determined.

Starting at *a*,

$$-120+(22\times0.2)+E_1+(4\times0.2)=0$$

$$E_1=114.8 \text{ volts.}$$

To find load voltage E_2, the algebraic sum of the voltages around circuit *dcefpd* is set equal to zero. Starting at *d*,

$$-114.8+(12\times0.2)+E_2+(0\times0.1)-(6\times0.1)=0$$

$$E_2=113 \text{ volts.}$$

To find load voltage E_3, the algebraic sum of the voltages around loop *feghf* is set equal to zero. Starting at *f*,

$$-113+(8\times0.2)+E_3-(4\times0.2)=0$$

$$E_3=112.2.$$

To find load voltage E_4, the algebraic sum of the voltages around loop *nadpfhkmn* is set equal to zero. Loop *mpfhkm* cannot be used because it would contain two unknown voltages, E_5 and E_4. Starting at *n*,

$$-120-(4\times0.2)+(6\times0.1)+(0\times0.1)+(4\times0.2)+E_4+(12\times0.3)+(18\times0.3)=0$$

$$E_4=110.4 \text{ volts.}$$

To find load voltage E_5, the algebraic sum of the voltages around loop *nadpmn* is set equal to zero. Starting at *n*,

$$-120-(4\times0.2)+(6\times0.1)+E_5+(18\times0.3)=0$$
$$E_5=114.8 \text{ volts.}$$

In each case, the equations used contain one unknown; and thus a simple solution is quickly obtained. It is necessary that the path traced include a completely closed loop and that all but one of the voltages within that loop be known. Simple transposition of the resulting equation gives the desired voltage.

QUIZ

1. In figure 5–1, A:
   - (a) If $R1$ is 2 ohms, $R2$ is 6 ohms, $R3$ is 12 ohms, and E_s is 24 volts, find I_t.
   - (b) If $R1$ is 1.5 ohms, $R2$ is 1 ohm, $R3$ is 1 ohm, and E_s is 20 volts, find I_t.
2. In figure 5–1, B:
   - (a) If $R1$, $R2$, and $R3$ are each equal to 2 ohms and E_s is 20 volts, find I_t.
   - (b) If $R1$ is 0.6 ohm, $R2$ is 0.2 ohm, $R3$ is 1 ohm, and E_s is 20 volts, find I_t.
   - (c) If $R1$ is 0.6 ohm, $R2$ is 0.2 ohm, $R3$ is 1 ohm, and E_s is 24 volts, find the current in the top branch.
3. In figure 5–2, A:
   - (a) If 20 volts are applied to the circuit, find the current in (1) $R1$, (2) $R6$, (3) $R2$, and (4) $R4$.
   - (b) If $R1$ is 0.1 ohm, $R2$ is 12 ohms, $R3$ is 1 ohm, $R4$ is 4 ohms, $R5$ is 3 ohms, and $R6$ is 0.1 ohm, find the current in (1) $R1$, (2) $R2$, and (3) $R4$.
4. In figure 5–3:
   - (a) If $R1$ is 1 ohm, $R2$ is 1 ohm, $R3$ is 0.5 ohm, $R4$ is 1 ohm, $R5$ is 1.5 ohms, and E_s is 50 volts, find I_t.
   - (b) If $R1$ is 0.5 ohm, $R2$ is 1 ohm, $R3$ is 0.667 ohm, $R4$ is 2 ohms, and $R5$ is 1.33 ohms, find the total resistance.
   - (c) If $R1$ is 0.5 ohm, $R2$ is 1 ohm, $R3$ is 0.667 ohm, $R4$ is 2 ohms, $R5$ is 1.33 ohms, and the battery voltage is 20 volts, find (1) the total current and (2) the total power supplied by the battery.
5. A series of three 10-K ohm resistors are connected across a 150-volt source.
   - (a) Find the current in milliamperes in each resistor.
   - (b) Find the voltage drop across each resistor.

6. In figure 5–4, if E_s is 300 volts, the current in resistor *A* is 10 milliamperes, the load voltages from left to right are 50, 100, and 200 volts, and the respective load currents are 5, 10, and 20 milliamperes, find the resistance of:
   (a) resistor A.
   (b) resistor B.
   (c) resistor C.
   (d) resistor D.
7. Assume that in figure 5–5 resistors *R*4, *R*5, *R*6, and *R*7 each has a resistance of 10-K ohms. Assume also that the load current in *R*1 is 4 milliamperes, the current in *R*2 is 6 milliamperes, the current in *R*3 is 8 milliamperes, and the source voltage, E_s is 600 volts.
   (a) Find the current in (1) *R*5, (2) *R*6, and (3) *R*7.
   (b) Find the voltage across (1) *R*1, (2) *R*2, and (3) *R*3.
8. What are the functions of an attenuator?
9. Attenuators that are not adjustable are commonly called __________.
10. In the L-type attenuator of figure 5–6, A:
   (a) What is the relation between R_s and the resistance measured between terminals *ab* when the source is removed?
   (b) Does the same relation hold for the resistance between terminals *cd* and the resistance, R_L?
   (c) If R_s and R_L are each 200 ohms, E_s is 40 volts, I_s is 0.1 ampere, and E_L is 4 volts, find (1) *R*1 and (2) *R*2.
11. In the T-type attenuator, figure 5–6, B:
   (a) What is the relation of the resistance between terminals *ab* and the resistance between terminals *ef*?
   (b) If R_s and R_L are each 60 ohms, E_s is 120 volts, I_s is 1 ampere, and E_L is 30 volts, find (1) *R*1 and *R*3, (2) find *R*2.
12. In figure 5–7:
   (a) If *R*1 is 4 ohms, *R*2 is 2 ohms, *R*3 is 8 ohms, *R*4 is 28 ohms, *R*5 is 6 ohms, and the total current delivered to the bridge is 5 amperes, find (1) I_1 and (2) I_2.
   (b) If *R*1 is 1 ohm, *R*2 is 1 ohm, *R*3 is 2 ohms, *R*4 is 1 ohm, *R*5 is 4.5 ohms, and the total current delivered to the bridge is 10 amperes, find (1) I_1 and (2) I_2.
13. In figure 5–8, A:
   (a) What is the relation between the voltage drops across *R*1 and *R*3 when the bridge is balanced?
   (b) If the bridge is balanced when *R*1 is 200 ohms, *R*2 is 100 ohms, and *R*3 is 100 ohms, find R_x.

14. In figure 5–8, B:
    (a) If the bridge is balanced, when $R2$ is 100 ohms, L_1 is 20 cm, and L_2 is 80 cm, find R_x.
    (b) Find R_x, if $R2=100$ ohms and $L_2=2L_1$.
15. In figure 5–9, if $E_{s1}=100$ volts, $E_{s2}=102.5$ volts, $R1=1$ ohm, $R2=2$ ohms, and $R_L=1$ ohm, find I_2.
16. In figure 5–10, assume that $E_s=120$ volts, the current in M is 50 amperes, and the total current supplied to the 5 loads, L, is 100 amperes. The resistance of the line wires *ab, bc, de,* and *ef* is 0.05 ohm each.
    (a) Find the total power consumed in the line wires.
    (b) If the current in the loads, L, is reduced to 60 amperes and the current in M remains at 50 amperes, find the total power consumed in the line wires.
17. In figure 5–11, A, E_{s1} is 120 volts, E_{s2} is 120 volts, and each line wire has a resistance of 0.5 ohm. If L_1 is 15 amperes and L_2 is 5 amperes, find:
    (a) E_1.
    (b) E_2.
18. In figure 5–11, B, if the source voltages and line resistances remain unchanged and L_1 is 5 amp, L_2 is 3 amp, L_3 is 6 amp, L_4 is 10 amp, and L_5 is 4 amp, find the following:
    (a) I_{hf}.
    (b) I_{fp}.
    (c) I_{ec}.
    (d) I_{pd}.
    (e) I_{mm}.
    (f) I_{ad}.
    (g) I_{cb}.
    (h) E across L_1.
    (i) E across L_2.
    (j) E across L_3.
    (k) E across L_5.
    (l) E across L_4.

CHAPTER 6

ELECTRICAL CHARACTERISTICS OF CONDUCTORS

INTRODUCTION

Because all electrical circuits depend upon conductors for their operation, an understanding of the electrical characteristics of conductors is essential.

To compare the resistance of one conductor with that of another, a standard, or unit, size of conductor must be established. A convenient unit of linear measurement, as far as the diameter of a piece of wire is concerned, is the MIL (0.001 of an inch); and a convenient unit of wire length is the foot. The standard unit of size in most cases is the MIL-FOOT. That is, a wire will have unit size if it has a diameter of 1 mil and a length of 1 foot. The resistance in ohms of a unit conductor of a given substance is called the SPECIFIC RESISTANCE, or SPECIFIC RESISTIVITY, of the substance.

Gage numbers are a further convenience in comparing the diameter of wires. The gage commonly used is the American wire gage (AWG), formerly the Brown and Sharpe (B and S) gage.

Large wires are usually stranded to increase their flexibility. Other reasons for this design will be described in chapter 13. Copper or aluminum conductors are commonly used, although other substances may be used in special cases. There are several factors that determine the size and type of conductor to be used in a given situation.

The effects of temperature on resistance must also be considered for a better understanding of circuit operation.

Likewise, it is desirable to consider conductor insulation and protection. Finally, it is also desirable to include a brief treatment of such operational features as making splices, soldering, and taping splices.

SQUARE MIL

The SQUARE MIL is a convenient unit of cross-sectional area for square or rectangular conductors such as bus bars. A square mil is the area of a square the sides of which are 1 mil, as shown in figure 6–1, A. To obtain the cross-sectional area in square mils of a square conductor, square one side measured in mils. To obtain the cross-sectional area in square mils of a rectangular conductor, multiply the length of one side by that of the other, each length being expressed in mils.

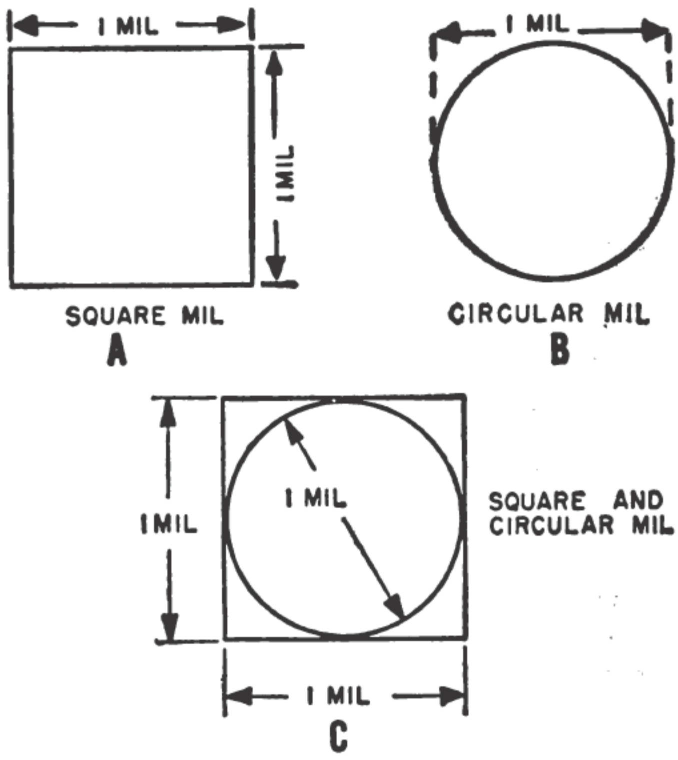


Figure 6–1.—The square mil and circular mil.

For example, find the cross-sectional area of a rectangular bus bar 3/8″ thick and 4″ wide. The thickness may be expressed in mils as $0.375 \times 1{,}000 = 375$ mils, and the width as $4 \times 1{,}000$, or 4,000 mils. The cross-sectional area is $375 \times 4{,}000$, or 1,500,000 square mils.

CIRCULAR MIL

The CIRCULAR MIL is the standard unit of wire cross-sectional area used in American and English wire tables. Because the

diameters of round conductors, or wires, used to conduct electricity may be only a small fraction of an inch, it is convenient to express these diameters in mils to avoid the use of decimals. For example, the diameter of a wire is expressed as 25 mils instead of 0.025 inch. A circular mil is the area of a circle having a diameter of 1 mil, as shown in figure 6–1, B. The area in circular mils of a round conductor is obtained by squaring the diameter measured in mils. Thus, a wire having a diameter of 25 mils has an area of 25^2, or 625, circular mils. By way of comparison, the basic formula for the area of a circle is $A=\pi R^2$ and in this example the area in square inches is

$$A=\pi R^2=3.14(0.0125)^2=0.00049 \text{ sq. in.}$$

If D is the diameter of a wire in mils, the area in square mils is

$$A=\pi\left(\frac{D}{2}\right)^2=\frac{3.1416}{4}D^2=0.7854D^2 \text{ sq. mils.}$$

Therefore, a wire 1 mil in diameter has an area of

$$A=0.7854\times 1^2=0.7854 \text{ sq. mils,}$$

which is equivalent to 1 circular mil. The cross-sectional area of a wire in circular mils is therefore determined as

$$A=\frac{0.7854D^2}{0.7854}=D^2 \text{ circular mils,}$$

where D is the diameter in mils. Thus, the constant $\frac{\pi}{4}$ is eliminated from the calculation.

In comparing square and round conductors it should be noted that the circular mil is a smaller unit of area than the square mil, and therefore there are more circular mils than square mils in any given area. The comparison is shown in figure 6–1, C. The area of a circular mil is equal to 0.7854 of a square mil. Therefore, to determine the circular-mil area when the square-mil area is given, divide the area in square mils by 0.7854. Conversely, to determine the square-mil area when the circular-mil area is given, multiply the area in circular mils by 0.7854.

For example, a No. 12 wire has a diameter of 80.81 mils. What is (1) its area in circular mils and (2) its area in square mils?

Solution:

(1) $A=D^2=80.81^2=6{,}530$ circular mils.
(2) $A=0.7854\times 6{,}530=5{,}128.6$ square mils.

A bus bar is 1.5 inches wide and 0.25 inch thick. (1) What is its area in square mils? (2) What size of round conductor in circular mils is necessary to carry the same current as the bus bar?
Solution:

(1) $1.5''=1.5\times 1{,}000=1{,}500$ mils
$0.25''=0.25\times 1{,}000=250$ mils
$A=1{,}500\times 250=375{,}000$ square mils.

(2) To carry the same current, the cross-sectional area of the bus bar and the cross-sectional area of the round conductor must be equal. There are more circular mils than square mils in this area, and therefore

$$A=\frac{375{,}000}{0.7854}=477{,}000 \text{ circular mils.}$$

A wire in its usual form is a slender rod or filament of drawn metal. In large sizes, wire becomes difficult to handle, and its flexibility is increased by stranding. The strands are usually single wires twisted together in sufficient numbers to make up the necessary cross-sectional area of the cable. The total area in circular mils is determined by multiplying the area of one strand in circular mils by the number of strands in the cable.

CIRCULAR-MIL-FOOT

A circular-mil-foot, as shown in figure 6–2, is actually a unit of volume. It is a unit conductor 1 foot in length and having a cross-sectional area of 1 circular mil. Because it is considered a unit conductor, the circular-mil-foot is useful in making comparisons between wires that are made of different metals. For example, a basis of comparison of the RESISTIVITY (to be treated later) of various substances may be made by determining the resistance of a circular-mil-foot of each of the substances.

In working with certain substances it is sometimes more convenient to employ a different unit volume. Accordingly, unit

volume may also be taken as the centimeter cube; and specific resistance becomes the resistance offered by a cube-shaped conductor 1 cm long and 1 sq. cm. in cross-sectional area. The inch cube may also be used. The unit of volume employed is given in tables of specific resistances.

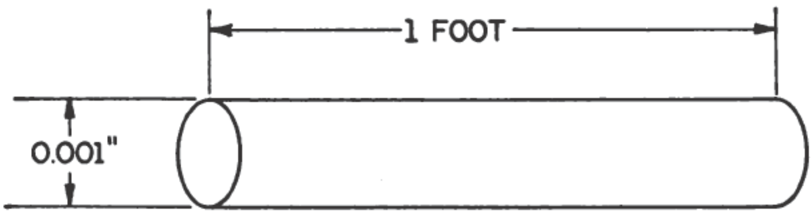


Figure 6–2.—Circular-mil-foot.

SPECIFIC RESISTANCE OR RESISTIVITY

Specific resistance, or resistivity, is the resistance in ohms offered by unit volume (the circular-mil-foot is used in many practical applications) of a substance to the flow of electric current. Resistivity is the reciprocal of conductivity. A substance that has a high resistivity will have a low conductivity, and vice versa.

Thus, the specific resistance of a substance is the resistance of a unit volume of that substance. Many tables of specific resistance are based on the resistance in ohms of a volume of the substance 1 foot long and 1 circular mil in cross-sectional area. The temperature at which the resistance measurement is made is also specified. If the kind of metal of which a conductor is made is known, the specific resistance of the metal may be obtained from a table. The specific resistances of some common substances are given in table 2.

The resistance of a conductor of uniform cross section varies directly as the product of the length and the specific resistance of the conductor and inversely as the cross-sectional area of the conductor. Therefore the resistance of a conductor may be calculated if the length cross-sectional area, and specific resistance of the substance are known. Expressed as an equation, the resistance, R, in ohms, of a conductor is

$$R=\rho\frac{L}{A},$$

TABLE 2.—*Specific resistance*

Substance	Specific resistance at 20° C.	
	Centimeter cube (*micro-ohms*)	Circular-mil-foot (*ohms*)
Silver	1.629	9.8
Copper (drawn)	1.724	10.37
Gold	2.44	14.7
Aluminum	2.828	17.02
Carbon (amorphous)	3.8 to 4.1	....
Tungsten	5.51	33.2
Brass	7.0	42.1
Steel (soft)	15.9	95.8
Nichrome	109.0	660.0

where ρ (Greek rho) is the specific resistance in ohms per circular-mil-foot, L the length in feet (in the direction of current flow), and A the cross-sectional area in circular mils.

For example, what is the resistance of 1,000 feet of copper wire having a cross-sectional area of 10,400 circular mils (No. 10 wire), the wire temperature being 20° C.? Solution:

The specific resistance, from table 2, is 10.37. Substituting the known values in the preceding equation, the resistance, R, is determined as

$$R=\rho\frac{L}{A}=10.37\times\frac{1{,}000}{10{,}400}=1 \text{ ohm approximately.}$$

If R, ρ, and A are known, the length may be determined by a simple mathematical transposition. This is of value in many applications. For example, when it is desired to locate a ground in a telephone line, special test equipment is used that operates on the principle that the resistance of a line varies directly with its length. Thus, the distance between the test point and a fault can be computed accurately.

As has been mentioned in preceding chapters, conductance (G) is the reciprocal of resistance. When R is in ohms, the conductance is expressed in mhos. Where resistance is opposition to flow, conductance is the ease with which the current flows. Conductance in mhos is equivalent to the number of amperes

flowing in a conductor per volt of applied emf. Expressed in terms of the specific resistance, length, and cross section of a conductor,

$$G=\frac{1}{R}=\frac{1}{\rho\frac{L}{A}}=\frac{A}{\rho L}.$$

The conductance, G, varies directly as the cross-sectional area, A, and inversely as the specific resistance, ρ, and the length, L. When A is in circular mils, ρ is in ohms per circular-mil-foot, L is in feet, and G is in mhos.

The relative conductance of several substances is given in table 3.

TABLE 3.—*Relative conductance*

Substance	Relative conductance (Silver=100%)	Substance	Relative conductance (Silver=100%)
Silver	100	Iron	16
Copper	98	Lead	15
Gold	78	Tin	9
Aluminum	61	Nickel	7
Tungsten	32	Mercury	1
Zinc	30	Carbon	0.05
Platinum	17		

WIRE MEASURE

Relation Between Wire Sizes

Wires are manufactured in sizes numbered according to a table, known as the American wire gage (AWG). As may be seen in table 4, the wire diameters become smaller as the gage numbers become larger. The largest wire size shown in the table is 0000 (read "4 naught") and the smallest is number 40. Larger and smaller sizes are manufactured but are not commonly used by the Navy. The ratio of the diameter corresponding to a given gage number to the diameter corresponding to the next higher gage number is a constant number, 1.123. The cross-sectional area varies as the square of the diameter. Therefore the ratio of the cross section corresponding to a given gage number to that corre-

TABLE 4.—*Standard annealed solid copper wire*

(American wire gage—B & S)

Gage number	Diameter (mils)	Cross section		Ohms per 1,000 ft.		Ohms per mile	Pounds per 1,000 ft.
		Circular mils	Square inches	25°C. (=77°F.)	65°C. (=149°F.)	25°C. (=77°F.)	
0000	460.0	212,000.0	0.166	0.0500	0.0577	0.264	641.0
000	410.0	168,000.0	.132	.0630	.0727	.333	508.0
00	365.0	133,000.0	.105	.0795	.0917	.420	403.0
0	325.0	106,000.0	.0829	.100	.116	.528	319.0
1	289.0	83,700.0	.0657	.126	.146	.665	253.0
2	258.0	66,400.0	.0521	.159	.184	.839	201.0
3	229.0	52,600.0	.0413	.201	.232	1.061	159.0
4	204.0	41,700.0	.0328	.253	.292	1.335	126.0
5	182.0	33,100.0	.0260	.319	.369	1.685	100.0
6	162.0	26,300.0	.0206	.403	.465	2.13	79.5
7	144.0	20,800.0	.0164	.508	.586	2.68	63.0
8	128.0	16,500.0	.0130	.641	.739	3.38	50.0
9	114.0	13,100.0	.0103	.808	.932	4.27	39.6
10	102.0	10,400.0	.00815	1.02	1.18	5.38	31.4
11	91.0	8,230.0	.00647	1.28	1.48	6.75	24.9
12	81.0	6,530.0	.00513	1.62	1.87	8.55	19.8
13	72.0	5,180.0	.00407	2.04	2.36	10.77	15.7
14	64.0	4,110.0	.00323	2.58	2.97	13.62	12.4
15	57.0	3,260.0	.00256	3.25	3.75	17.16	9.86
16	51.0	2,580.0	.00203	4.09	4.73	21.6	7.82
17	45.0	2,050.0	.00161	5.16	5.96	27.2	6.20
18	40.0	1,620.0	.00128	6.51	7.51	34.4	4.92
19	36.0	1,290.0	.00101	8.21	9.48	43.3	3.90
20	32.0	1,020.0	.000802	10.4	11.9	54.9	3.09
21	28.5	810.0	.000636	13.1	15.1	69.1	2.45
22	25.3	642.0	.000505	16.5	19.0	87.1	1.94
23	22.6	509.0	.000400	20.8	24.0	109.8	1.54
24	20.1	404.0	.000317	26.2	30.2	138.3	1.22
25	17.9	320.0	.000252	33.0	38.1	174.1	0.970
26	15.9	254.0	.000200	41.6	48.0	220.0	0.769
27	14.2	202.0	.000158	52.5	60.6	277.0	0.610
28	12.6	160.0	.000126	66.2	76.4	350.0	0.484
29	11.3	127.0	.0000995	83.4	96.3	440.0	0.384
30	10.0	101.0	.0000789	105.0	121.0	554.0	0.304
31	8.9	79.7	.0000626	133.0	153.0	702.0	0.241
32	8.0	63.2	.0000496	167.0	193.0	882.0	0.191
33	7.1	50.1	.0000394	211.0	243.0	1,114.0	0.152
34	6.3	39.8	.0000312	266.0	307.0	1,404.0	0.120
35	5.6	31.5	.0000248	335.0	387.0	1,769.0	0.0954
36	5.0	25.0	.0000196	423.0	488.0	2,230.0	0.0757
37	4.5	19.8	.0000156	533.0	616.0	2,810.0	0.0600
38	4.0	15.7	.0000123	673.0	776.0	3,550.0	0.0476
39	3.5	12.5	.0000098	848.0	979.0	4,480.0	0.0377
40	3.1	9.9	.0000078	1,070.0	1,230.0	5,650.0	0.0299

sponding to the next higher gage number is the square of 1.123, or 1.261. Because the cube of 1.261 is very nearly 2, the cross-sectional area is approximately halved, or doubled, every three gage numbers. Also because 1.261 raised to the 10th power is very nearly equal to 10, the cross-sectional area is increased or decreased 10 times every 10 gage numbers.

A No. 10 wire has a diameter of approximately 100 mils, a cross-sectional area of approximately 10,400 circular mils, and a resistance of approximately 1 ohm per 1,000 feet. From these facts it is possible to estimate quickly the cross-sectional area and the resistance of any size copper wire without referring directly to a wire table.

For example, to estimate the cross-sectional area and the resistance of 1,000 feet of No. 17 wire the following reasoning might be employed. A No. 17 wire is 3 sizes removed from a No. 20 wire and therefore has twice the cross-sectional area of a No. 20 wire. A No. 20 wire is 10 sizes removed from a No. 10 wire and therefore has one-tenth the cross section of a No. 10 wire. Therefore, the cross-sectional area of a No. 17 wire is $2 \times 0.1 \times 10{,}000 = 2{,}000$ circular mils. Since resistance varies inversely with the cross-

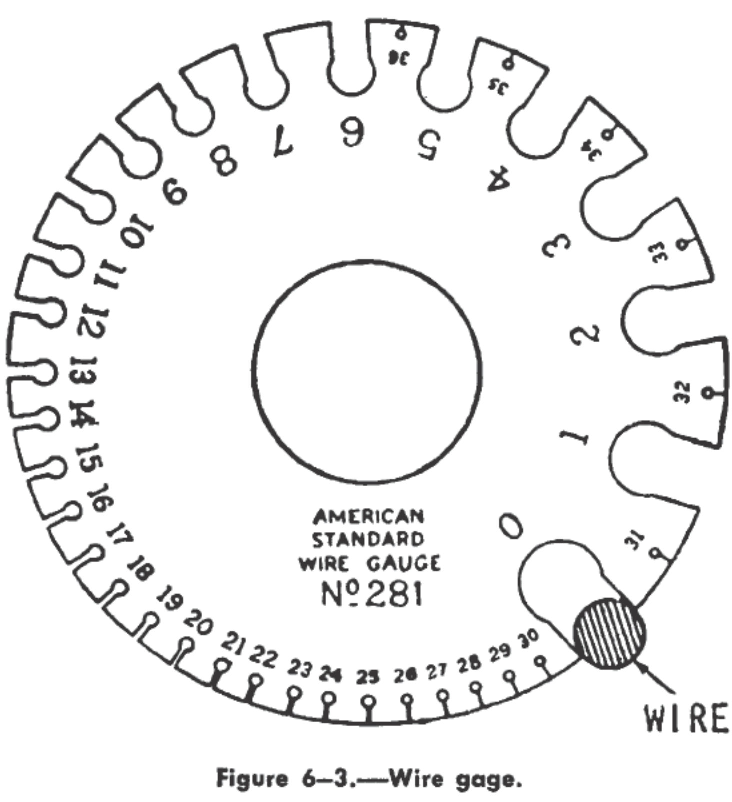


Figure 6–3.—Wire gage.

sectional area the resistance of a No. 17 wire is $10 \times 1 \times 0.5 = 5$ ohms per 1,000 feet.

A wire gage is shown in figure 6–3. It will measure wires ranging in size from number 0 to number 36. The wire whose size is to be measured is inserted in the smallest slot that will just accommodate the bare wire. The gage number corresponding to that slot indicates the wire size. The slot has parallel sides and should not be confused with the semicircular opening at the end of the slot. The opening simply permits the free movement of the wire all the way through the slot.

Stranded Wires and Cables

A wire is a SINGLE, SOLID conductor. A cable may be a STRANDED conductor (single-conductor cable), or a GROUP of conductors insulated from each other (multiconductor cable). Some typical wires and cables are shown in figure 6–4.

Relatively small conductors, like lamp cord, are stranded for increased flexibility; large conductors are stranded for the same reason. For conductors having a cross section greater than 0000 (American wire gage) stranding is a practical necessity.

Cross-sectional views of a solid and a stranded conductor are

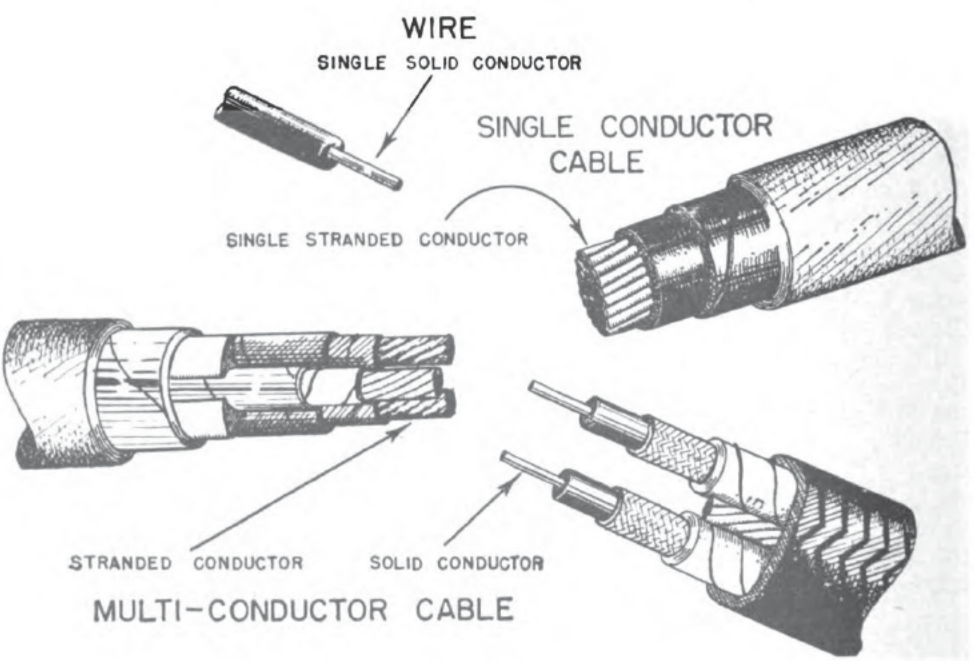


Figure 6–4.—Cables and wire.

shown in figure 6–5. If all of the individual strands are of equal diameter, the wires are arranged geometrically in concentric-lay cables as follows:

The first layer of wires around the center wire is made up of 6 wires. The second layer is made up of 12 wires, the third layer

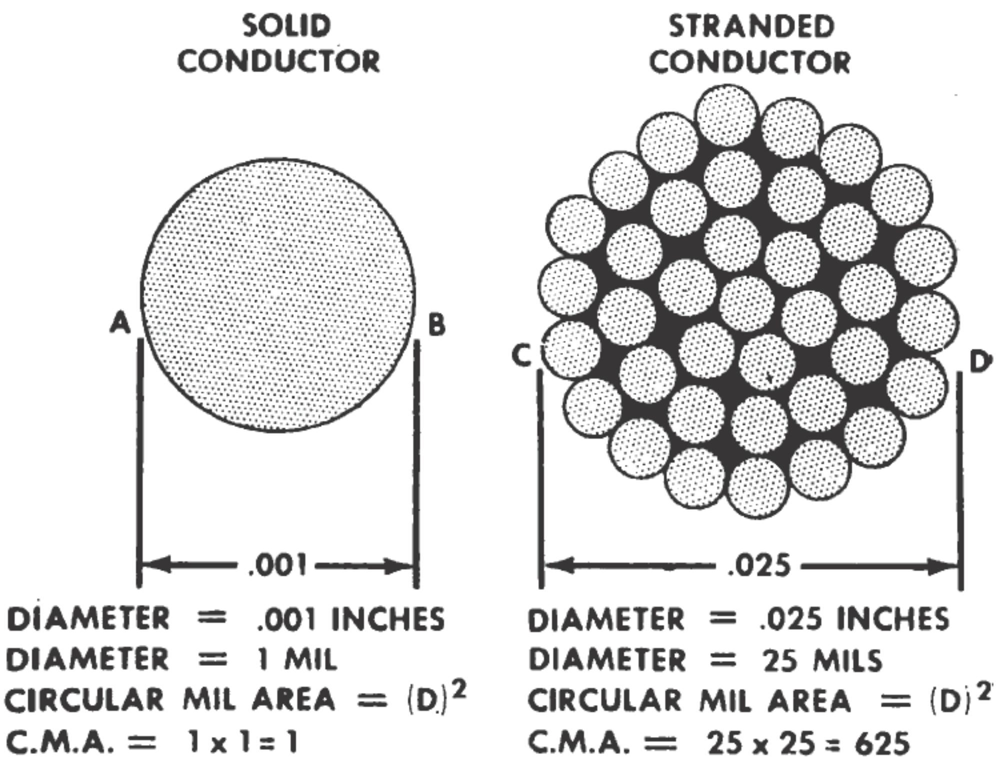


Figure 6–5.—Solid and stranded conductors.

is made up of 18 wires, and so forth. Thus, standard cables are composed of 1, 7, 19, 37, etc. strands.

The over-all flexibility may be increased by stranding the individual strands.

Factors Governing the Selection of Wire Size

Several factors must be considered in selecting the size of wire to be used for transmitting and distributing electric power.

One factor is the allowable power loss (I^2R loss) in the line. This loss represents electrical energy converted into heat. The use of large conductors will reduce the resistance and therefore the I^2R loss. However, large conductors are more expensive

initially than small ones; they are heavier and require more substantial supports.

A second factor is the permissible voltage drop (*IR* drop) in the line. If the source maintains a constant voltage at the input to the line, any variation in the load on the line will cause a variation in line current, and a consequent variation in the *IR* drop in the line. A wide variation in the *IR* drop in the line causes poor voltage regulation at the load. The obvious remedy is to reduce either *I* or *R*. A reduction in load current lowers the amount of power being transmitted, whereas a reduction in line resistance increases the size and weight of conductors required. A compromise is generally reached whereby the voltage variation at the load is within tolerable limits and the weight of line conductors is not excessive.

A third factor is the current-carrying ability of the line. When current is drawn through the line, heat is generated. The temperature of the line will rise until the heat radiated, or otherwise dissipated, is equal to the heat generated by the passage of current through the line. If the conductor is insulated, the heat generated in the conductor is not so readily removed as it would be if the conductor were not insulated. Thus, to protect the insulation from too much heat, the current through the conductor must be maintained below a certain value. Rubber insulation will begin to deteriorate at relatively low temperatures. Varnished cloth insulation retains its insulating properties at higher temperatures; and other insulation—for example, asbestos or silicone—is effective at still higher temperatures.

Electrical conductors may be installed in locations where the ambient temperature is relatively high, in which case the heat generated by external sources constitutes an appreciable part of the total conductor heating. Due allowance must be made for the influence of external heating on the allowable conductor current and each case has its own specific limitations. The maximum allowable operating temperature of insulated conductors is specified in tables and varies with the type of conductor insulation being used.

Tables have been prepared by the National Board of Fire Underwriters giving the safe current ratings for various sizes and types of conductors covered with various types of insulation.

For example, the allowable current-carrying capacities of copper conductors at not over 30° C. room temperature for single conductors in free air are given in table 5.

TABLE 5.—*Current-carrying capacities (in amperes) of single copper conductors at ambient temperature of below 30° C.*

Size	Rubber or thermo-plastic	Thermoplastic asbestos, var-cam, or asbestos var-cam	Impregnated asbestos	Asbestos	Slow-burning or weather-proof
0000	300	385	475	510	370
000	260	330	410	430	320
00	225	285	355	370	275
0	195	245	305	325	235
1	165	210	265	280	205
2	140	180	225	240	175
3	120	155	195	210	150
4	105	135	170	180	130
6	80	100	125	135	100
8	55	70	90	100	70
10	40	55	70	75	55
12	25	40	50	55	40
14	20	30	40	45	30

COPPER VS. ALUMINUM CONDUCTORS

Although silver is the best conductor, its cost limits its use to special circuits where a substance with high conductivity is needed.

The two most generally used conductors are copper and aluminum. Each has characteristics that make its use advantageous under certain circumstances. Likewise, each has certain disadvantages.

Copper has a higher conductivity; it is more ductile (can be drawn out), has relatively high tensile strength, and can be easily soldered. It is more expensive and heavier than aluminum.

Although aluminum has only about 60 percent of the conductivity of copper, it is used extensively for high-voltage transmission lines. Its lightness makes possible long spans, and its relatively large diameter for a given conductivity reduces corona—that is, the discharge of electricity from the wire when it has a

high potential. The discharge is greater when smaller diameter wire is used than when larger diameter wire is used. Some bus bars are made of aluminum instead of copper, where for the same conductance there is a greater radiating surface. However, aluminum conductors are not easily soldered, and aluminum's relatively large size for a given conductance does not permit the economical use of an insulation covering.

A comparison of some of the characteristics of copper and aluminum is given in table 6.

TABLE 6.—*Characteristics of copper and aluminum*

Characteristic	Copper	Aluminum
Tensile strength (lb/in^2)	55, 000	25, 000
Tensile strength for same conductivity (lb)	55, 000	40, 000
Weight for same conductivity (lb)	100	48
Cross section for same conductivity (C. M.)	100	160
Specific resistance (Ω/mil ft)	10. 6	17

TEMPERATURE COEFFICIENT OF RESISTANCE

The resistance of pure metals—such as silver, copper, and aluminum—increases as the temperature increases. However, the resistance of some alloys—such as constantan and manganin—changes very little as the temperature changes. Measuring instruments use these alloys because the resistance of the circuits must remain constant if accurate measurements are to be achieved.

In table 2 the resistance of a circular-mil-foot of wire (the specific resistance) is given at a specific temperature, 20° C. in this case. It is necessary to establish a standard temperature because, as has been stated, the resistance of pure metals increase with an increase in temperature; and a true basis of comparison cannot be made unless the resistances of all the substances being compared are meaured at the same temperature. The amount of increase in the resistance of a 1-ohm sample of the conductor per degree rise in temperature above 0° centigrade (the assumed standard) is called the TEMPERATURE COEFFICIENT OF RESISTANCE. For copper, the value is approximately 0.00427 ohms. For pure metals, the temperature coefficient of resistance ranges between 0.003 and 0.006 ohms.

Thus, a copper wire having a resistance of 50 ohms at an initial temperature of 0° C. will have an increase in resistance of 50×0.00427, or 0.214 ohms for the entire length of wire for each degree temperature rise above 0° C. At 20° C. the increase in resistance is approximately 20×0.214, or 4.28 ohms. The total resistance at 20° C. is $50+4.28$, or 54.28 ohms.

Figure 6–6 shows how the resistance of copper wire varies with temperature. At temperatures likely to be encountered under ordinary circumstances resistance increases uniformly with temperature. However, at very low temperatures resistance does not increase uniformly with temperature, as is shown by the curved portion of the solid line. If the straight-line portion of the curve in the vicinity of 0° C. is extended along the dotted line to point A, it will be seen that copper acts as if it had zero resistance at −234.5° C. This value is numerically equal to the reciprocal of the temperature coefficient of resistance of copper at 0° C., or $\frac{1}{0.00427}$.

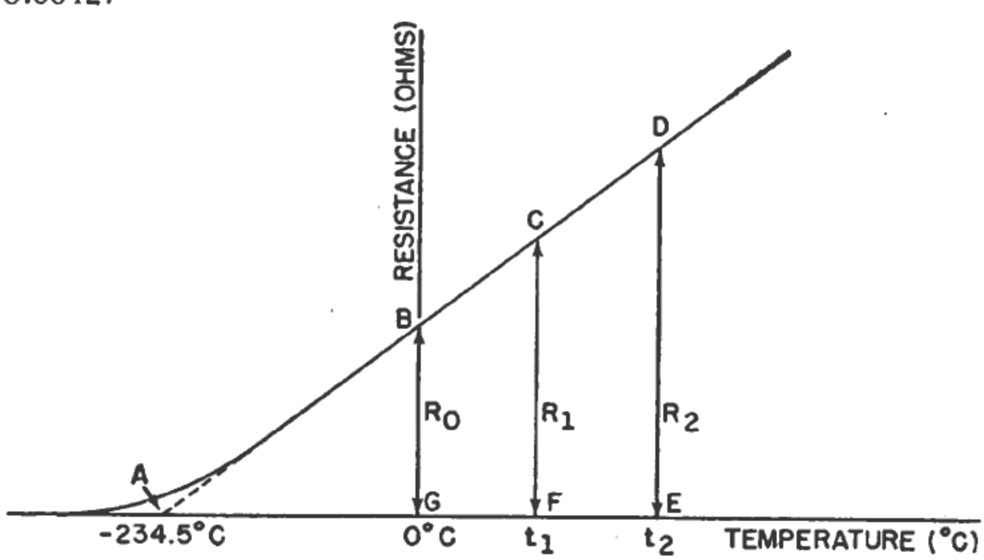


Figure 6–6.—Graph of resistance versus temperature for a copper conductor.

Assuming the graph of temperature versus resistance to be a straight line, the effect of temperature on resistance may be estimated over the normal range of operating temperatures with a fair degree of accuracy. By the law of similar triangles,

$$\frac{CF}{AF}=\frac{DE}{AE}.$$

By substituting the resistance and temperature values,

$$\frac{R_1}{234.5+t_1}=\frac{R_2}{234.5+t_2},$$

or

$$\frac{R_1}{R_2}=\frac{234.5+t_1.}{234.5+t_2}.$$

This equation applies to copper wire. However, it is also applicable for any other type of wire provided the proper constant (the last column of table 7) is substituted for 234.5 in the preceding equation. Table 7 indicates the temperature coefficient of resistance at 0° C. for some of the metals in general use, and the projected temperature at which they would appear to have zero resistance.

TABLE 7.—*Temperature coefficient of resistance*

Substance	Temperature coeff. per degree centigrade per ohm at 0° C.	Reciprocal of temperature coeff.
Silver	0. 00411	−243. 3
Copper	0. 00427	−234. 5
Gold	0. 00365	−274
Aluminum	0. 00420	−238. 1
Tungsten	0. 0049	−204
Steel (soft)	0. 00458	−218. 3

The following examples illustrate the practical value of considering the temperature coefficient of resistance in circuits where there is an appreciable change in temperature during operation.

Example 1. Assume that the resistance of an aluminum wire is 100 ohms at 20° C. What is the resistance at 80° C.?

Solution:

$$\frac{R_1}{R_2}=\frac{238.1+t_1}{238.1+t_2}$$

$$\frac{100}{R_2}=\frac{238.1+20}{238.1+80}$$

$$R_2 = \frac{318.1 \times 100}{258.1} = 123.25 \text{ ohms.}$$

Example 2. Assume that the resistance of a copper coil is 100 ohms at 20° C. What is the average operating temperature of the coil when the resistance of the coil has increased to 116 ohms?

Solution:

$$\frac{R_1}{R_2} = \frac{234.5 + t_1}{234.5 + t_2}$$

$$\frac{100}{116} = \frac{234.5 + 20}{234.5 + t_2}$$

$$t_2 = 60° \text{ (approx.)}$$

Example 3. The resistance of copper wire increases about 1 percent for each 2.5° C. rise in temperature. Using this relation, find the resistance of a copper wire at 60° C. if its resistance at 20° C. is 30 ohms.

Solution: The temperature rise is 60°–20°, or 40° C. The percentage of increase in resistance corresponding to a 40° C. rise in temperature is $\frac{40}{2.5}$, or 16 percent. The resistance at 60° C. is equal to 30×1.16, or 34.8 ohms.

Example 4. The resistance of a transformer winding (copper wire) before loading measured 10 ohms (at 20° C.). After 2 hours of operation at full load the transformer was shut down and the resistance of the winding taken again. This time it measured 13 ohms. If the allowable temperature rise is 50° C., did this transformer meet the temperature specification? If not, what was the excess temperature?

Solution:

$$\frac{R_1}{R_2} = \frac{234.5 + t_1}{234.5 + t_2}$$

$$\frac{10}{13} = \frac{234.5 + 20}{234.5 + t_2}$$

$$t_2 = \frac{963.5}{10} = 96.35° \text{ C.}$$

$$\text{Excess temperature} = 96.35 - 70 = 26.35° \text{ C.}$$

CONDUCTOR INSULATION

Two fundamental properties of insulation materials (for example, rubber, glass, asbestos, and plastic) are insulation resistance and dielectric strength. These are entirely different and distinct properties.

INSULATION RESISTANCE is the resistance to current leakage through and over the surface of insulation materials. Insulation resistance can be measured by means of a megger without damaging the insulation, and data so obtained serves as a useful guide in appraising the general condition of insulation. However, the data obtained in this manner may not give a true picture of the condition of the insulation. Clean-dry insulation having cracks or other faults may show a high value of insulation resistance but would not be suitable for use.

DIELECTRIC STRENGTH is the ability of the insulator to withstand potential difference and is usually expressed in terms of the voltage at which the insulation fails because of the electrostatic stress. Maximum dielectric strength values can be measured by raising the voltage of a TEST SAMPLE until the insulation breaks down.

In many cases, wires that are strung on poles are not covered with an insulation material. However, they are supported by means of insulators (glass or porcelain) attached to the crossarm. Wires and cables that are used underground or in buildings, ships, or aircraft are placed relatively close together for convenience and space conservation. This means that each wire or cable must be covered with a nonconducting or insulating material to prevent short circuits. The type of insulation that is used depends on the type (a-c or d-c) and the amount of voltage applied to the line, the current through the line, and the physical surroundings in which the line is to be operated.

Because of the expense of insulation and its stiffening effect, together with the great variety of physical and electrical conditions under which the conductors are operated, only the necessary minimum of insulation is applied for any particular type of cable designed to do a specific job. Therefore there is a wide variety of insulated conductors available to meet the requirements of any job.

Rubber

One of the most common types of insulation is rubber. The voltage that may be applied to a rubber-covered pair of conductors (twisted pair) is dependent on the thickness and the quality of the rubber covering. Other factors being equal, the thicker the insulation the higher may be the applied voltage. Figure 6–7 shows two types of rubber-covered wire. One is a single, solid conductor, and the other is a 2-conductor cable in which each stranded conductor is covered with rubber insulation. In each case the rubber serves the same purpose—to confine the current to its conductor.

It may be seen from the enlarged cross-sectional view that a thin coating of tin separates the copper conductor from the rubber insulation. If the thin coating of tin were not used, chemical action would take place and the rubber would become soft and gummy where it makes contact with the copper. When small, solid, or stranded conductors are used, a winding of cotton threads is applied between the conductors and the rubber insulation.

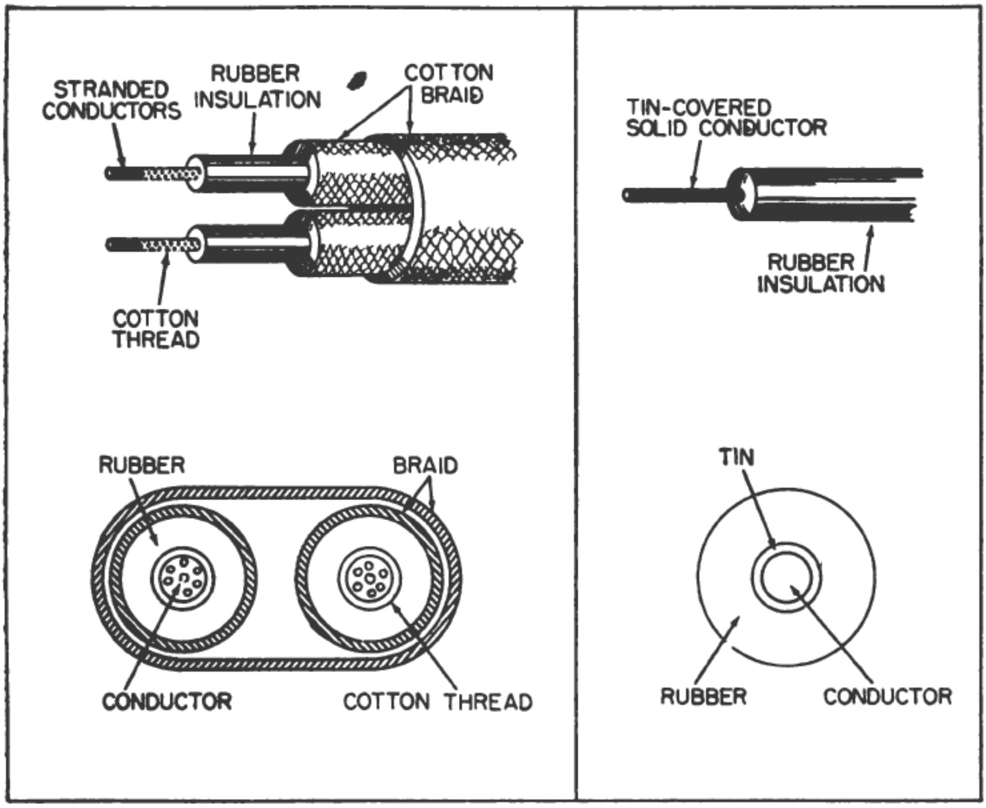


Figure 6–7.—Rubber insulation.

Varnished Cambric

Heat is developed when current flows through a wire, and when a large amount of current flows, considerable heat may be developed. The heat can be dissipated if air is circulated freely around the wire. If a cover of insulation is used, the heat is not removed so readily and the temperature may reach a high value.

Rubber is a good insulator at relatively low voltage as long as the temperature remains low. Too much heat will cause even the best grade of rubber insulation to become brittle and crack. VARNISHED CAMBRIC insulation will stand much higher temperatures than rubber insulation. Varnished cambric is cotton cloth that has been coated with an insulating varnish. Figure 6–8 shows some of the detail of a cable covered with varnished cambric insulation. The varnished cambric is in tape form and is wound around the conductor in layers. An oily compound is applied between each layer of the tape. This compound prevents water from seeping through the insulation. It also acts as a lubricant between the layers of tape, so they will slide over each other when the cable is bent.

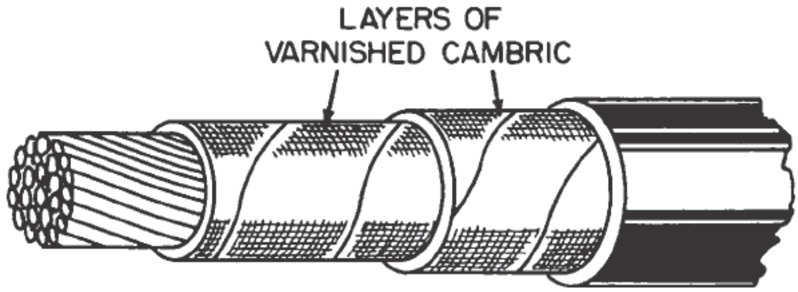


Figure 6–8.—Varnished cambric insulation.

This type of insulation is used on high-voltage conductors associated with switch gear in substations and power houses and other locations subjected to high temperatures. It is also used on high-voltage generator coils and leads, and also on transformer leads because it is unaffected by oils or grease and because it has a high dielectric strength. Varnished cambric and paper insulation for cables are the two types of insulating materials most widely used at voltages above 15,000 volts but such cables are always lead covered to keep the moisture out.

Asbestos

Even varnished cambric may break down when the temperature goes above 85° C. When the combined effects of a high ambient (surrounding) temperature and a high internal temperature due to large current flow through the wire makes the total temperature of the wire go above 85° C., ASBESTOS insulation is used.

Asbestos is a good insulation for wires and cables used under very high-temperature conditions. It is fire resistant and does not change with age. One type of asbestos-covered wire is shown in figure 6–9. It consists of a stranded copper conductor covered

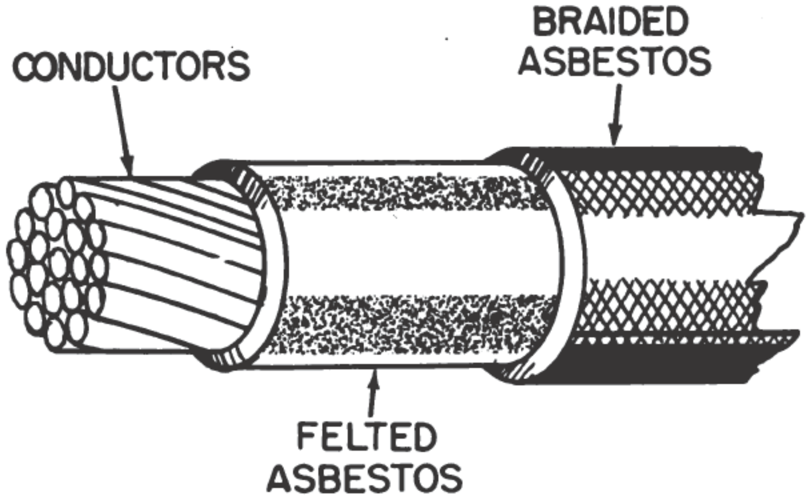


Figure 6–9.—Asbestos insulation.

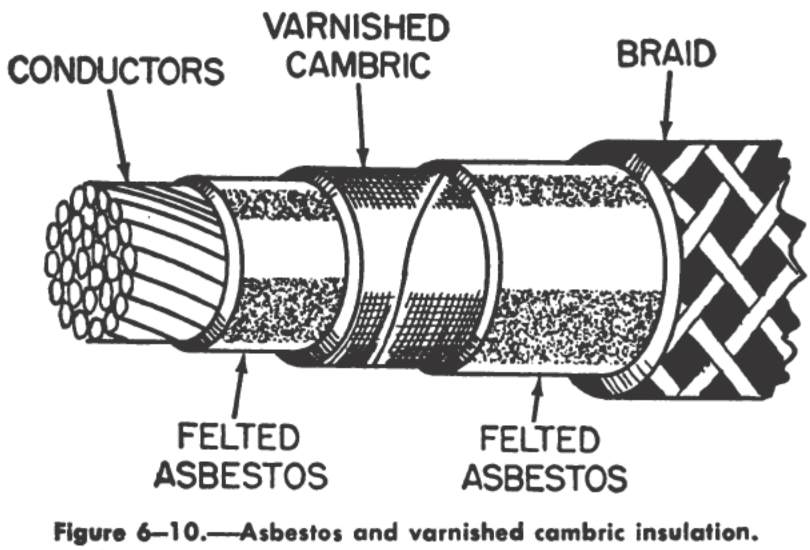


Figure 6–10.—Asbestos and varnished cambric insulation.

with felted asbestos, which is in turn covered with asbestos braid. This type of wire is used in motion-picture projectors, arc lamps, spotlights, heating element leads, and so forth.

Another type of asbestos-covered cable is shown in figure 6–10. It serves as leads for motors and transformers that sometimes must operate in hot, wet locations. The varnished cambric covers the inner layer of felted asbestos and prevents moisture from reaching the innermost layer of asbestos. Asbestos loses its insulating properties when it becomes wet, and will in fact become a conductor. The varnished cambric prevents this from happening because it resists moisture. Although this insulation will withstand some moisture, it should not be used on conductors that may at times be partly immersed in water, unless the insulation is protected with an outer lead sheath.

Paper

Paper has little insulation value alone, but when impregnated with a high grade of mineral oil it serves as a satisfactory insulation for high-voltage cables. The oil has a high dielectric strength, and tends to prevent breakdown of paper insulation when the paper is thoroughly saturated with it. The thin paper tape is wrapped in many layers around the conductors, and it is then soaked with oil.

The 3-conductor cable shown in figure 6–11 consists of paper insulation on each conductor with a spirally wrapped nonmagnetic tape over the insulation. The space between conductors is filled

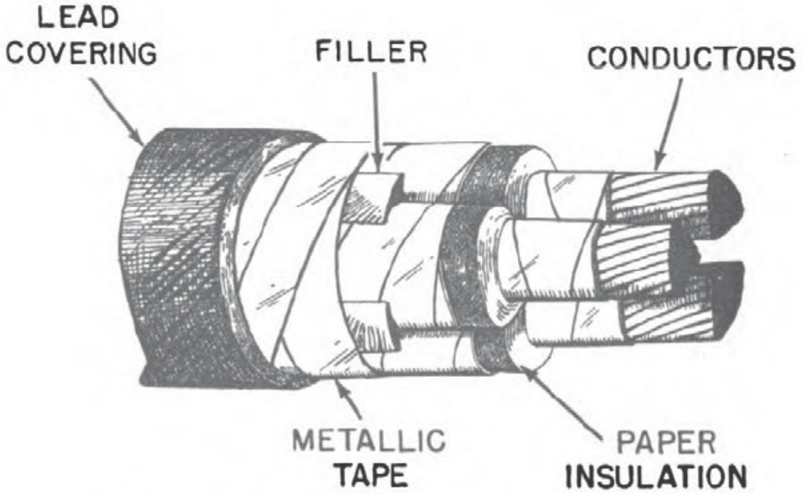


Figure 6–11.—Paper insulated power cables.

with a suitable spacer to round out the cable and another non-magnetic metal tape is used to secure the entire cable, and then a lead sheath is applied over all. This type of cable is used on voltages from 10,000 volts to 35,000 volts.

Silk and Cotton

In certain types of circuits—for example, communications circuits—a large number of conductors are needed, perhaps as many as several hundred. Figure 6–12 shows a cable containing many conductors, each insulated from the others by silk and cotton threads.

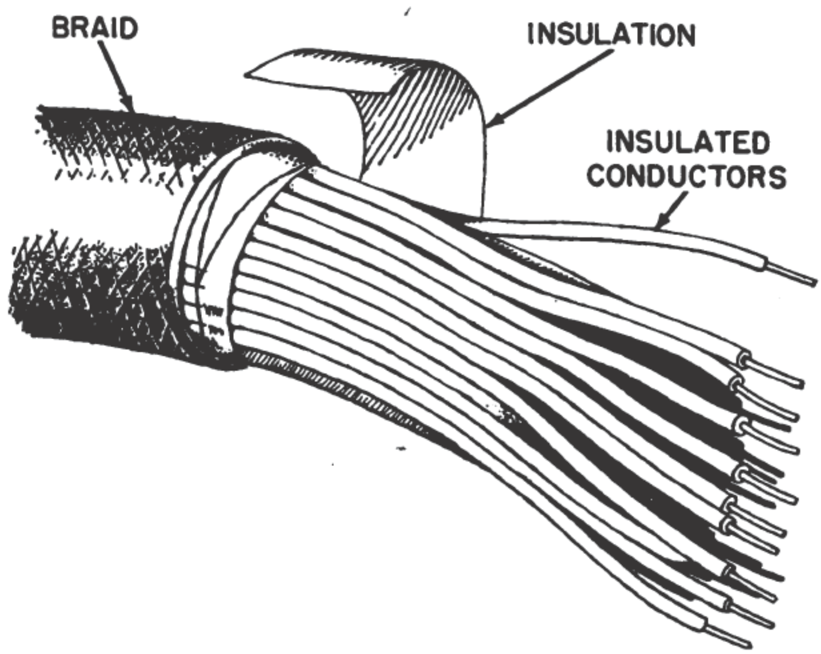


Figure 6–12.—Silk and cotton insulation.

The use of silk and cotton as insulation keeps the size of the cable small enough to be handled easily. The silk and cotton threads are wrapped around the individual conductors in reverse directions, and the covering is then impregnated with a special wax compound.

Because the insulation in this type of cable is not subjected to high voltage, thin layers of silk and cotton are used.

Enamel

The wire used on the coils of meters, relays, small transformers, and so forth, is called MAGNET WIRE. This wire is insulated with

an enamel coating. The enamel is a synthetic compound of cellulose acetate (wood pulp and magnesium). In the manufacture, the bare wire is passed through a solution of the hot enamel and then cooled. This process is repeated until the wire acquires from 6 to 10 coatings. Enamel has a higher dielectric strength than rubber for equal thickness. It is not practical for large

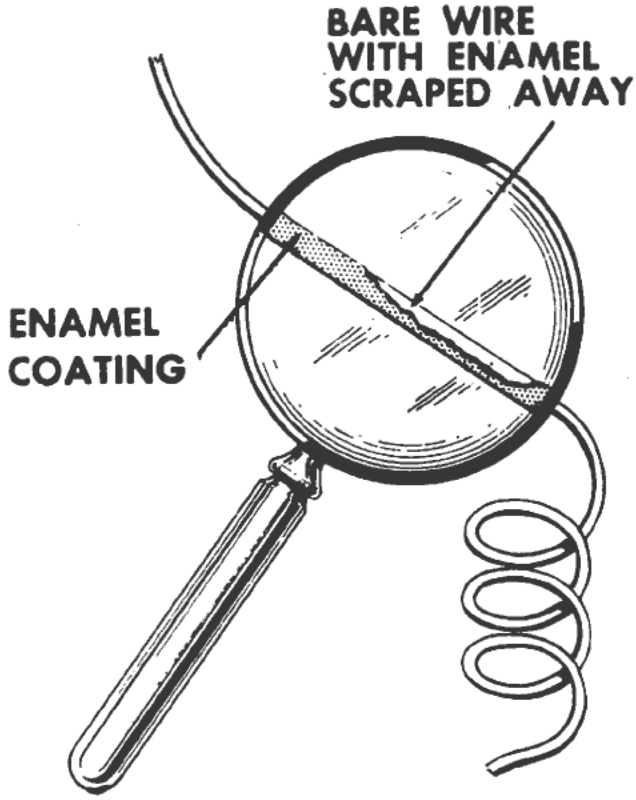


Figure 6–13.—Enamel insulation.

wires because of the expense and because the insulation is readily fractured when large wires are bent.

Figure 6–13 shows an enamel-coated wire. Enamel is the thinnest insulating coating that can be applied to wires. Hence, enamel-insulated magnet wire makes smaller coils. Enameled wire is frequently covered with one or more layers of cotton covering to protect the enamel from nicks, cuts, or abrasions.

CONDUCTOR PROTECTION

Wires and cables are generally subject to abuse. The type and amount of abuse depends on how and where they are installed and on the way they are used. Cables buried directly in the ground must resist moisture, chemical action, and abrasion. Wires installed in buildings must be protected against mechanical injury and overloading. Wires strung on crossarms on poles are kept far enough apart so that they do not touch; but snow, ice, and strong winds necessitate the use of conductors having high tensile strength and substantial supporting frame structures.

Generally, except for overhead transmission lines, wires or cables are protected by some form of covering. The covering may be some type of insulator like rubber or plastic. Over this an outer covering of fibrous braid may be applied. If conditions require, a metallic outer covering may be used. The type of outer covering used depends on how and where the wire or cable is to be used.

Fibrous Braid

Cotton, linen, silk, rayon, and jute are types of FIBROUS BRAIDS. They are used for outer covering under conditions where the wires or cables are not exposed to heavy mechanical injury. Interior wiring for lights or power is usually done with impregnated cotton braid-covered rubber-insulated wire. Generally, the wire will be further protected by a flame-resistant nonmetallic outer covering or by a flexible or rigid conduit.

Figure 6–14 shows a typical building wire. In this instance two braid coverings are used for extra protection. The outer braid is soaked with a compound that resists moisture and flame.

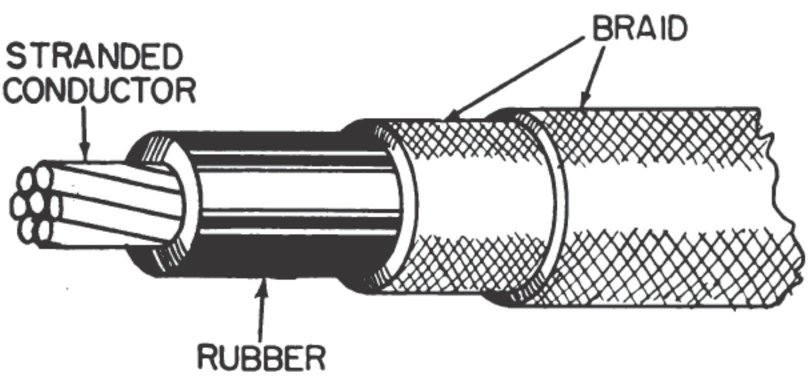


Figure 6–14.—Fibrous braid covering.

Impregnated cotton braid is used as a covering for outdoor overhead conductors to afford protection against abrasion. For example, the service wires from the transformer secondary mains to the service entrance and also the high voltage primary mains to the transformer are protected in this manner.

Lead Sheath

Subway-type cables or wires that are continually subjected to water must be protected by a watertight cover. This watertight cover is made either of a continuous lead jacket or a rubber sheath molded around the cable.

Figure 6–15 is an example of a lead-sheathed cable used in power work. The cable shown is a stranded 3-conductor type. Each conductor is insulated with rubber and then wrapped with a layer of rubberized tape. The conductors are twisted together and fillers or rope are added to form a rounded core. Over this is wrapped a second layer of tape called the SERVING, and finally the lead sheath is molded around the cable.

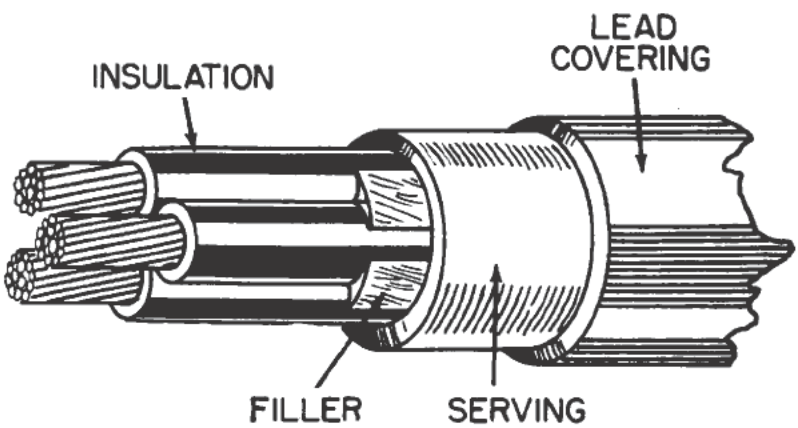


Figure 6–15.—Lead-sheathed cable.

Metallic Armor

Metallic armor provides a tough protective covering for wires or cables. The type, thickness, and kind of metal used to make the armor depend on the use of the conductors, the circumstances under which the conductors are to be used, and on the amount of rough treatment that is to be expected.

Four types of metallic armor for cables are shown in figure 6–16.

Wire braid armor is used wherever light, flexible protection is needed. This type of armor is used almost exclusively aboard ship. The individual wires that are woven together to form the metal braid may be made of steel, copper, bronze, or aluminum. Besides mechanical protection, the wire braid also presents a static shield. This is important in radio work aboard ship to prevent interference from stray fields.

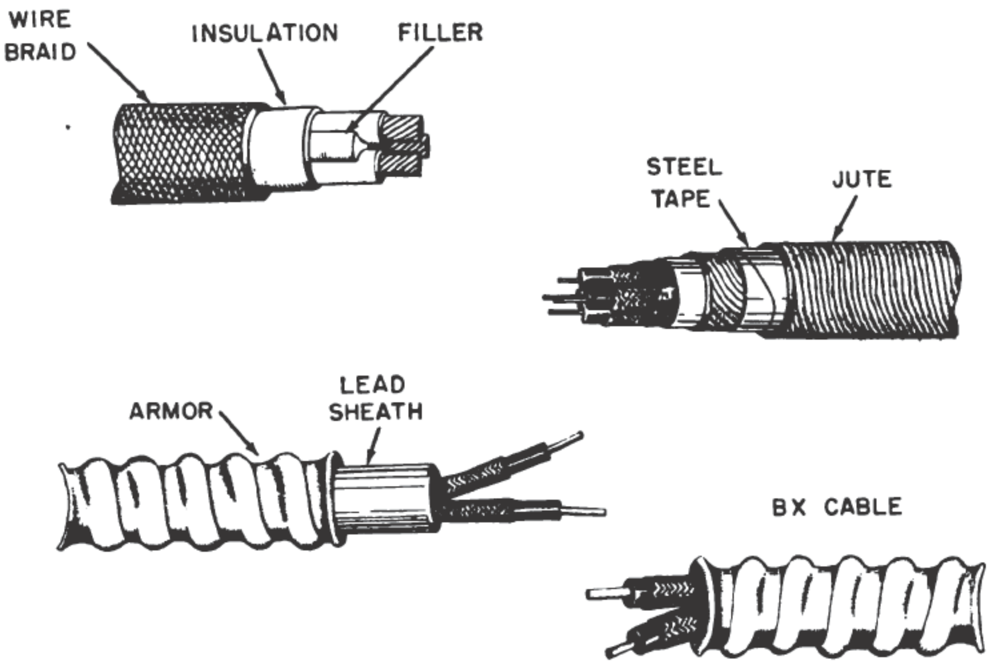


Figure 6–16.—Metallic armor.

When cables are buried directly in the ground, they might be injured from two sources—moisture, and abrasion. They are protected from moisture by a lead sheath, and from abrasion by steel tape or interlocking armor covers. The steel tape covering, as shown in figure 6–16, is wrapped around the cable and then covered with a serving of jute. It is known as Parkway cable. The interlocking armor covering can withstand impacts better than steel tape. Interlocking armor has other uses besides underground work. In wiring the interior of buildings, interlocking armor-covered wire (BX cable) without the lead sheath is frequently used.

Armor wire is the best type of covering to withstand severe wear and tear. Underwater leaded cable usually has an outer armor wire cover.

All wires and cables do not have the same type of protective covering. Some coverings are designed to withstand moisture, others to withstand mechanical strain, and so forth. A cable may have a combination of each type, each doing its own job.

CONDUCTOR SPLICES

The splicing of conductors is important because if a splice fails, the electrical system fails. A good splice is one that has the same strength and electrical conductivity as the wires it joins. The splice is strong when it can stand the same pull as any other part of the wire; it has good electrical conductivity when there is no increase in resistance because of the splice.

The first, and perhaps the most important, step in making a splice is preparing the wire. In skinning the insulation from a wire, the knife is handled in much the same manner as in sharpening a pencil—that is, the blade is moved at a small angle with the wire to avoid "nicking" it. This produces a taper on the cut insulation, as shown in the lower figure of figure 6–17. The

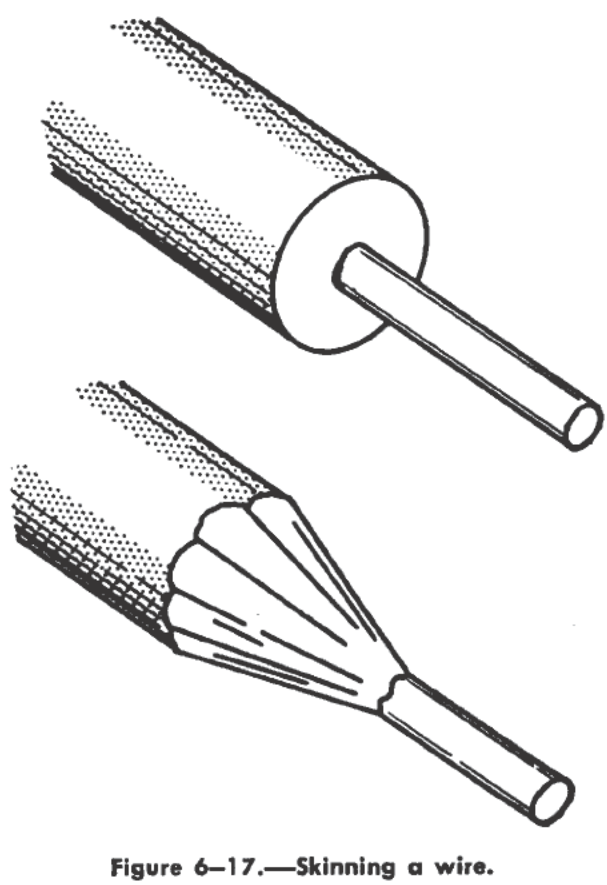


Figure 6–17.—Skinning a wire.

insulation is not cut at right angles to the conductor, as shown in the upper figure of figure 6–17, because the sharp edge of the knife will nick the conductor. If there is an outer covering, it should be cut back a short distance from the end of the insulation.

The back edge of the knife is used to scrape all traces of dirt and insulation from the bare conductors. This procedure ensures good electrical contact and makes soldering easier. When a stranded wire is being spliced, each individual wire should be cleaned in order to ensure a good soldering job.

Western Union Splice

Small, solid conductors may be joined together by a simple connection known as the WESTERN UNION SPLICE. In most instances the wires may be twisted together with the fingers and the ends clamped into position with a pair of pliers.

Figure 6–18 shows the steps in making a Western Union splice. First, the wires are prepared for splicing by removing about three inches of insulation and cleaning the conductor. Next, the wires are brought to a crossed position about one inch from the insulation and a long twist or bend is made in each wire. Then one of the wire ends is wrapped four or five times around the straight portion of the wire. The other end wire is wrapped in

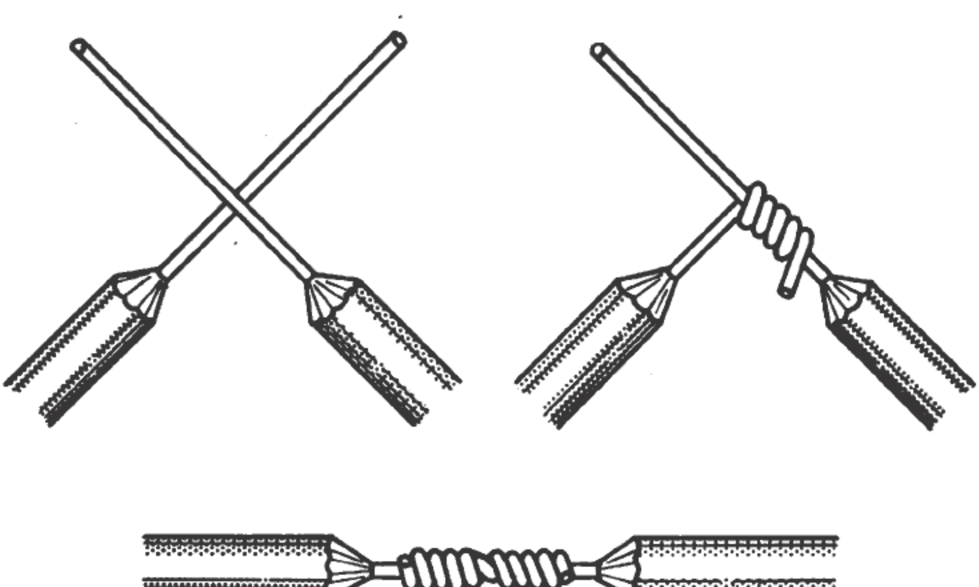


Figure 6–18.—Western Union splice.

a similar manner. Finally, the ends of the wires should be pressed down as close as possible to the straight portion of the wire. This prevents the sharp ends from puncturing the tape covering that is wrapped over the splice.

Staggered Splice

Joining small, multiconductor cables together presents somewhat of a problem. Each conductor must be spliced and taped; and if the splices are directly opposite each other, the over-all size of the joint becomes large and bulky. A smoother and less bulky joint may be made by staggering the splices.

Figure 6–19 shows how a 2-conductor cable is joined to a similar cable by means of the staggered splice. First, about eight inches of the outer braid is removed. The insulation is then stripped back to give unequal lengths of bare conductors. The Western Union splice is then used, as shown. Care should be exercised to ensure that a short bare wire is connected to a long bare wire, and that the sharp ends are clamped firmly down on the conductor.

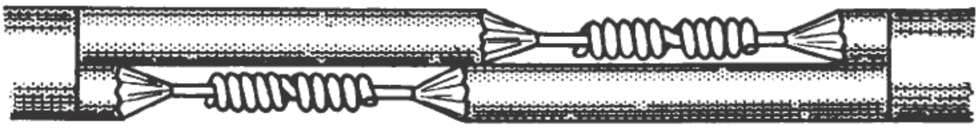


Figure 6–19.—Staggered splice.

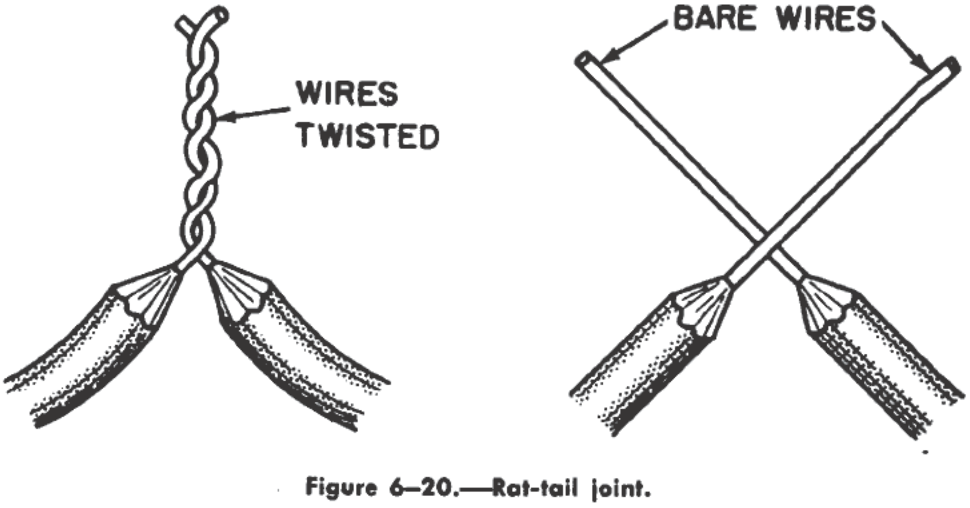


Figure 6–20.—Rat-tail joint.

Rat-Tail Joint

Wiring that is installed in buildings is usually placed inside long lengths of steel pipe or conduit. Whenever branch circuits are required, junction or pull boxes are inserted in the conduit. One type of splice that is used for branch circuits is the rat-tail joint shown in figure 6–20.

The ends of the conductors to be joined are stripped of about two inches of insulation. The wires are then twisted to form the rat-tail effect.

Fixture Joint

A fixture joint is used to connect a light fixture to the branch circuit of an electrical system where the fixture wire is smaller in diameter than the branch wire. Like the rat-tail joint, it will not stand much mechanical strain.

The first step is to remove the insulation from the wires to

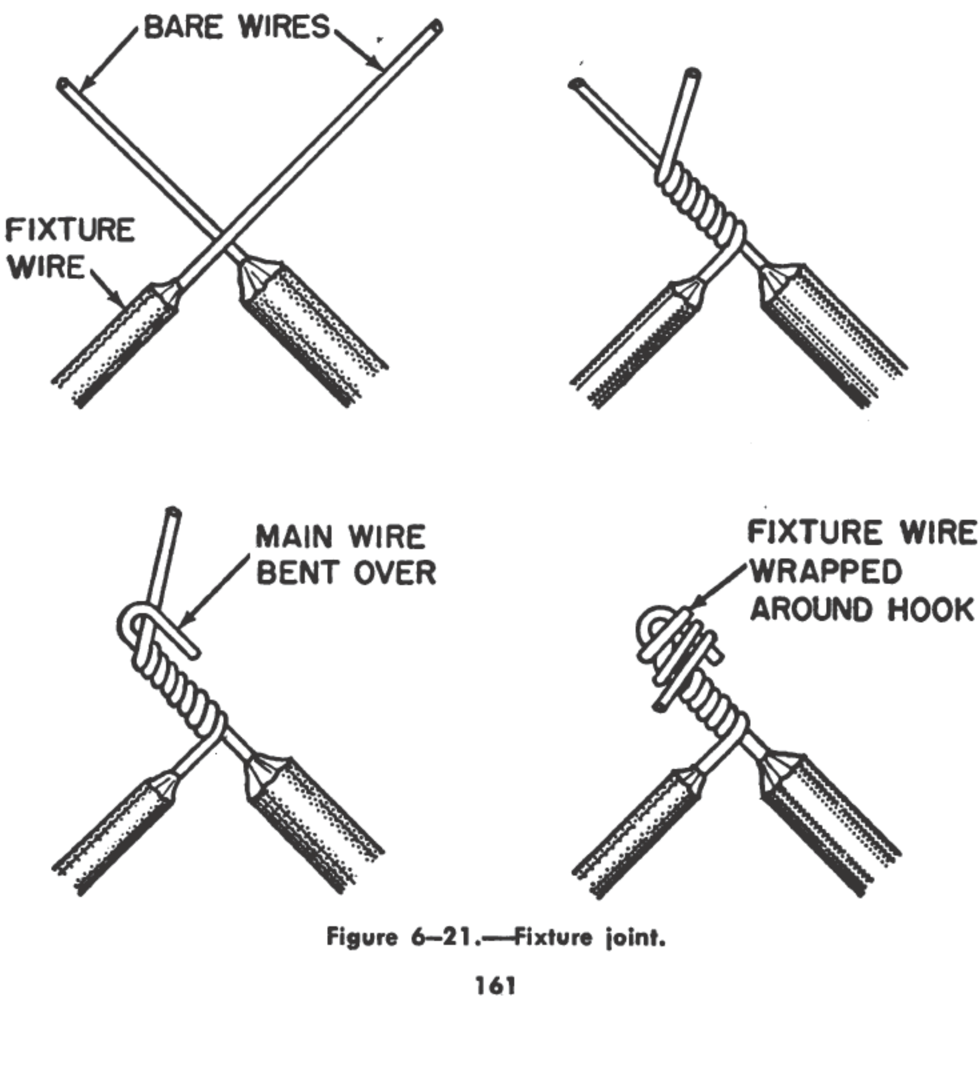


Figure 6–21.—Fixture joint.

be joined. About three inches of insulation is stripped from the fixture wires and one inch from the branch wires. Figure 6–21 shows the steps in making a fixture joint.

After the wires are prepared, the fixture wire is wrapped four or five times around the branch wire, as shown in the figure. The wires are not twisted, as in the rat-tail joint. The end of the branch wire is then bent over the completed turns. The remainder of the bare fixture wire is then wrapped over the bent branch wire. Soldering and taping completes the job.

Knotted Tap Joint

All of the splices considered up to this point are known as BUTTED splices. Each was made by joining the FREE ends of the conductors together. Sometimes, however, it is necessary to join a conductor to a CONTINUOUS wire, and such a junction is called a TAP joint.

The main wire, to which the branch wire is to be tapped, has about one inch of insulation removed. The branch wire is stripped of about three inches of insulation. The steps in making the tap are shown in figure 6–22.

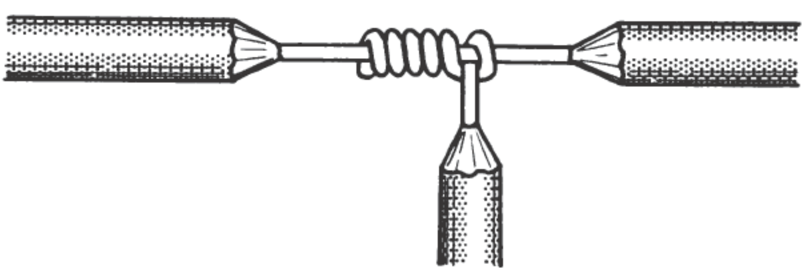


Figure 6–22.—Knotted tap joint.

The branch wire is crossed over the main wire, as shown in the figure, with about three-fourths of the bare portion of the branch wire extending above the main wire. The end of the branch wire is bent over the main wire, brought under the main wire, around the branch wire, and then over the main wire to form a knot. It is then wrapped around the main conductor in short, tight turns and the end is trimmed off.

The knotted tap is used where the splice is subject to strain or slip. When there is no mechanical strain the knot may be eliminated.

SOLDERING THE SPLICE

A splice is never completely finished until it has been soldered and taped. Soldering a splice helps to protect it from corrosion; provides an increased area of contact, thus increasing the conductivity; and increases the mechanical rigidity of the joint, although the solder must not be depended upon to provide the mechanical strength necessary to keep the wires or cables from pulling apart.

One of the most important steps in soldering a splice is TINNING THE COPPER, or the ELECTRIC SOLDERING IRON, whichever is used. Although the tip of an electric soldering iron is made of copper, the instrument is commonly called a soldering iron.

Tinning the Copper

The copper is heated, often by means of a blowtorch, until it is hot enough to melt solder. The beveled sides of the copper are filed smooth, flux is added, and an even coating of solder is applied. A damp cloth is used to wipe off the excess solder. If the soldering copper is heated too hot on a job, it will be necessary to repeat the tinning process.

Soldering coppers should be kept clean and bright. In order to keep them in top condition, frequent tinning may be necessary.

Electric soldering irons also require careful attention. They are tinned in much the same way that soldering coppers are tinned. Electric soldering irons generally are supplied with an assortment of tips to suit the needs of a particular soldering job. Tips should be loosened occasionally to ensure that the threads will not become coated with oxide to such an extent that the tip cannot be easily removed.

Where exceedingly precise soldering work is done—for example, in electronic instrument repair shops—special tinning and soldering techniques are employed.

Solder and Fluxes

Solder is a low-melting-point alloy made of tin and lead, and comes in bar or wire form. In the wire form, the flux is often contained in the core.

Rosin or some other noncorrosive flux is used in soldering electrical connections. The flux cleans the surface of the metal to be soldered and prevents oxides from forming during the soldering operation. Oxides will prevent the solder from adhering properly to the surface of the metal. Acid flux is NOT used in making electrical connections, because it will cause the joint to corrode.

Soldering Procedure

After the iron has been prepared for soldering—tinned and heated to soldering temperature—the actual soldering operation is performed as follows:

1. If the flux is not contained in the core of the solder, a small amount of flux paste is applied to the joint.
2. The soldering copper or soldering iron is held in close contact with the joint to be soldered until the joint is heated enough to make the applied solder flow. If convenient, the iron may be held under the joint, and the solder applied to the top of the joint.
3. Excess solder should not be left on a joint. Leave the soldering copper or iron on the joint long enough to make the solder flow evenly through the joint, but do not apply more solder than is necessary.
4. In summary: The splice itself must be hot enough to melt the solder; and time should be allowed for the splice to heat up. The wire has reached the right temperature if when the soldering iron is placed on one side of the joint and solder on the other, the solder melts and flows into all of the cracks and hidden spaces in the joint. Do not use more solder than is necessary to secure a firm joint.

A gasoline blowtorch or alcohol torch is used for certain soldering jobs. The blowtorch is used for soldering splices of large wire where it would be impracticable to use a soldering copper. It is also used for soldering conductors into terminal lugs.

After the splice has been fluxed, the blowtorch is applied until the temperature of the splice is above the melting point of the solder being used. The solder is applied from the top until it flows freely into the spaces between the wires and over the surface of the wires. Care should be taken to avoid burning the insulated portions of the wire adjacent to the splice.

Terminal lugs are attached as follows:

1. After the necessary amount of insulation is removed, the wire is covered with flux and the blowtorch is applied. When the wire is hot enough, the end of the wire is tinned by applying the solder. Care should be taken to avoid burning the insulation.
2. Flux is applied to the lug, and it is tinned by holding it (by means of pliers) in the flame and applying solder until the lug is completely filled. The tinned portion of the conductor is then inserted into the lug while it is still held in the flame.
3. The conductor and the lug are removed from the flame. The joint should be cooled rapidly, for example, by touching the lug with a damp rag in order to set the solder before any relative motion between the lug and conductor can occur. Such motion during the time the solder is cooling would create a poor connection.
4. In order to preserve a neat appearance, the excess solder is removed from the exterior of the lug.

An alcohol torch (having a very small flame) is sometimes used for making soldered connections when its use is more convenient than the gasoline blowtorch. Because the flame is more concentrated, it may be used in smaller spaces; and insulation is less likely to be burned.

TAPING A SPLICE

Taping a splice is the final step (when a metal shielding is not used) in joining wires and cables.

Rubber Tape

When rubber-insulated, braid-covered wires are joined together, the insulation must first be restored to the splice. To fill in the insulation a rubber SPLICING COMPOUND is used. This compound is in the form of a RUBBER TAPE. The tape is applied to the splice with a light tension so that each layer presses tightly against the one underneath it. This pressure causes the rubber tape to blend into a solid mass. When the application is completed, an insulation similar to the original is restored.

Between each layer on the roll of rubber tape there is a layer

of paper or treated cloth that prevents the layers of rubber from fusing together when they are still on the roll. The paper or cloth is discarded when the tape is applied to the splice.

Figure 6–23 shows the correct way to cover a splice with rubber insulation. The tape is started at one end of the splice, beginning at the point where the original insulation ends. Be sure to apply the overlapping turns of tape at a slight angle to the conductor, and keep the tape under tension. The width of the tape gives an indication of the tension. The tape should be stretched to about three-quarters of its original width, and is wrapped until the end of the splice is reached. The direction of wrap is reversed, and a second layer is added. It may be necessary to add a third layer, depending on the thickness of the original insulation. In any case, the thickness of the applied insulation should be slightly greater than that of the original insulation. The applied tape should be lapped a slight amount over on the original insulation.

Figure 6–23.—Applying rubber tape.

Friction Tape

Putting rubber tape over the splice means that the insulation has been restored to a great degree. It is also necessary to re-

store the protective covering. Friction tape is used for this purpose; it also affords a minor degree of electrical insulation.

Friction tape is a cotton cloth that has been treated with a sticky rubber compound. It comes in rolls, similar to rubber tape except that no paper or cloth separator is used. Friction tape is applied like rubber tape, however, it does not stretch.

The friction tape should be started slightly back on the original braid covering. Wind the tape so that each turn overlaps the

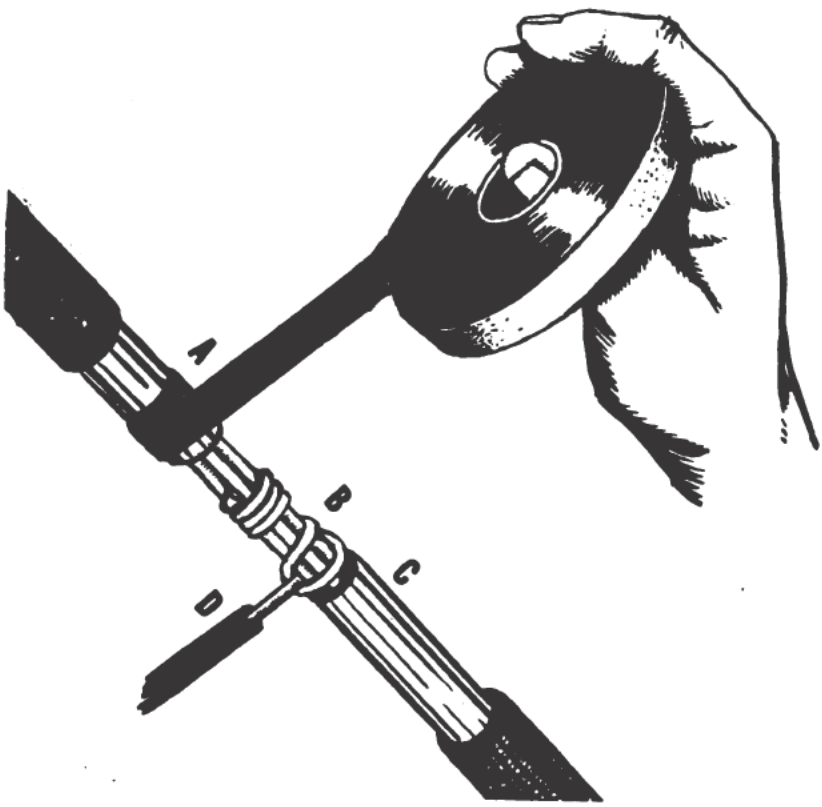


Figure 6–24.—Taping a tapped splice.

one before it; and extend the tape over onto the braid covering at the other end of the splice. From this point a second layer is wound back along the splice until the original starting point is reached. Cutting the tape and firmly pressing down the ends complete the job. When proper care is taken, the splice can take as much abuse as the rest of the wire.

Weatherproof wire has no rubber insulation, just a braid covering. In that case, no rubber tape is necessary, only friction tape need be used.

Tapped splices require more care in taping than butted joints. The main wire must be completely covered as well as the branch wire that is tapped into it. Figure 6–24 shows a simple tapped splice with the tape in place.

The best way to tape this kind of splice is to start at point *A* and wrap a layer of tape to point *C*. The tape is then wrapped back to point *B* and down the branch wire to point *D*. From *D* the tape is wrapped back to point *B*, and thus to point *A*. The rubber and friction tape are wrapped in the same manner, except that the friction tape is extended farther along the wire.

Taping a rat-tail joint also requires some care to ensure that all parts of the splice are completely covered. This applies equally well to all splices. If the wraps are tight and neat and excess tape is not used, a good job will be ensured.

Plastic Electrical Tape

Plastic electrical tape has come into wide use in recent years. It has certain advantages over rubber and friction tape. For example, it will withstand higher voltages for a given thickness. Single thin layers of certain commercially available plastic tape will stand several thousand volts without breaking down. However, to provide an extra margin of safety several layers are usually wound over the splice. Because the tape is very thin the extra layers add only a very small amount of bulk; but at the same time the added protection, normally furnished by friction tape, is provided by the additional layers of plastic tape. In the choice of plastic tape, the factor of expense must be balanced against the other factors involved.

Plastic electric tape normally has a certain amount of stretch so that it easily conforms to the contour of the splice without adding unnecessary bulk. The lack of bulkiness is especially important in some junction boxes where space is at a premium.

For high temperatures—for example, above 175° F.—a special type of tape backed with glass cloth is used.

QUIZ

1. **Define a unit size conductor of 1 mil-foot.**
2. **A mil is what fractional part of an inch?**

3. How many square mils are in the cross-sectional area of a rectangular bus bar ¼ inch thick and 3.5 inches wide?
4. What is the cross-sectional area in circular mils of a round conductor having a diameter of 1 mil?
5. What is the cross-sectional area in circular mils of a round wire having a diameter of 0.1 inch?
6. A bus bar is 3 inches wide and 0.375 inch thick.
   (a) What is its cross-sectional area in square mils?
   (b) What is its cross-sectional areas in circular mils?
   (c) How many round conductors, each 0.1 inch in diameter, will be required to provide an equivalent total cross-sectional area?
7. What is the specific resistance (circular-mil-foot) at 20° C. of:
   (a) Drawn copper?
   (b) Aluminum?
   (c) Nichrome?
8. Give the formula for the resistance of a conductor in terms of the specific resistance, ρ, in ohms per circular-mil foot; the length, L, in feet; and the cross-sectional area, A, in circular mils.
9. What is the resistance of 1,000 feet of No. 14 drawn copper wire that has a cross-sectional area of 4,110 circular mils, the temperature being 20° C.?
10. If the cross-sectional area of a No. 10 wire is approximately 10,000 circular mils, what is the approximate cross-sectional area of:
    (a) No. 13 wire?
    (b) No. 20 wire?
    (c) No. 0 wire?
11. If the approximate resistance of 1,000 feet of No. 10 copper wire is 1 ohm, what is the approximate resistance of 1,000 feet of:
    (a) No. 40 copper wire?
    (b) No. 23 copper wire?
    (c) No. 0 copper wire?
12. If the resistance of 1,000 feet of No. 18 copper wire is approximately 6.3 ohms, what is the approximate resistance of 1,000 feet of No. 17 copper wire?
13. Give three factors that must be considered in selecting the size of wire to be used for transmitting and distributing electric power.
14. What is the temperature coefficient of resistance of copper at zero degrees centigrade?
15. If the resistance of an aluminum wire is 50 ohms at 18° C., what is the resistance at 50° C.?

16. If the resistance of a copper coil is 90 ohms at 23° C., what is the average operating temperature of the coil when the resistance of the coil has increased to 105 ohms?
17. Using the relation that the resistance of copper wire increases about 1 percent for each 2.5° C. rise in temperature, find the resistance of a copper wire at 80° C. if its resistance at 20° C. is 15 ohms.
18. The resistance of a d-c generator field winding (copper wire) before being energized measured 100 ohms at 23° C. After two hours of operation at full excitation the machine was shut down and the resistance of the winding taken again. This time it measured 115 ohms. What was the temperature rise in the field?
19. What is the name given to the quality that opposes current leakage through and over the surface of insulating materials?
20. What is the name that applies to the ability of an insulator to withstand high electric stress as created by a high potential difference across the insulator?
21. What precaution is taken when rubber insulation is placed on copper wires?
22. What type of circuits employ varnished cambric insulation?
23. What type of circuits employ oil-impregnated paper insulation?
24. What type of circuits employ enamel-coated magnet wire?
25. What type of electric circuits employ a lead sheath?
26. What is the name of the splice used to join small solid conductors of approximately the same size and aligned on the same axis?
27. What is the name of the splice used to connect a lighting outlet to a branch circuit?
28. What are the three main reasons for soldering a splice?
29. Distinguish between the function of rubber tape and friction tape, as applied to splices.
30. What type of tape is replacing rubber and friction tape for ordinary splices?

CHAPTER 7

MAGNETISM AND MAGNETIC CIRCUITS

INTRODUCTION

Before considering the theory of magnetism or the principles of magnetic circuits it is important to consider the wide and varied uses made of the property of magnetism.

A magnet is an essential part of the instruments used by navigators and surveyors, and it is the heart of most electric motors and instruments. Without the magnet the telephone receiver would be impossible in its present form, and electronic equipment would be nonexistent. Practically all electrical equipment employs the property of magnetism either within itself or in the circuits that supply it. Therefore, it is important for the technician who works with electrical equipment to have a clear understanding of some of the basic principles of magnetism.

A substance is said to be a magnet if it has the property of magnetism—that is, if it has the power to attract such substances as iron, steel, nickel, or cobalt, which are known as MAGNETIC MATERIALS. A steel knitting needle magnetized by a method to be described later, exhibits two points of maximum attraction (one at each end) and no attraction at its center. The points of maximum attraction are called MAGNETIC POLES. All magnets have at least two poles. If the needle is suspended so that it rotates freely in a horizontal plane about its center, the needle comes to rest in an approximately north-south line of direction with the same pole always pointing to the north and with the other always pointing toward the south. The magnetic pole that points northward is called the NORTH POLE, and the other the SOUTH POLE.

A MAGNETIC FIELD exists around a simple bar magnet. The field consists of imaginary lines along which a MAGNETIC FORCE acts. These lines emanate from the north pole of the magnet, and enter the south pole, returning to the north pole through the magnet itself, thus forming closed loops.

A MAGNETIC CIRCUIT is a complete path through which magnetic lines of force may be established under the influence of a magnetizing force. Most magnetic circuits are composed largely of magnetic materials in order to contain the magnetic flux. These circuits are similar to the ELECTRIC CIRCUIT, which is a complete path through which current is caused to flow under the influence of an electromotive force.

Magnets may be conveniently divided into three groups—(1) NATURAL MAGNETS, found in the natural state in the form of a mineral called magnetite; (2) PERMANENT MAGNETS, bars of hardened steel (or some form of alloy such as alnico) that have been permanently magnetized; and (3) ELECTROMAGNETS, composed of soft-iron cores around which are wound coils of insulated wire. When an electric current flows through the coil the core becomes magnetized. When the current ceases to flow, the core loses most of its magnetism. Permanent magnets and electromagnets are sometimes called ARTIFICIAL MAGNETS to further distinguish them from natural magnets.

NATURAL MAGNETS

For many centuries it has been known that certain stones (magnetite, Fe_3O_4) have the ability to attract small pieces of iron. Because many of the best of these stones (natural magnets) were found near Magnesia in Asia Minor, the Greeks called the substance MAGNETITE, or MAGNETIC.

Before this, ancient Chinese observed that when similar stones were suspended freely or floated on a light substance in a container

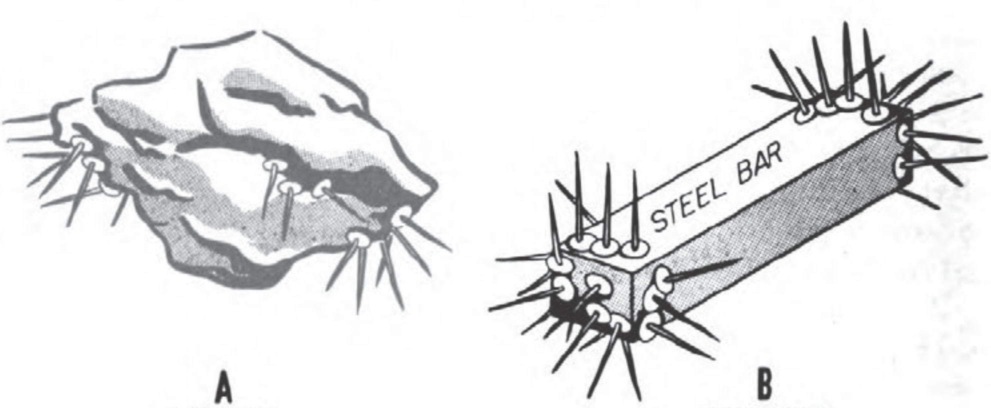


Figure 7–1.—Natural and artificial magnets.

of water they tended to assume a nearly north-and-south position. Probably Chinese navigators used bits of magnetite floating on wood in a liquid-filled vessel as crude compasses. At that time it was not known that the earth itself acts like a magnet, and these stones were regarded with considerable superstitious awe. Because bits of this substance were used as compasses they were called LOADSTONES (or lodestones), which means "leading stones."

Natural magnets are also found in the United States, Norway, and Sweden. A natural magnet, demonstrating the attractive force at the poles, is shown in figure 7–1, A.

ARTIFICIAL MAGNETS

Natural magnets no longer have any practical value because more powerful and more conveniently shaped permanent magnets can be produced artificially. Commercial magnets are made from special steels and alloys—for example, alnico, made principally of aluminum, nickel, and cobalt. The name is derived from the first two letters of the three principal elements of which it is composed. An artificial magnet is shown in figure 7–1, B.

An iron, steel, or alloy bar can be magnetized by inserting the bar into a coil of insulated wire and passing a heavy direct current through the coil, as shown in figure 7–2, A. This aspect of magnetism is treated later in the chapter. The same bar may also be magnetized if it is stroked with a bar magnet, as shown in

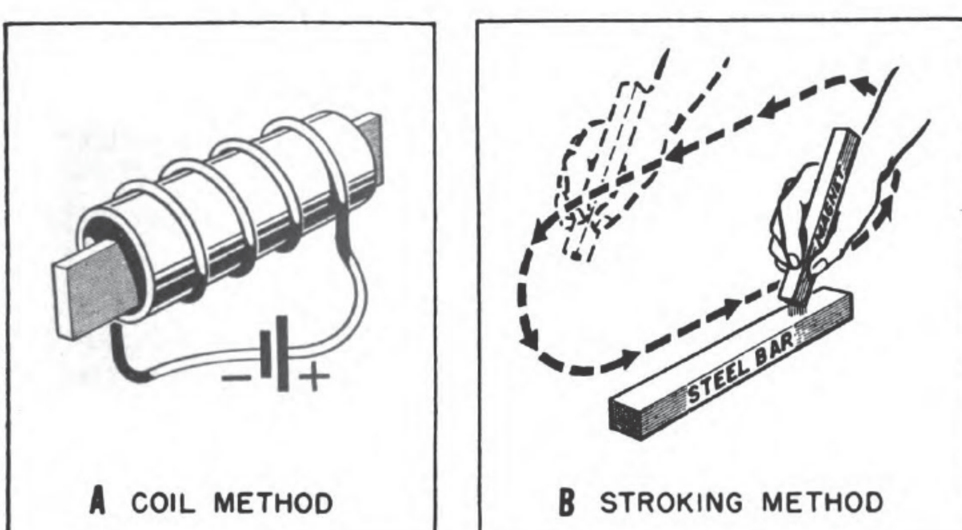


Figure 7–2.—Methods of producing artificial magnets.

figure 7–2, B. It will then have the same magnetic property that the magnet used to induce the magnetism has—namely, there will be two poles of attraction, one at either end. This process produces a permanent magnet by INDUCTION—that is, the magnetism is induced in the bar by the influence of the stroking magnet.

Artificial magnets may be classified as "permanent" or "temporary" depending on their ability to retain their magnetic strength after the magnetizing force has been removed. Hardened steel and certain alloys are relatively difficult to magnetize and are said to have a LOW PERMEABILITY because the magnetic lines of force do not easily permeate, or distribute themselves readily through the steel. Once magnetized, however, these materials retain a large part of their magnetic strength and are called PERMANENT MAGNETS. Permanent magnets are used extensively in electric instruments, meters, telephone receivers, permanent-magnet loudspeakers, and magnetos. Conversely, substances that are relatively easy to magnetize—such as soft iron and annealed silicon steel—are said to have a HIGH PERMEABILITY. Such substances retain only a small part of their magnetism after the magnetizing force is removed and are called TEMPORARY MAGNETS. Silicon steel and similar materials are used in transformers where the magnetism is constantly changing and in generators and motors where the strengths of the fields can be readily changed.

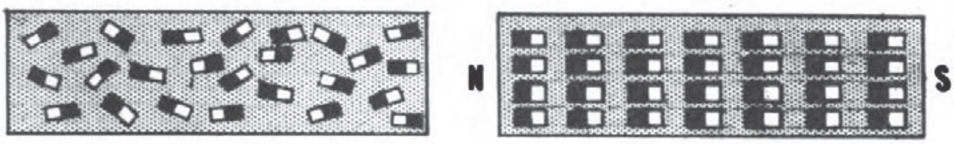
The magnetism that remains in a temporary magnet after the magnetizing force is removed is called RESIDUAL MAGNETISM. The fact that temporary magnets retain even a small amount of magnetism is an important factor in the build-up of voltage in self-excited d-c generators.

NATURE OF MAGNETISM

Weber's theory of the nature of magnetism is based on the assumption that each of the molecules of a magnet is itself a tiny magnet. The molecular magnets that compose an unmagnetized bar of iron or steel are arranged at random, as shown by the simplified diagram of figure 7–3, A. With this arrangement, the magnetism of each of the molecules is neutralized by that of adjacent molecules, and no external magnetic effect is produced.

When a magnetizing force is applied to an unmagnetized iron or steel bar, the molecules become aligned so that the north poles point one way and the south poles point the other way, as shown in figure 7–3, B.

If a bar magnet is broken into several parts, as in figure 7–4, each part constitutes a complete magnet. The north and south poles of these small magnets are in the same respective directions as those of the original magnet. If each of these parts is again



UNMAGNETIZED STEEL
A

MAGNETIZED STEEL
B

Figure 7–3.—Molecular theory of magnetism.

broken, the resulting parts are likewise complete magnets, and the magnetic orientation is the same. If this breaking process could be continued, smaller and smaller pieces would retain their magnetism until each part was reduced to a molecule. It is therefore logical to assume that each of these molecules is a magnet.

A further justification for this assumption results from the fact that when a bar magnet is held out of alinement with the earth's field and is repeatedly jarred, heated, or exposed to a powerful alternating field, the molecular alignment is disarranged and the magnet becomes demagnetized. For example, electric measuring instruments become inaccurate if their permanent magnets lose some of their magnetism because of severe jarring or exposure to opposing magnetic fields.

A theory of magnetism that is perhaps more adequate than the

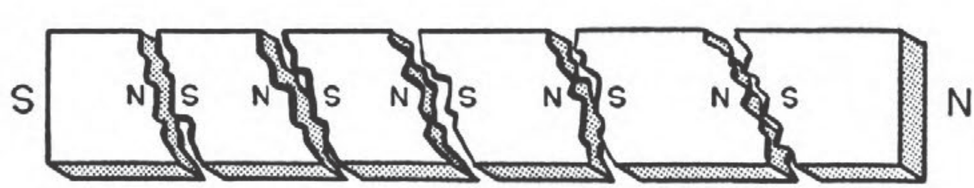


Figure 7–4.—Magnetic poles in a broken magnet.

MOLECULAR theory is the DOMAIN theory. Much simplified, this theory may be stated as follows:

In magnetic substances the "atomic" magnets, produced by the movement of the planetary electrons around the nucleus, have a strong tendency to line up together in groups of from 10^{14} to 10^{15} atoms without the influence of any external magnetic field. These groups of atoms having their poles orientated in the same direction are called DOMAINS. Therefore, throughout each domain an intense magnetic field is produced. These fields are normally in a miscellaneous arrangement so that no external field is apparent when the substance as a whole is unmagnetized. Each tiny domain (10^6 of them may be contained in 1 cubic millimeter) is always magnetized to saturation, and the addition of an external magnetic field does not increase the inherent magnetism of the individual domains.

However, if an external field that is gradually increased in strength is applied to the magnetic substance the domains will line up one by one (or perhaps several at a time) with the external field.

The usual curve of magnetization versus magnetic field strength (fig. 7-5) is shown as a smooth line. If the field strength is

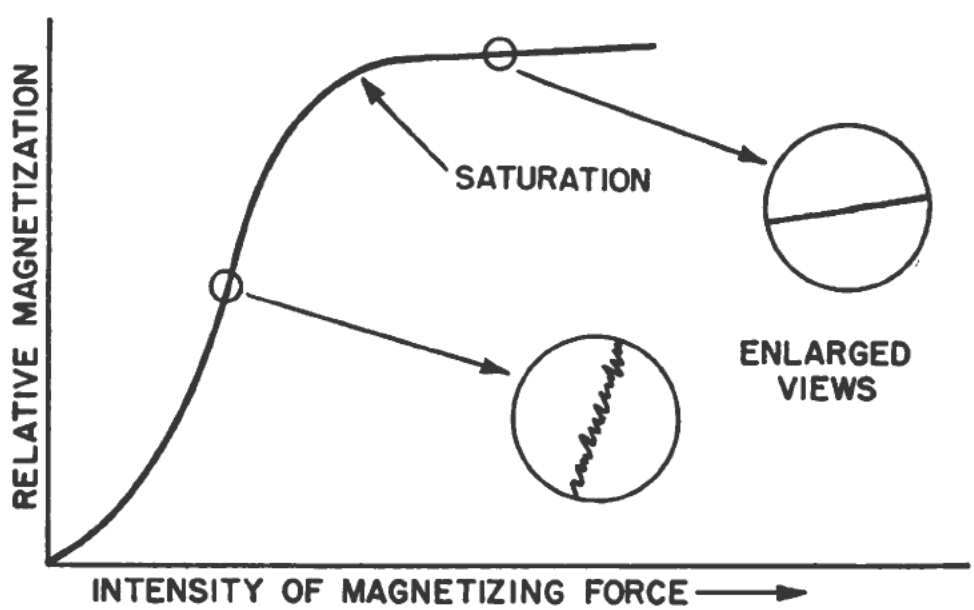


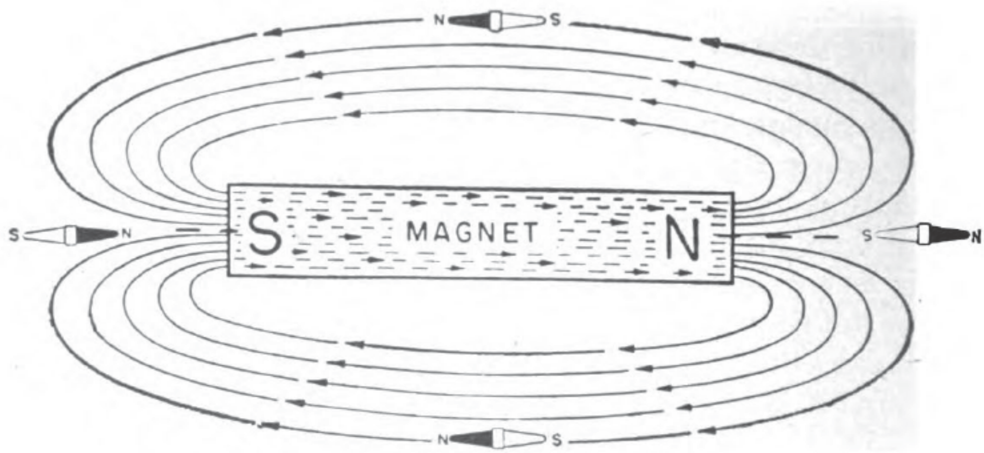
Figure 7-5.—Typical magnetization curve.

increased in very small increments and the increase in magnetization is observed in sufficient detail, it can be shown that the curve is very irregular up to the point of saturation. The explanation for the irregularity is that entire domains turn into alignment only after appreciable magnetizing force has been applied, and consequently there is a series of sudden increases in magnetization as whole domains, or groups of domains, respond to the magnetizing force. As the magnetizing force is gradually increased there may be an interval in which there is no appreciable gain in magnetization. Then suddenly there will be a sharp gain as more domains move into alignment. These sudden and irregular increases in magnetization continue up to the point of saturation—that is, the point at which essentially all of the domains are lined up in the direction of the external magnetizing force.

MAGNETIC FIELDS AND LINES OF FORCE

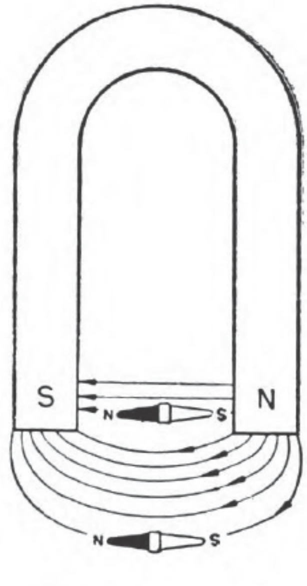
If a bar magnet is dipped into iron filings, many of the filings are attracted to the ends of the magnet, but none are attracted to the center of the magnet. As mentioned previously, the ends of the magnet where the attractive force is the greatest are called the POLES of the magnet. By using a compass, the line of direction of the magnetic force at various points near the magnet may be observed. The compass needle itself is a magnet. The north end of the compass needle always points toward the south pole, *S*, as shown in figure 7–6, A, and thus the sense of direction (with respect to the polarity of the bar magnet) is also indicated. At the center, the compass needle points in a direction that is parallel to the bar magnet.

When the compass is placed successively at several points in the vicinity of the bar magnet the compass needle aligns itself with the field at each position. The direction of the field is indicated by the arrows and represents the direction in which the north pole of the compass needle will point when the compass is placed in this field. Such a line along which a compass needle aligns itself is called a MAGNETIC LINE OF FORCE. As mentioned previously, the magnetic lines of force are assumed to emanate from the north pole of a magnet, pass through the surrounding space, and enter the south pole. The lines of force then pass from the south pole to the north pole inside the magnet to form a closed loop. Each line of force forms an independent closed loop and does not merge



BAR MAGNET

A



HORSESHOE MAGNET

B

Figure 7–6.—Magnetic lines of force.

with or cross other lines of force. The lines of force between the poles of a horseshoe magnet are shown in figure 7–6, B.

The space surrounding a magnet, in which the magnetic force acts, is called a MAGNETIC FIELD. Michael Faraday was the first scientist to visualize the magnet field as being in a state of stress and consisting of uniformly distributed lines of force. The entire quantity of magnetic lines surrounding a magnet is called MAGNETIC FLUX. Flux in a magnetic circuit corresponds to current in an electric circuit.

The number of lines of force per unit area is called FLUX DENSITY and is measured in lines per square inch or lines per square centimeter. Flux density is expressed by the equation

$$B=\frac{\Phi}{A},$$

where B is the flux density, Φ (Greek phi) is the total number of lines of flux, and A is the cross-sectional area of the magnetic circuit. If A is in square centimeters, B is in lines per square centimeter, or GAUSS. The terms FLUX and FLOW of magnetism are frequently used in textbooks. However, magnetism itself is not thought to be a stream of particles in motion, but is simply a field of force exerted in space. The number of lines of force per unit area can be measured by rotating a small wire loop at a fixed speed through an air gap in the magnetic circuit in such a manner that the loop cuts across the magnetic lines of force comprising the field. As the wire loop cuts across the magnetic lines, a voltage is generated in the loop. This voltage is proportional to the flux density of the field.

A visual representation of the magnetic field around a magnet can be obtained by placing a plate of glass over a magnet and sprinkling iron filings onto the glass. The filings arrange themselves in definite paths between the poles. This arrangement of the filings shows the pattern of the magnetic field around the magnet, as in figure 7–7.

The magnetic field surrounding a symmetrically shaped magnet has the following properties:

1. The field is symmetrical unless disturbed by another magnetic substance.
2. The lines of force have direction and are represented as emanating from the north pole and entering the south pole.

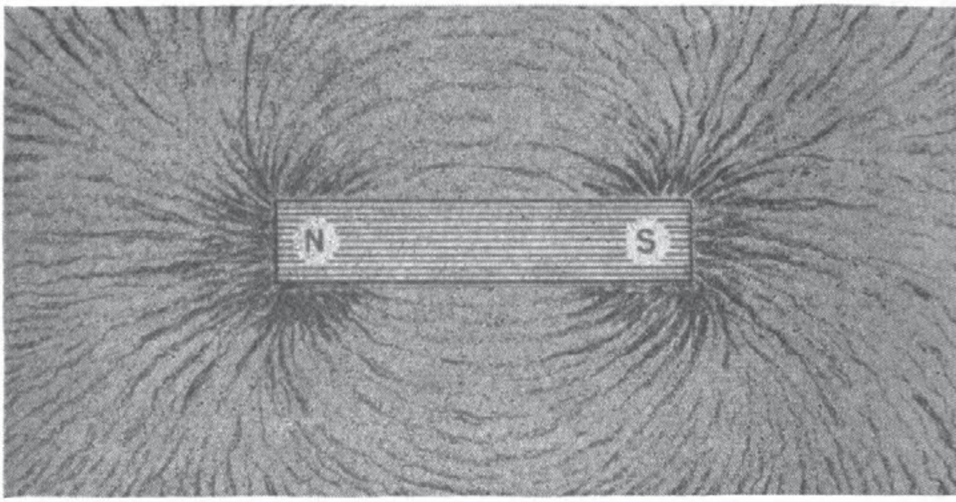


Figure 7-7.—Magnetic field around a magnet.

3. A compass needle placed anywhere in the magnetic field is always deflected so that its north end points in the direction of the lines of force (toward the south pole).
4. The greatest field intensity occurs near the pole surfaces and diminishes with increased distance from the poles.

LAWS OF ATTRACTION AND REPULSION

If a magnetized needle is suspended near a bar magnet, as in figure 7-8, it will be seen that a north pole repels a north pole and a south pole repels a south pole. Opposite poles, however,

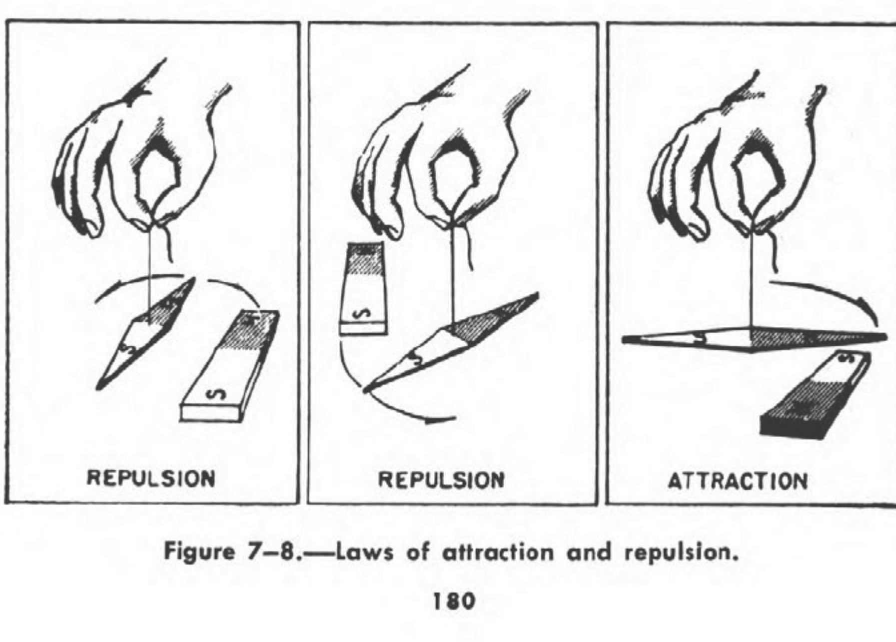


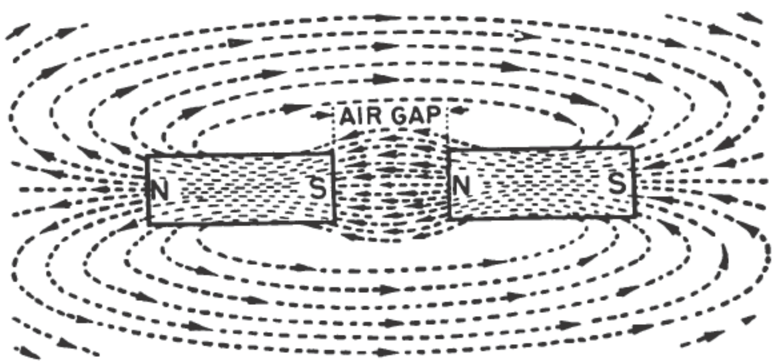
Figure 7-8.—Laws of attraction and repulsion.

will attract each other. Thus, the first two laws of magnetic attraction and repulsion are:

The *FIRST LAW* states that like magnetic poles repel each other.

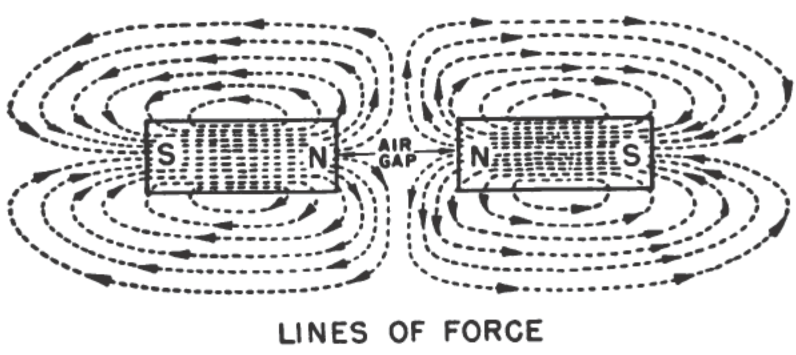
The *SECOND LAW* states that unlike magnetic poles attract each other.

The flux patterns between adjacent unlike poles of bar magnets, as indicated by lines, are shown in figure 7–9, A. Similar patterns for adjacent like poles are shown in figure 7–9, B. The



UNLIKE POLES ATTRACT

A



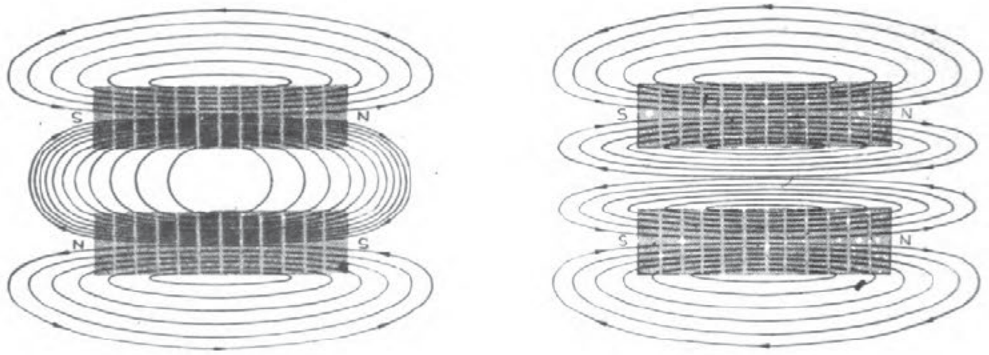
LIKE POLES REPEL

B

Figure 7–9.—Lines of force between unlike and like poles.

lines do not cross each other at any point and they act as if they repel each other.

Figure 7–10 shows the flux pattern (indicated by lines) around two bar magnets placed close together and parallel with each other. Figure 7–10, A, shows the flux pattern when opposite poles are adjacent; and figure 7–10, B, shows the flux pattern when like poles are adjacent.



FLUX PATTERN-ATTRACTION
A

FLUX PATTERN-REPULSION
B

Figure 7–10.—Flux patterns of adjacent parallel bar magnets.

The *THIRD LAW* of magnetic attraction and repulsion states in effect that the force of attraction or repulsion existing between two magnetic poles decreases rapidly as the poles are separated from each other. Actually, the force of attraction or repulsion varies directly as the product of the separate pole strengths and inversely as the square of the distance separating the magnetic poles, provided the poles are small enough to be considered as points. For example, if the distance between two north poles is increased from 2 feet to 4 feet, the force of repulsion between them is decreased to one-fourth of its original value. If either pole strength is doubled, the distance remaining the same, the force between the poles will be doubled.

THE EARTH'S MAGNETISM

As has been stated, the earth is a huge magnet; and surrounding the earth is the magnetic field produced by the earth's magnetism. The magnetic polarities of the earth are as indicated in figure 7–11. The geographic poles are also shown at each end of the

axis of rotation of the earth. The magnetic axis does not coincide with the geographic axis, and therefore the magnetic and geographic poles are not at the same place on the surface of the earth.

The early users of the compass regarded the end of the compass needle that points in a northerly direction as being a north pole. The other end was regarded as a south pole. On some

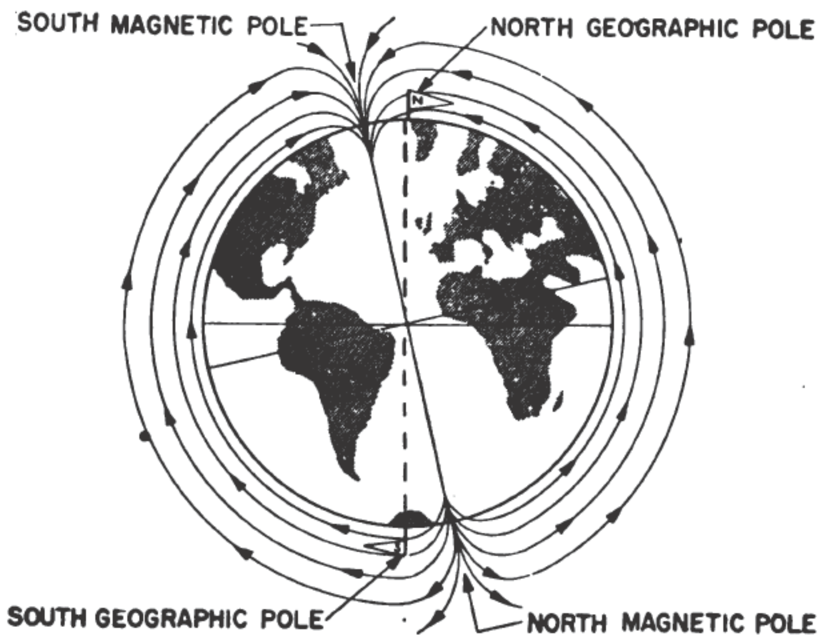


Figure 7–11.—Earth's magnetic poles.

maps the magnetic pole of the earth towards which the north pole of the compass pointed was designated a north magnetic pole. This magnetic pole was obviously called a north pole because of its proximity to the north geographic pole.

When it was learned that the earth is a magnet and that opposite poles attract, it was necessary to call the magnetic pole located in the northern hemisphere a south magnetic pole and the magnetic pole located in the southern hemisphere a north magnetic pole. The matter of naming the poles was arbitrary. Obviously, the polarity of the compass needle that points toward the north must be opposite to the polarity of the earth's magnetic pole located there.

As has been stated, magnetic lines of force are assumed to emanate from the north pole of a magnet and to enter the south

pole as closed loops. Because the earth is a magnet, lines of force emanate from its north magnetic pole and enter the south pole as closed loops. The compass needle aligns itself in such a way that these lines of force enter at its south pole and leave at its north pole. Because the north pole of the needle is defined as the end that points in a northerly direction it follows that the magnetic pole in the vicinity of the north geographic pole is in reality a south magnetic pole, and vice versa.

Because the magnetic poles and the geographic poles do not coincide, a compass will not (except at certain positions on the earth) point in a true (geographic) north-south direction—that is, it will not point in a line of direction that passes through the north and south geographic poles, but in a line of direction that makes an angle with it. This angle is called the angle of VARIATION OF DECLINATION.

MAGNETIC FIELD AROUND A CURRENT-CARRYING CONDUCTOR

In 1819 Hans Christian Oersted, a Danish physicist, found that a definite relation exists between magnetism and electricity. He discovered that an electric current is accompanied by certain magnetic effects and that these effects obey definite laws. If a compass is placed in the vicinity of a current-carrying conductor, the needle aligns itself at right angles to the conductor, thus in-

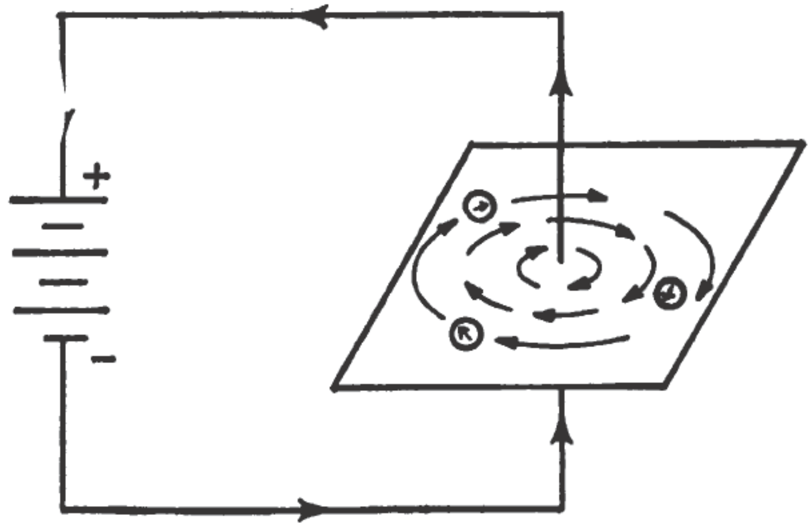
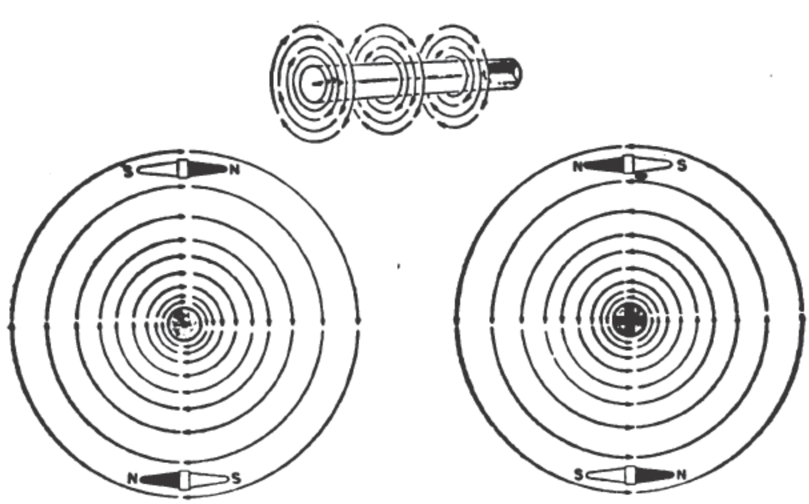


Figure 7–12.—Magnetic field around a current-carrying conductor.

dicating the presence of a magnetic force. The presence of this force can be demonstrated by passing an electric current through a vertical conductor which passes through a horizontal piece of cardboard, as illustrated in figure 7–12. The magnitude and direction of the force are determined by setting a compass at various points on the cardboard and noting the deflection. The direction of the force is assumed to be the direction the north pole of the compass points. These deflections show that a magnetic field exists in circular form around the conductor. When the current flows upward, the field direction is clockwise, as viewed from the top, but if the polarity of the supply is reversed so that the current flows downward, the direction of the field is counterclockwise.

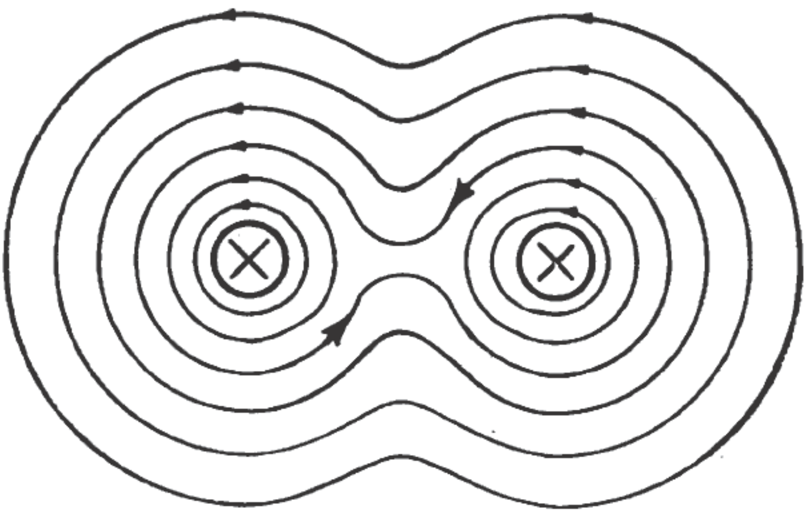
The relation between the direction of the magnetic lines of force around a conductor and the direction of current flow along the conductor may be determined by means of the LEFT-HAND RULE FOR A CONDUCTOR. If the conductor is grasped in the left hand with the thumb extended in the direction of electron flow



CURRENT
FLOWING OUT
A

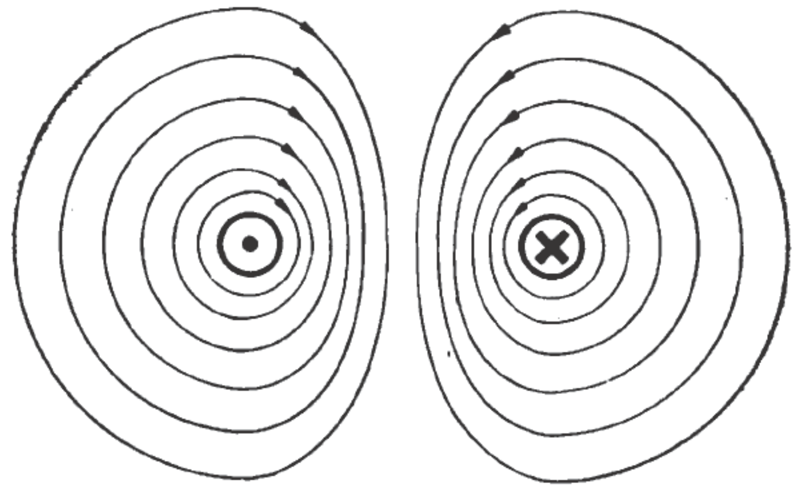
CURRENT
FLOWING IN
B

Figure 7–13.—Magnetic field around a current-carrying conductor—detailed view.



CURRENTS FLOWING IN
THE SAME DIRECTION

A



CURRENTS FLOWING IN
THE OPPOSITE DIRECTION

B

Figure 7–14.—Magnetic field around two parallel conductors.

(− to +), the fingers will point in the direction of the magnetic lines of force (the direction in which the north pole of a compass points).

Arrows generally are used in electric diagrams to denote the direction of current flow along the length of wire. Where cross sections of wire are shown, a special view of the arrow is used. A cross-sectional view of a conductor that is carrying current toward the observer is illustrated in figure 7–13, A. The direction of current is indicated by a dot, which represents the head of the arrow. A conductor that is carrying current away from the observer is illustrated in figure 7–13, B. The direction of current is indicated by a cross, which represents the tail of the arrow.

When two parallel conductors carry current in the same direction, the magnetic fields tend to encircle both conductors, drawing them together with a force of attraction, as shown in figure 7–14, A. Two parallel conductors carrying currents in opposite directions are shown in figure 7–14, B. The field around one conductor is opposite in direction to the field around the other conductor. The resulting lines of force are crowded together in the space between the wires, and tend to push the wires apart. Therefore, two parallel adjacent conductors carrying currents in the same direction attract each other and two parallel conductors carrying currents in opposite directions repel each other.

MAGNETIC FIELD ABOUT A COIL

Polarity—Left-Hand Rule

If a conductor is formed into a coil, or SOLENOID (fig. 7–15), most of the lines of force thread the entire coil and return on the outside of the coil to the other end. Note that practically no magnetic lines (none in the figure) encircle the individual turns of wire of the coil. The current flow is away from the observer at *A, B,* and *C,* and toward the observer at *D, E,* and *F.*

The field on the right side of *A* neutralizes the field on the left side of *B* because the flux lines are in opposite directions and the turns *A* and *B* are close to each other. The same is true for the flux lines between all adjacent turns. Because the magnetic lines cannot pass readily between the loops they continue through the entire length of coil. This reaction produces a field that is similar

to the field around a bar magnet. The end from which the lines emerge is a north pole and the end in which the lines enter is a south pole.

The relation between the direction of current and the direction of magnetic flux may be determined by means of the LEFT-HAND RULE FOR A COIL. Grasp the coil in the left hand with the fingers

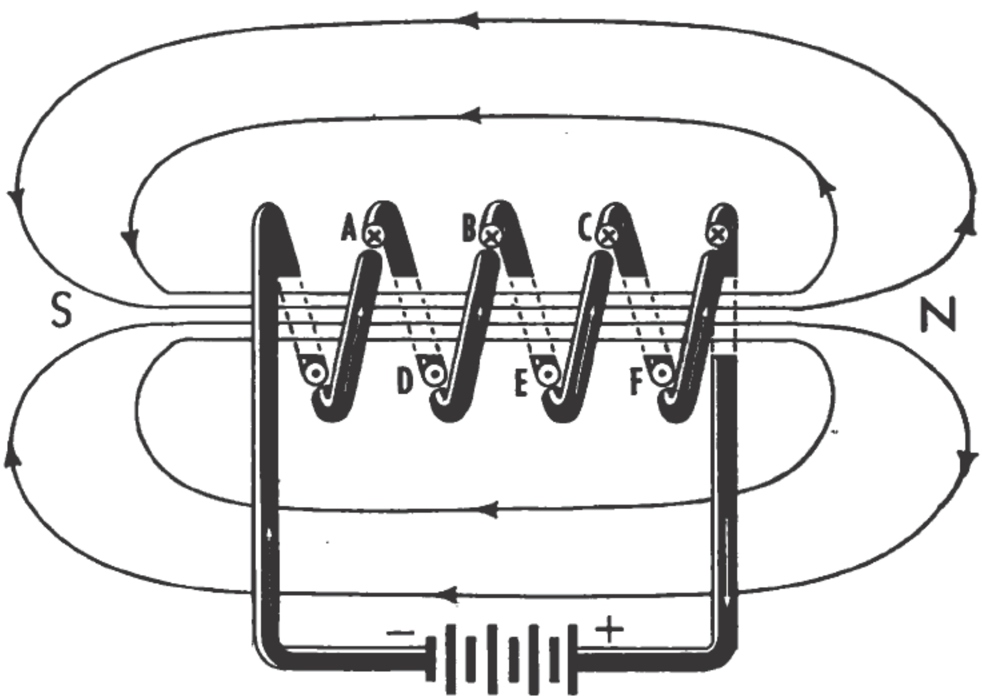


Figure 7–15.—Magnetic field around a coil.

pointing in the direction of the current flow in the coil. The thumb extended longitudinally along the coil will point in the direction of the flux and the north pole of the coil. The magnetic circuit comprises the complete path of the magnetic field: inside the coil, emanating at the north pole, and returning outside the coil to reenter the coil at the south pole as closed loops of magnetic flux.

Effect of Number of Turns and Current on Field Strength

The field strength (amount of flux threading through the coil at its center) of the coil in figure 7–15 can be increased by increasing the number of turns (one turn being one complete wrap of a conductor around the core) on the coil, by increasing the current

through the coil, or by increasing both the number of turns and the current. The flux may also be increased by using a core material that will permit the flux to pass through more easily. However, the effect of core material on field strength is treated later. For an air-core coil the flux is directly proportional to current flowing in the coil and the number of turns of wire comprising the coil. The product of the current in amperes and the number of turns is called the AMPERE-TURNS of the coil. For example, suppose 1,000 ampere-turns will produce the required field strength in air. Among other possibilities, 50 turns through which 20 amperes flow will suffice. Likewise, the same field strength can be produced by 500 turns and 2 amperes, or 1,000 turns and 1 ampere, and so forth.

MAGNETIC SHIELDING

There is no known INSULATOR for magnetic flux. If a nonmagnetic material is placed in a magnetic field, there is no appreciable change in flux—that is, the flux penetrates the nonmagnetic material. For example, a glass plate placed between the poles of a horseshoe magnet will have no appreciable effect on the field although glass itself is a good insulator in an electric circuit. If a magnetic material (for example, soft iron) is placed in a magnetic field, the flux may be redirected to take advantage of the greater permeability of the magnetic material, as shown in figure 7–16. In all probability the amount of flux would be increased; certainly, it would not be reduced.

The sensitive mechanisms of electric instruments and meters can be influenced by stray magnetic fields which will cause errors

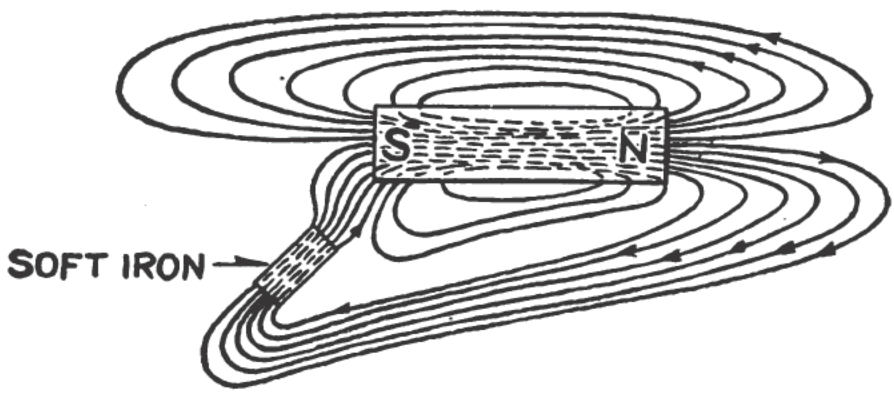


Figure 7–16.—Effect of a magnetic substance in a magnetic field.

in their readings. Because instrument mechanisms cannot be insulated against magnetic flux, it is necessary to employ some means of directing the flux around the instrument. This is accomplished by placing a soft-iron case, called a MAGNETIC SCREEN OR SHIELD, about the instrument. Because the flux is established more readily through the iron (even though the path is longer) than through the air inside the case, the instrument is effectively shielded, as shown by the watch and soft-iron shield in figure 7–17.

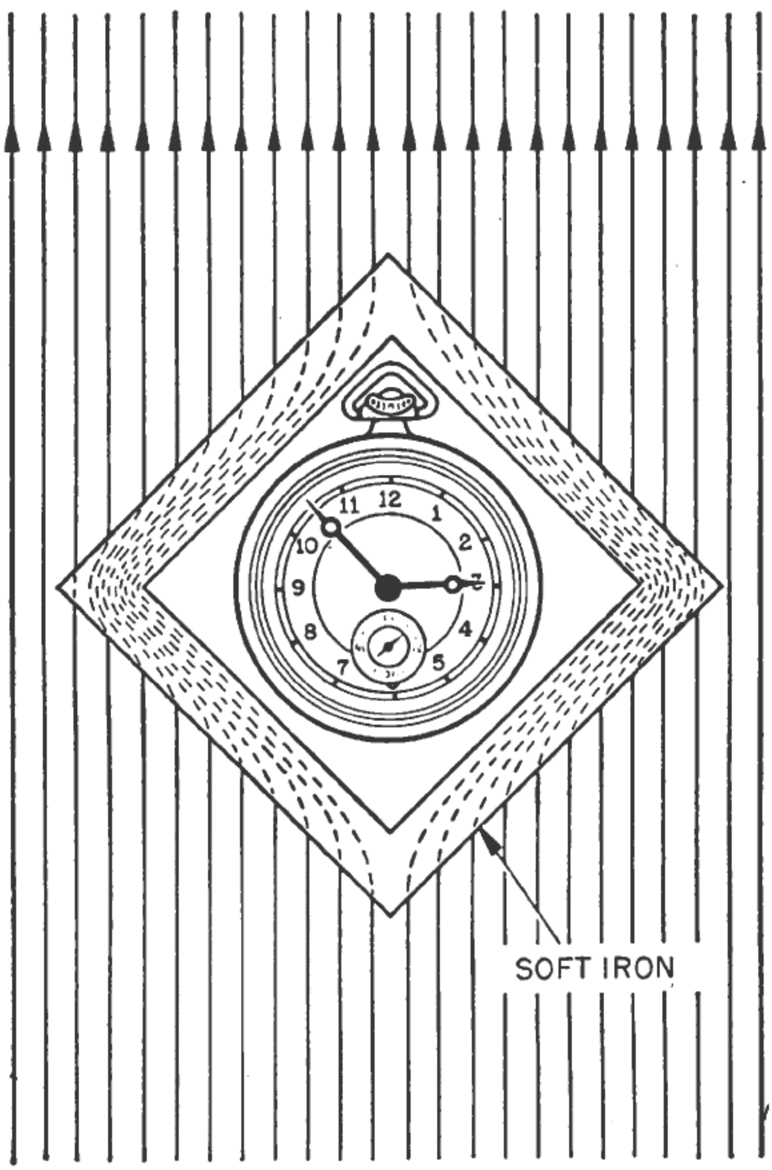


Figure 7–17.—Magnetic shield.

OHM'S LAW EQUIVALENT FOR MAGNETIC CIRCUITS

The law of current flow in the electric circuit is similar to the law for the establishing of flux in the magnetic circuit.

Ohm's law for ELECTRIC circuits states that the current is directly proportional to the applied voltage and inversely proportional to the resistance offered by the circuit. Expressed mathematically,

$$I=\frac{E}{R}.$$

Rowland's law for MAGNETIC circuits states, in effect, that the number of lines of magnetic flux in maxwells (Φ) is directly proportional to the magnetomotive force in gilberts (F) and inversely proportional to the reluctance ($\mathcal{R}$) offered by the circuit. The unit of reluctance sometimes used is the REL, $\mathcal{R}$. Expressed mathematically,

$$\Phi=\frac{F}{\mathcal{R}}.$$

The similarity of Ohm's law and Rowland's law is apparent. However, the units used in the expression for Rowland's law need to be explained.

The MAGNETIC FLUX, Φ, is similar to current in the Ohm's law formula, and comprises the total number of lines of force existing in the magnetic circuit. The MAXWELL is the unit of flux—that is, 1 line of force is equal to 1 maxwell. However, the maxwell is often referred to as simply a *line of force, line of induction,* or *line.*

The MAGNETOMOTIVE FORCE, F, or mmf, comparable to electromotive force in the Ohm's law formula, is the force that produces the flux in the magnetic circuit. The practical unit of magnetomotive force is the AMPERE-TURN. Another unit of magnetomotive force sometimes used is the GILBERT, designated by the capital letter, F. The gilbert is the magnetomotive force required to establish 1 maxwell in a magnetic circuit having 1 unit of reluctance (1 rel). The magnetomotive force in gilberts is expressed in terms of ampere-turns as

$$F=1.257IN,$$

where F is in gilberts, I is in amperes, and N is the number of complete turns of wire encircling the circuit.

The UNIT OF INTENSITY of magnetizing force per unit of length is designated as H, and is sometimes expressed as gilberts per centimeter of length. Expressed mathematically,

$$H=\frac{1.257IN}{l},$$

where l is the length in centimeters.

The RELUCTANCE, $\mathcal{R}$, similar to resistance in the Ohm's law formula, is the opposition offered by the magnetic circuit to the passage of magnetic flux. The unit of reluctance has not been named officially. However, the REL has been proposed, and the symbol $\mathcal{R}$ is commonly used. The unit of reluctance is the reluctance of 1 centimeter-cube of air. The reluctance of a magnetic substance varies directly as the length of the flux path and inversely as the cross-sectional area and the permeability, μ, of the substance. Expressed mathematically,

$$\mathcal{R}=\frac{l}{\mu A},$$

where l is the length in centimeters, and A is the cross-sectional area in square centimeters.

PERMEABILITY, designated by the Greek letter MU, μ, is treated under a separate heading. However, it is defined here to permit a fuller interpretation of Rowland's law and also a practical application of this law. Permeability is a measure of the relative ability of a substance to conduct magnetic lines of force as compared with air. The permeability of air is taken as 1. Permeability is indicated as the ratio of the flux density in lines per square centimeter (gauss, B) to the intensity of the magnetizing force in gilberts per centimeter of length, indicated by H. Expressed mathematically,

$$\mu=\frac{B}{H}.$$

Another term used in magnetic circuits is PERMEANCE. Permeance, indicated by the symbol, P, is the reciprocal of reluctance—that is,

$$P=\frac{1}{\mathcal{R}}.$$

TABLE 8.—*Comparison of units in electric and magnetic circuits*

	Electric circuit	Magnetic circuit
Force.........	Volt, E, or emf	Gilberts, F, or mmf
Flow..........	Ampere, I	Flux, Φ, in maxwells
Opposition.....	Ohms, R	Reluctance, $\mathcal{R}$, or rels
Law..........	ohm's law, $I=\frac{E}{R}$	Rowland's law, $\Phi=\frac{F}{\mathcal{R}}$
Intensity of force.	Volts per cm of length	$H=\frac{1.257IN}{l}$, gilberts per centimeter of length.
Density........	Current density—for example, amperes per cm².	Flux density—for example, lines per cm², or gausses.

Values of B, H, and μ for common magnetic substances are given in table 9.

Permeance is like conductance in electric circuits, and is defined as the property of a magnetic circuit that permits lines of magnetic flux to pass through the circuit.

A comparison of the units, symbols, and equations used in applying Ohm's law to electric circuits and Rowland's law to magnetic circuits is given in table 8.

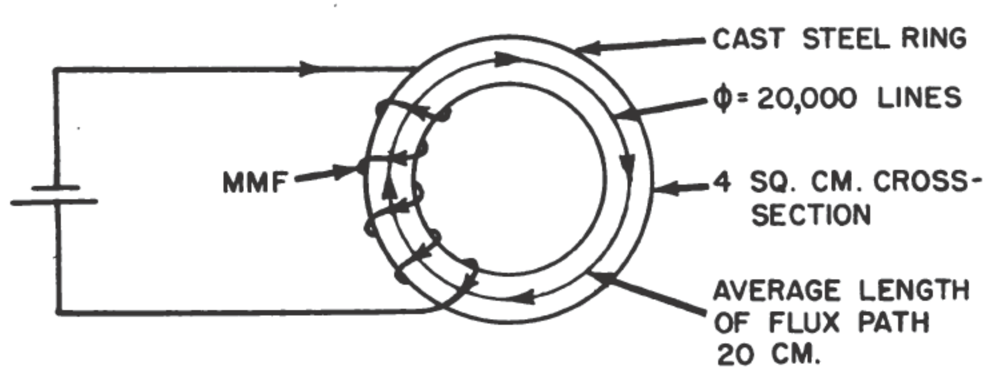


Figure 7–18.—Determination of ampere-turns in a magnetic circuit.

TABLE 9.—*B, H, and μ for common magnetic materials**

B	Sheet steel		Cast steel		Wrought iron		Cast iron	
	H	μ	H	μ	H	μ	H	μ
3,000	1.3	2,310	2.8	1,070	2.0	1,500	5.0	600
4,000	1.6	2,500	3.4	1,177	2.5	1,600	8.5	471
5,000	1.9	2,630	3.9	1,281	3.0	1,666	14.5	347
6,000	2.3	2,605	4.5	1,332	3.5	1,716	24.0	250
7,000	2.6	2,700	5.1	1,371	4.0	1,750	38.5	182
8,000	3.0	2,666	5.8	1,380	4.5	1,778	60.0	133
9,000	3.5	2,570	6.5	1,382	5.0	1,800	89.0	101
10,000	3.9	2,560	7.5	1,332	5.6	1,782	124.0	80.6
11,000	4.4	2,500	9.0	1,222	6.5	1,692	166.0	66.4
12,000	5.0	2,400	11.5	1,042	7.9	1,520	222.0	54.1
13,000	6.0	2,166	16.0	813	10.0	1,300	290.0	44.8
14,000	9.0	1,558	21.5	651	15.0	934	369.0	38.0
15,000	15.5	970	32.0	469	25.0	600	......	......
16,000	27.0	594	49.0	327	49.0	327	......	......
17,000	52.5	324	74.0	230	93.0	183	......	......
18,000	92.0	196	115.0	156	152.0	118	......	......
19,000	149.0	127	175.0	108	229.0	83	......	......
20,000	232.0	86	285.0	70	......	......	......	......

*B=flux density in lines per square centimeter; H=gilberts per centimeter of length; μ=permeability; $\mu=\frac{B}{H}$.

As a practical application of Rowland's law, let it be required to find the ampere-turns (IN) necessary to produce 20,000 lines of flux in a CAST STEEL ring having a cross-sectional area of 4 square centimeters and an average length of 20 centimeters, as shown in figure 7–18.

Flux density B is expressed as

$$B=\frac{\Phi}{A}=\frac{20{,}000}{4}=5{,}000 \text{ lines/cm}^2,$$

and from table 9 the corresponding value of H for cast steel is 3.9. The formula for H has previously been given as

$$H=\frac{1.257IN}{l},$$

from which

$$IN=\frac{Hl}{1.257}.$$

Substituting 3.9 for H and 20 for l in the preceding equation,

$$IN=\frac{3.9\times 20}{1.257}=62 \text{ ampere-turns.}$$

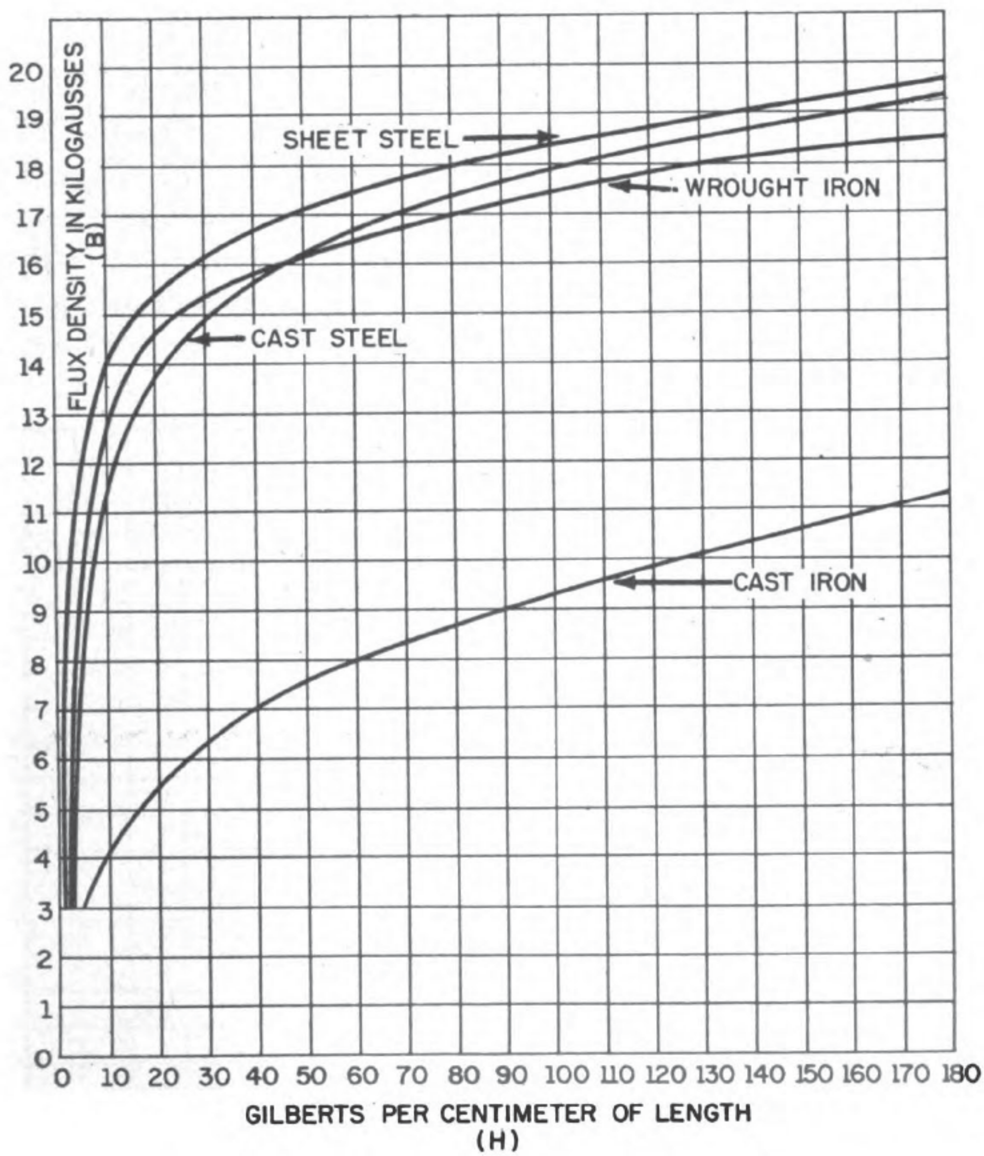


Figure 7–19.—Magnetization curves for four magnetic materials.

Permeability

When an annealed sheet steel core is used in an electromagnet it produces a stronger magnet than if a cast iron core is used. This is true because annealed sheet steel is more readily acted upon than hard cast iron by the magnetizing force of the coil. In other words, soft sheet steel is said to have greater permeability because the magnetic lines are established more easily in it than in cast iron. The ratio of the flux produced by a coil when the core is iron (or some other substance) to the flux produced when the core is air is called the PERMEABILITY of the iron (or whatever substance is used), the current in the coil being the same in each case. The permeability of a substance is thus a measure of the relative ability to conduct magnetic lines of force, or its magnetic conductivity. The permeability of air is 1. The permeability

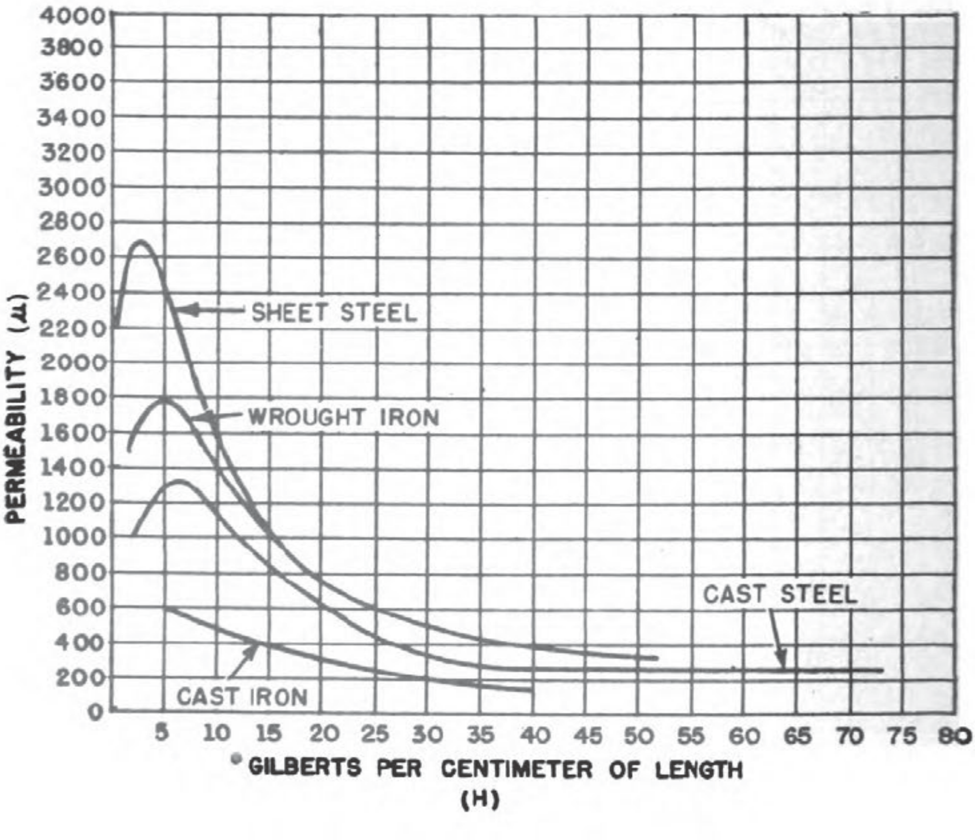


Figure 7–20.—Permeability curves.

of nonmagnetic materials, such as wood, aluminum, copper, and brass is essentially unity, or the same as for air.

Magnetization curves for the four magnetic materials listed in table 9 are given in figure 7–19.

The permeability of magnetic materials varies with the degree of magnetization, being smaller for high values of flux density, as is indicated in table 9 and figure 7–20.

Hysteresis

Perhaps the simplest method of illustrating the property of hysteresis is by graphical means such as the hysteresis loop shown in figure 7–21.

In this figure the magnetizing force is indicated in gilberts per centimeter of length along the plus and minus *H* axis, and the flux density is indicated in gausses along the plus and minus *B* axis. The intensity of the magnetizing force, *H*, applied by means of a current-carrying coil of wire around the sample of magnetic material, is varied uniformly through one cycle of operation, starting at zero. The force, *H*, is increased in the positive direction (current flowing in a given direction through the coil) to 11 gilberts per centimeter. During this time the flux density, *B*, increases from zero to 14,000 at point *A*. If *H* is decreased to zero, the descending curve of flux density does not return to zero via its rise path. Instead, it returns to point *B*, where the flux density is 13,000. The magnetic flux indicated by the length of line *OB* represents the RETENTIVITY of the magnetic substance.

Retentivity is the ability of a magnetic substance to retain its magnetism after the magnetizing force has been removed. Retentivity is most apparent in hard steel and is least apparent in soft iron.

The value of the RESIDUAL, or remaining, MAGNETISM when *H* has been reduced to zero, depends on the substance used and the degree of flux density attained. In this example the residual magnetism is 13,000 gausses.

If current is now sent through the coil in the opposite direction, so that the intensity of the magnetizing force becomes $-H$, the force will have to be increased to point *C* before the residual magnetism is reduced to zero. The magnetizing force, *OC*, necessary to reduce the residual magnetism to zero is called the COERCIVE FORCE. In this example the coercive force is 6 gilberts per centimeter.

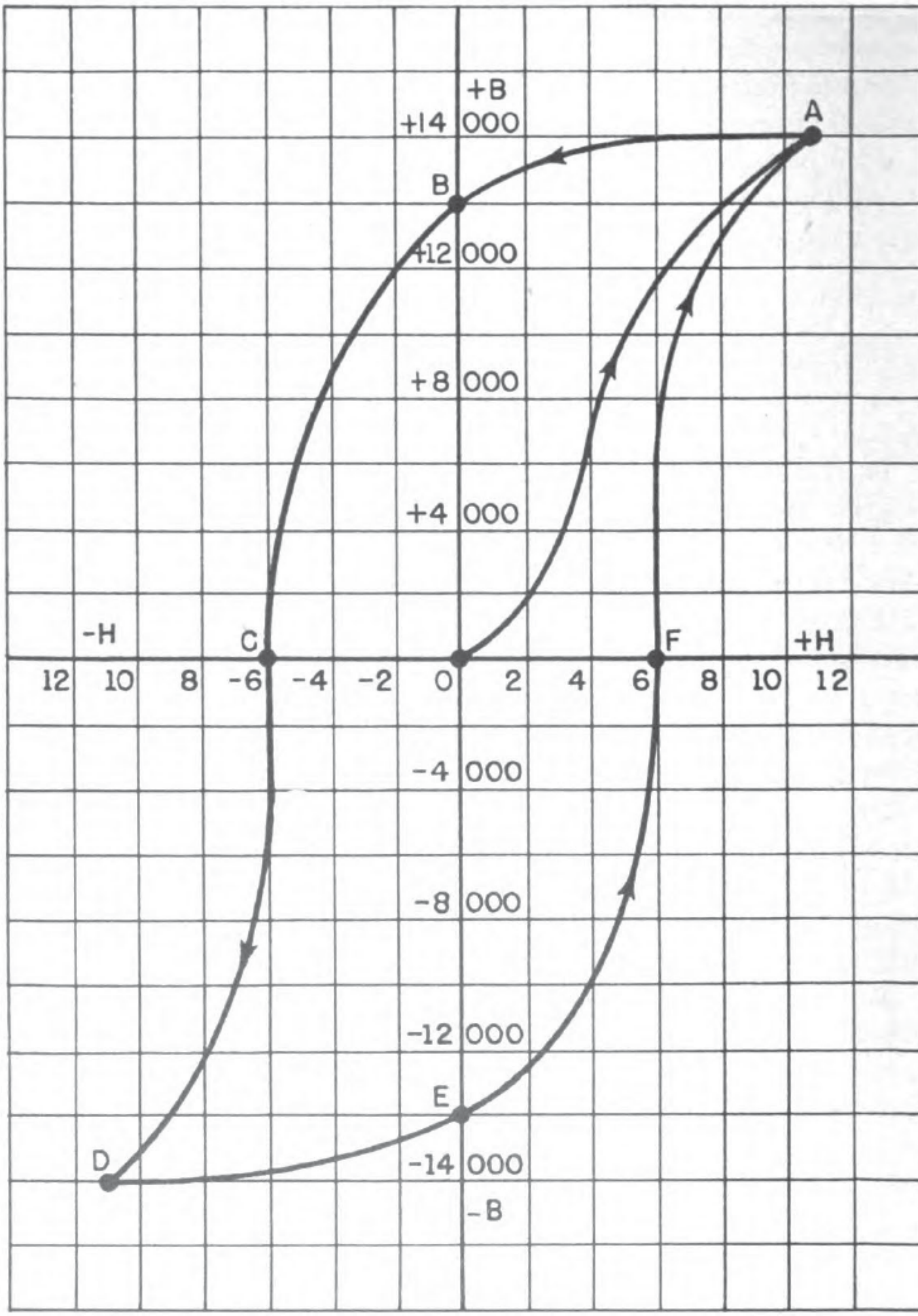


Figure 7–21.—Hysteresis loop.

If the magnetizing force is continued to -11 gilberts per centimeter, the curve descends from C to D, magnetizing the sample of magnetic material with the opposite polarity. If the

magnetizing force is reduced again to zero, the flux density is reduced to point E. The magnetic flux indicated by the length of line OE represents the retentivity of the magnetic substance, as did line OB. The residual magnetism is again 13,000 gausses.

If the current through the coil is again reversed (sent through in the original direction), the magnetization curve moves to zero when the magnetizing force is increased to point F.

Thus, when the magnetizing force goes through a complete cycle, the resulting magnetization likewise goes through a complete cycle.

From the foregoing analysis it is apparent that hysteresis is the property of a magnetic substance that causes the magnetization to lag behind the force that produces it. The lag of magnetization behind the force that produces it is caused by molecular

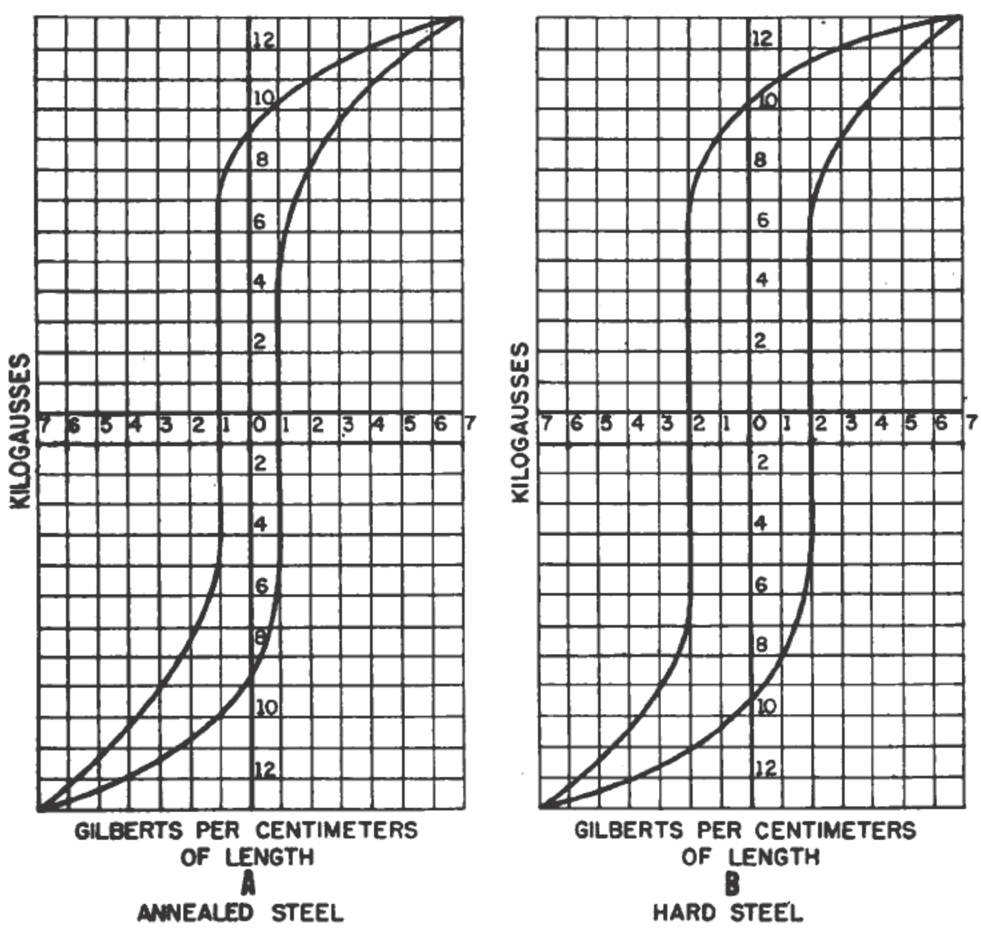


Figure 7–22.—Comparison of hysteresis loops.

friction. Energy is needed to move the molecules (or domains) through a cycle of magnetization. If the magnetization is reversed slowly, the energy loss may be negligible. However, if the magnetization is reversed rapidly, as when commercial alternating current is used, considerable energy may be dissipated. If the molecular friction is great, as when hard steel is used, the losses may be very great. Another factor that determines hysteresis loss is the maximum density of the flux established in the magnetic material.

A comparison of the hysteresis loops for annealed steel and hard steel is shown in figure 7–22. The area within each loop is a measure of the hysteresis energy loss per cycle of operation. Thus, as shown in the figure, more energy is dissipated in molecular friction in hard steel than in annealed steel. It is therefore important that substances having low hysteresis loss be used for transformer cores and similar a-c applications.

ELECTROMAGNETS

Introduction

An electromagnet is composed of a coil of wire wound around a core of soft iron. When direct current flows through the coil the core will become magnetized with the same polarity that the coil (solenoid) would have without the core. If the current is reversed, the polarity of both the coil and the soft-iron core is reversed.

The polarity of the electromagnet is determined by the left-hand rule in the same manner that the polarity of the solenoid in figure 7–15 was determined. If the coil is grasped in the left hand in such a way that the fingers curve around the coil in the direction of electron flow ($-$ to $+$), the thumb will point in the direction of the north pole.

The addition of the soft-iron core does two things for the current-carrying coil, or solenoid. First, the magnetic flux is increased because the soft-iron core is more permeable than the air core; second, the flux is more highly concentrated. The permeability of soft iron is many times that of air, and therefore the flux density is increased considerably when a soft-iron core is inserted in the coil.

The magnetic field around the turns of wire making up the coil

influences the molecules in the iron bar causing, in effect, the individual molecular magnets, or domains, to line up in the direction of the field created by the coil. Essentially the same effect is produced in a soft-iron bar when it is under the influence of a permanent magnet.

The magnetomotive force resulting from the current flow around the coil does not increase the magnetism that is inherent in the iron core, it merely reorientates the "atomic" magnets that were present before the magnetizing force was applied. If substantial numbers of the tiny magnets are orientated in the same direction, the core is said to be magnetized.

When soft iron is used, most of the atomic magnets return to what amounts to a miscellaneous orientation upon removal of the magnetizing current, and the iron is said to be demagnetized. If hard steel is used, more of them will remain in alignment with the direction of the flux produced by the flow of current through the coil, and the metal is said to be a permanent magnet. Soft iron and other magnetic materials having high permeability and low retentivity are generally used in electromagnets.

It is known from experience that a piece of soft iron is attracted to either pole of a permanent magnet. A soft-iron bar is likewise attracted by a current-carrying coil, if the coil and bar are orientated as in figure 7–23. As shown in the figure, the lines of force extend through the soft iron and magnetize it by induction. Because unlike poles attract, the iron bar is pulled toward the coil. If the bar is free to move, it will be drawn into the coil to a position near the center where the field is the strongest.

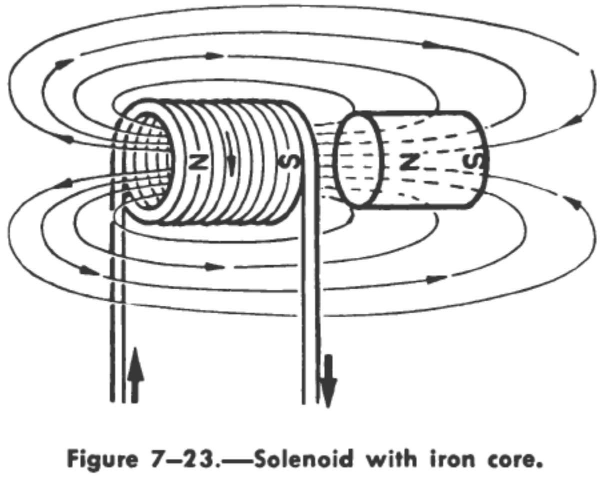


Figure 7–23.—Solenoid with iron core.

The SOLENOID-AND-PLUNGER TYPE of magnet in various forms is employed extensively aboard ships and aircraft to operate the feeding mechanism of carbon-arc searchlights; to open circuit-breakers automatically when the load current becomes excessive; to close switches for motorboat starting; to fire guns; and to operate flood valves, magnetic brakes, and many other devices.

The ARMATURE-TYPE of electromagnet also has extensive applications. In this type of magnet the coil is wound on and insulated from the iron core. The core is not movable. When current flows through the coil the iron core becomes magnetized and causes a pivoted soft-iron armature located near the electromagnet, to be attracted toward it. This type of magnet is used in door bells, relays, circuit breakers, telephone receivers, and so forth.

Applications of Electromagnets

ELECTRIC BELL.—The electric bell is one of the most common devices employing the electromagnet. A simple electric bell is shown in figure 7–24. Its operation is explained as follows:

1. When the switch is closed, current flows from the negative terminal of the battery, through the contact points, the spring, the two coils, and back to the positive terminal of the battery.
2. The cores are magnetized, and the soft-iron armature (magnetized by induction) is pulled down, thus causing the hammer to strike the bell.

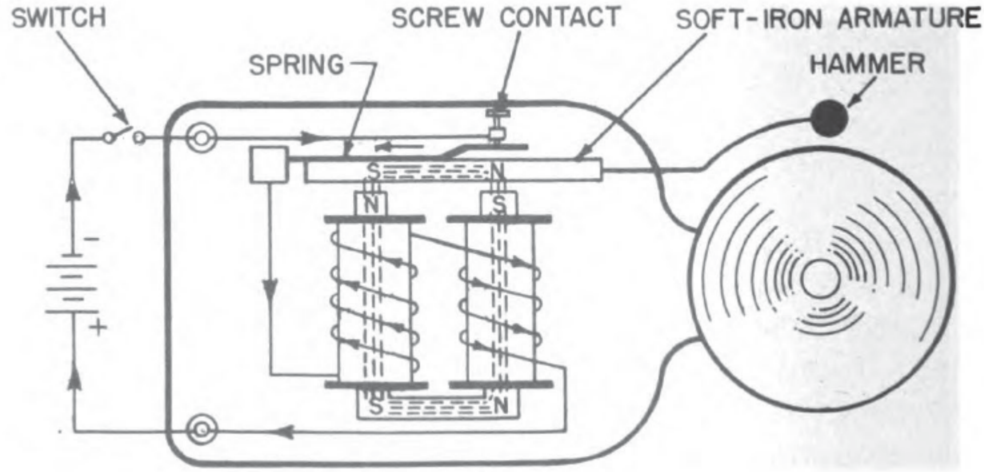


Figure 7–24.—Electric bell.

3. At the instant the armature is pulled down, the contact is broken, and the electromagnet loses its magnetism. The spring pulls the armature up so that contact is reestablished, and the operation is repeated. The speed with which the hammer is moved up and down depends on the stiffness of the spring and the mass of the moving element.

The magnetomotive forces of the two coils are in series addition and therefore the magnetization of the core is increased over that produced by one coil alone.

CIRCUIT BREAKER.—A circuit breaker, like a fuse, protects a circuit against short circuits and overloading. In this device the winding of an electromagnet is connected in series with the load circuit to be protected and with the switch contact points. The principle of operation is shown in figure 7–25. Excessive current through the magnet winding causes the switch to be tripped, and the circuit to both breaker and load is opened by

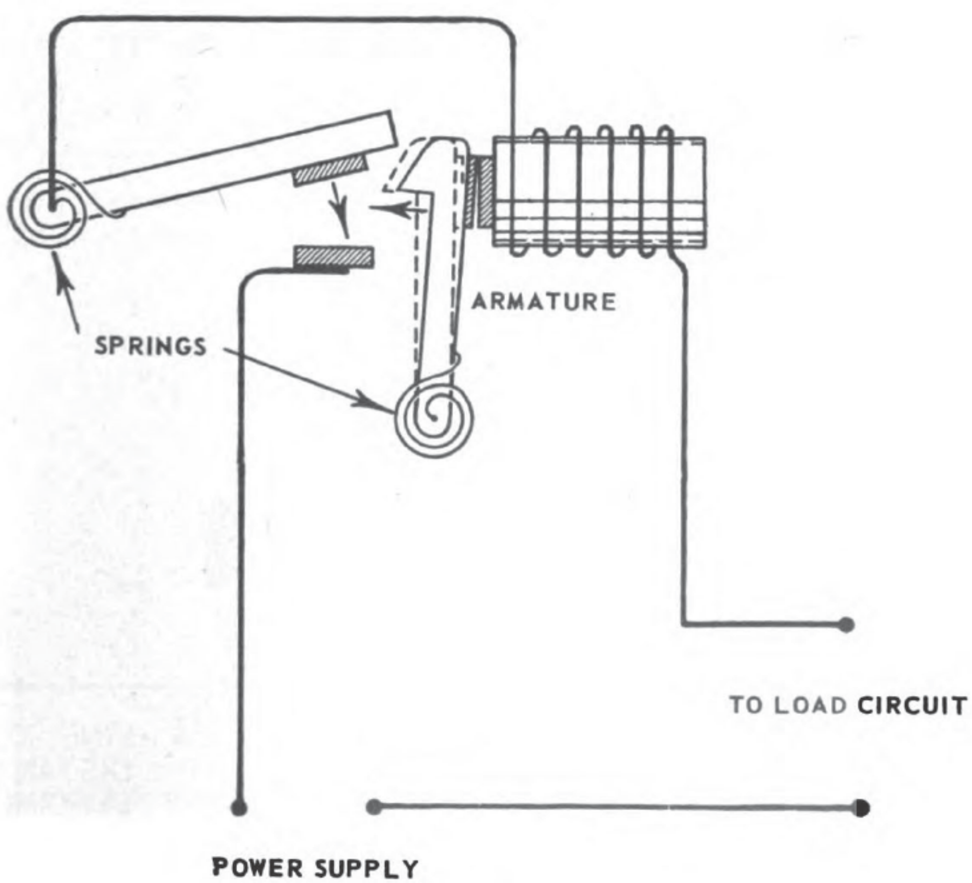


Figure 7–25.—Circuit breaker.

a spring. When the circuit fault has been cleared, the circuit is closed again by manually resetting the circuit breaker.

TELEGRAPH RELAY.—A SIMPLE telegraph consists of a sounder electromagnet, arranged so that its armature will make an audible click when current flows through the coil; a key for closing and opening the circuit; suitable conductors; and a voltage source. The simple telegraph is used where the resistance of the line is low enough to permit sufficient current to energize the sounder electromagnet.

However, when the resistance of the line is great (for example, when a long line with a ground return is used), the current is

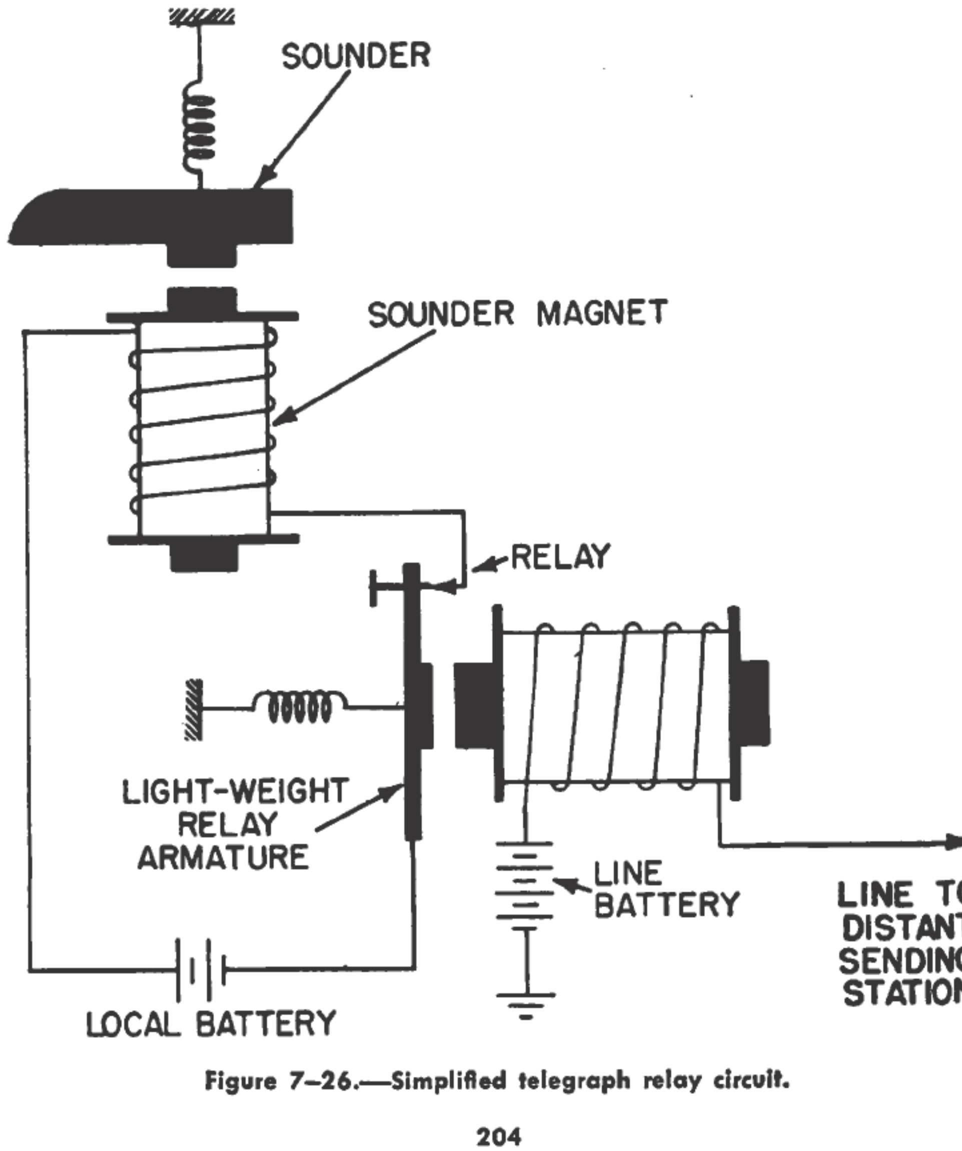


Figure 7-26.—Simplified telegraph relay circuit.

not sufficient to activate a heavy sounder even when a relatively high voltage is employed; and a RELAY must be used.

A simplified telegraph relay circuit is shown in figure 7–26. When the key (switch) at the distant station is closed, current from the line battery flows successively through ground (perhaps the rails of a railroad), the key, the ungrounded side of the line, the relay magnet, and returns to the positive terminal of the line battery. The relatively feeble line current is sufficient to pull over the light-weight relay armature which closes the sounder magnet circuit. Current from the local battery then energizes the sounder electromagnet and the sounder is activated.

VOLTAGE REGULATOR.—In order to maintain a relatively constant voltage at the output of a generator (treated in chapter 10) some type of voltage regulator is employed. Briefly, the output voltage depends in part on the amount of current that flows in the field coil. Therefore, the output voltage may be regulated by varying the amount of current that flows through the field coil.

A simplified voltage regulator circuit is shown in figure 7–27. This circuit consists of a carbon-pile variable resistor inserted in the field circuit of the generator. Generator field current must flow through a series of carbon wafers held together by a spring. The resistance from one end of this series of wafers to the other end is determined by the tension of the spring. When the spring tension is released, the wafers move apart. In this position, the

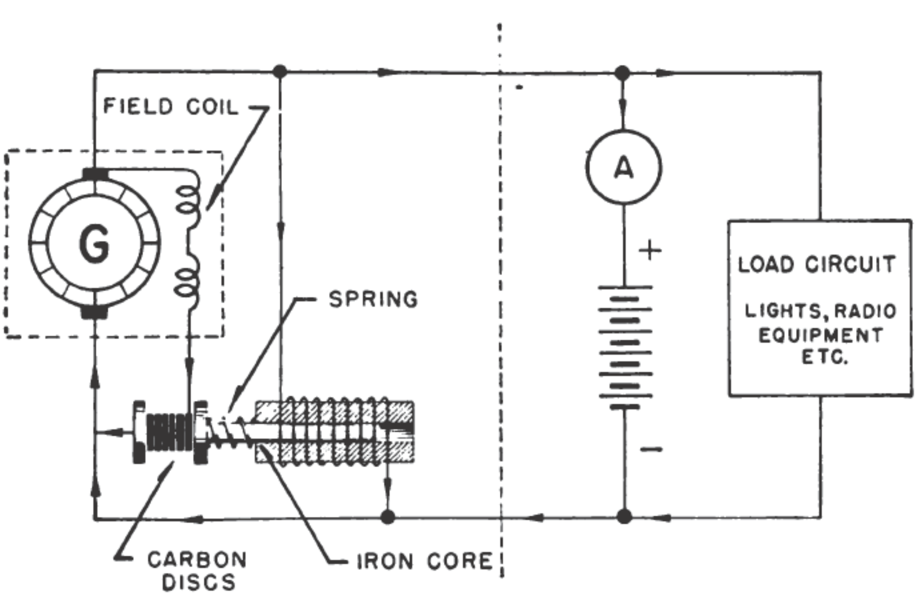


Figure 7–27.—Simplified voltage regulator circuit.

wafers only touch each other at certain points, and the resistance is increased. Conversely, when the spring tension is high, the wafers are pressed together firmly and the over-all resistance decreases. The tension of the spring is controlled by a solenoid connected across the generator terminals.

If, because of an increase in speed, the voltage of the generator increases above the prescribed limit, the current in the solenoid also increases. The solenoid then attracts the iron core, and thus eases the tension on the carbon-pile spring. The resulting increased resistance of the carbon pile reduces the field current and as a result the magnetic flux is reduced. The increased generator speed and the decreased field strength tend to maintain the output voltage nearly constant.

Reverse-current cutout relay.—As has been shown, the electromagnet in the voltage regulator helps to prevent the gen-

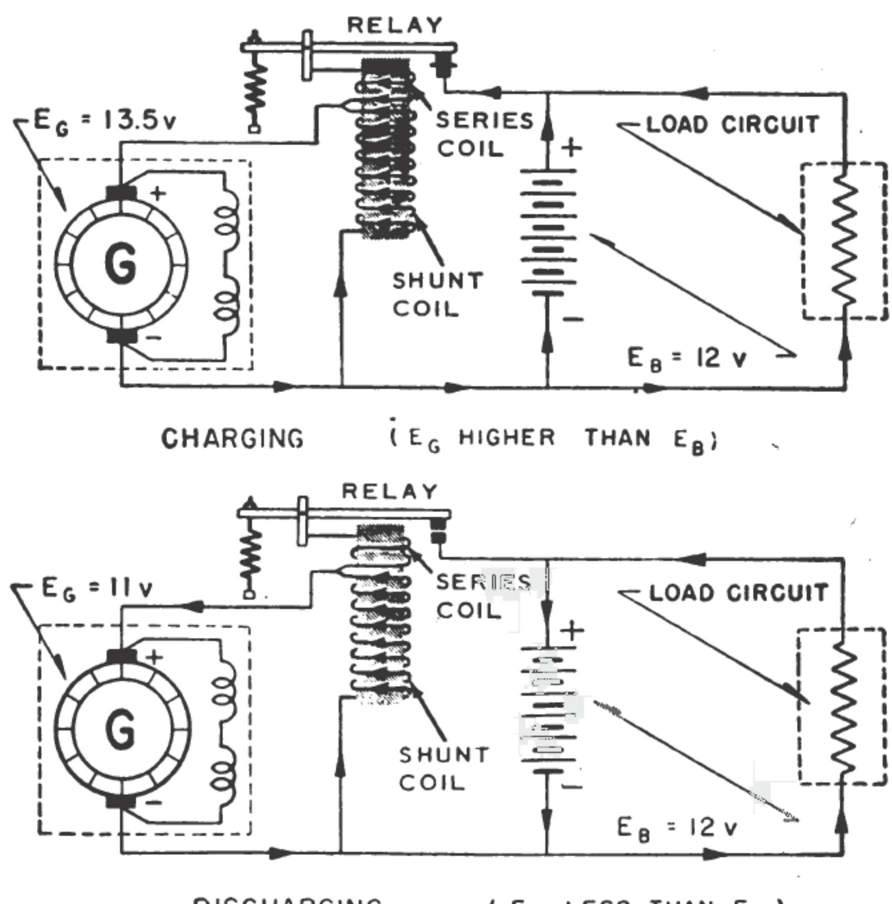


Figure 7–28.—Reverse-current cutout relay.

erator of an aircraft electrical system from exceeding the voltage required for operation of the electrical devices in the load. However, it cannot prevent the voltage of the generator from falling if the engine speed is insufficient. Furthermore, when the engine is stopped the generator has no voltage whatsoever. At such times, the battery would discharge through the generator. Thus, current would be wasted, and serious damage to the generator armature might result. To prevent this effect, a magnetic cutout relay is placed in the electrical system to disconnect the battery from the generator whenever the generator voltage falls below that of the battery.

The action of a typical reverse-current cutout relay is shown in figure 7–28. Normally, the contact points of the cutout relay are held open by a spring. Under this condition, the generator is disconnected from the line. As the engine speed increases, the generator voltage increases and sends a current through the shunt coil of the relay. The shunt coil of the relay is always across the generator terminals. The current in the shunt coil is proportional to the generator voltage. The tension of the contact spring is so adjusted that the relay contacts close when the generator voltage slightly exceeds the battery voltage.

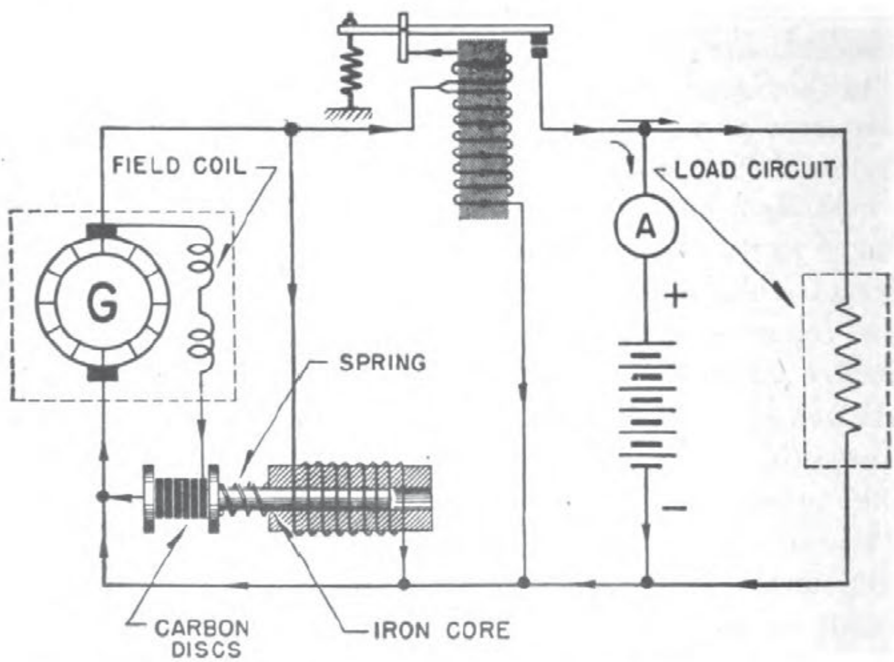
The action of closing the contact points connects the generator to the line. Charging current from the generator now flows through the series coil, the contact points, and the battery circuit. The generator continues to supply a relatively small current to the shunt coil of the relay.

Now the direction of current in both the shunt and series coils is such that the coils assist each other in magnetizing the core. If the load current or the battery-charging current increases, the relay contact points are held more firmly together. Thus, the relay does not regulate the charging rate or affect the load current in any way as long as its contacts remain closed.

When the engine speed decreases, the generator speed also decreases and the generator voltage drops. When the point is reached where the generator voltage is less than the battery voltage, the battery momentarily discharges through the generator. The discharge current passes through the series coil in the opposite direction to that of the charging current. Current in the shunt coil continues in the original direction. The series coil now op-

poses the shunt coil and cancels the magnetism of the core. This action permits the spring to separate the contacts and disconnect the generator from the battery.

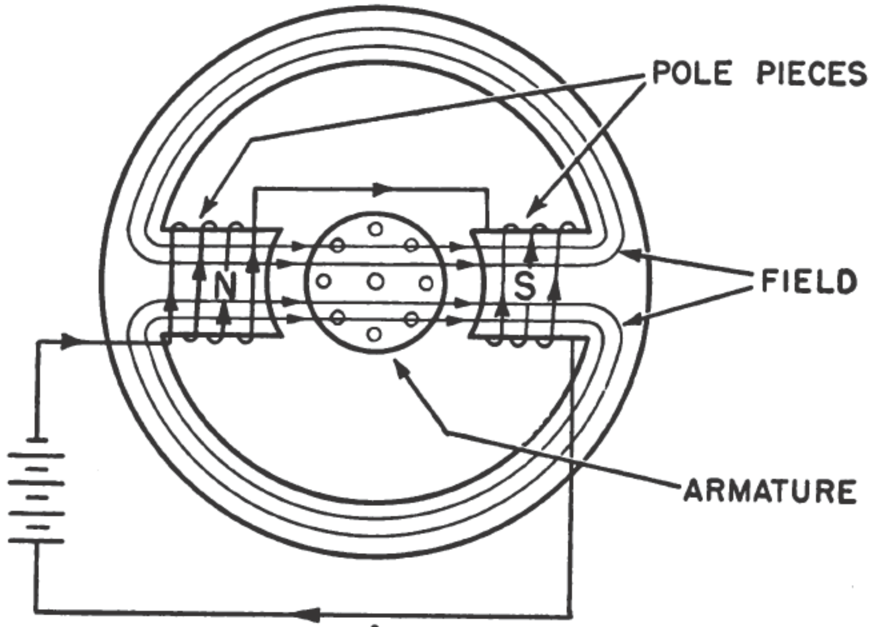
Figure 7–29 shows the circuit connections of 2 electromagnets composing a simple carbon-pile voltage regulator and a reverse current cutout, both in the same electrical system. Similar arrangements are used in other installations.



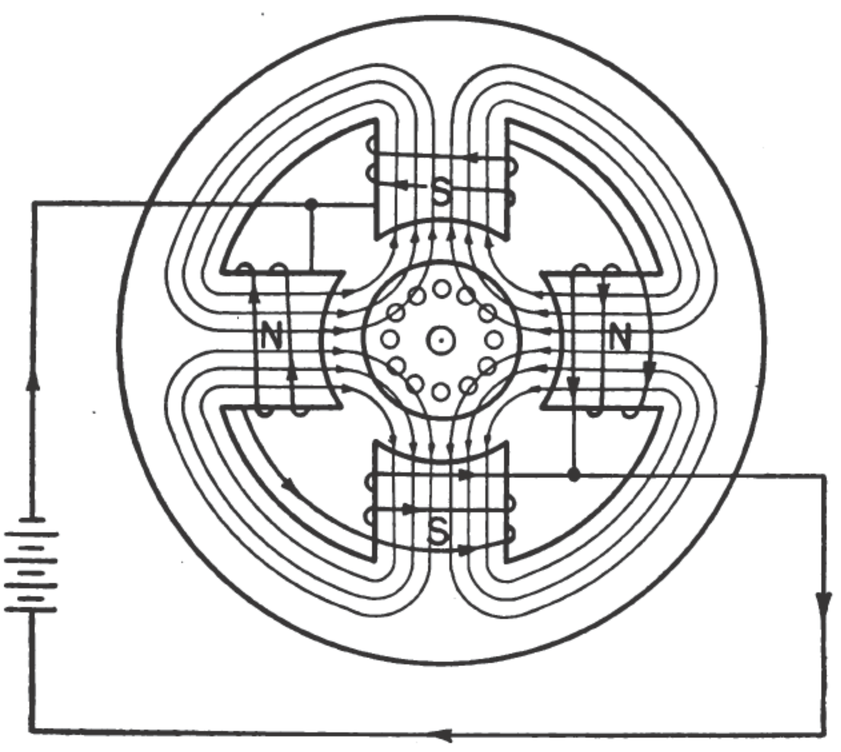
Fgure 7–29.—Use of electromagnets in aircraft electrical systems.

Generator field magnets.—The field magnets of d-c electric motors and generators (treated later in the text) represent another important use of electromagnets. Figure 7–30, A, shows in simplified form the field magnets of a 2-pole d-c generator or motor. The 2 field coils are in series with a d-c source. The pole pieces concentrate the flux in the air gap between the rotating and stationary members, and the flux path is completed through the iron frame. The ampere-turns in both windings contribute to the establishment of the magnetic flux across the air gap in the direction indicated in the figure.

When 4 poles are used, as in figure 7–30, B, the windings and flux distributions are as shown. As in the 2-pole motor or generator, each flux path includes the iron frame, the pole pieces,



A
TWO POLES



B
FOUR POLES

Figure 7–30.—Field magnets.

the air gap, and the rotating member, all in series. There are as many flux paths as there are poles in the field.

EARPHONES.—Earphones utilize permanent and electromagnets. The basic components of earphones are shown in figure 7–31. When no signal currents are present, the permanent magnet exerts a steady pull on the soft-iron diaphragm. Signal currents flowing through the coils wound on the soft-iron pole pieces develop a magnetomotive force that either adds to or subtracts from the field of the permanent magnet. The dia-

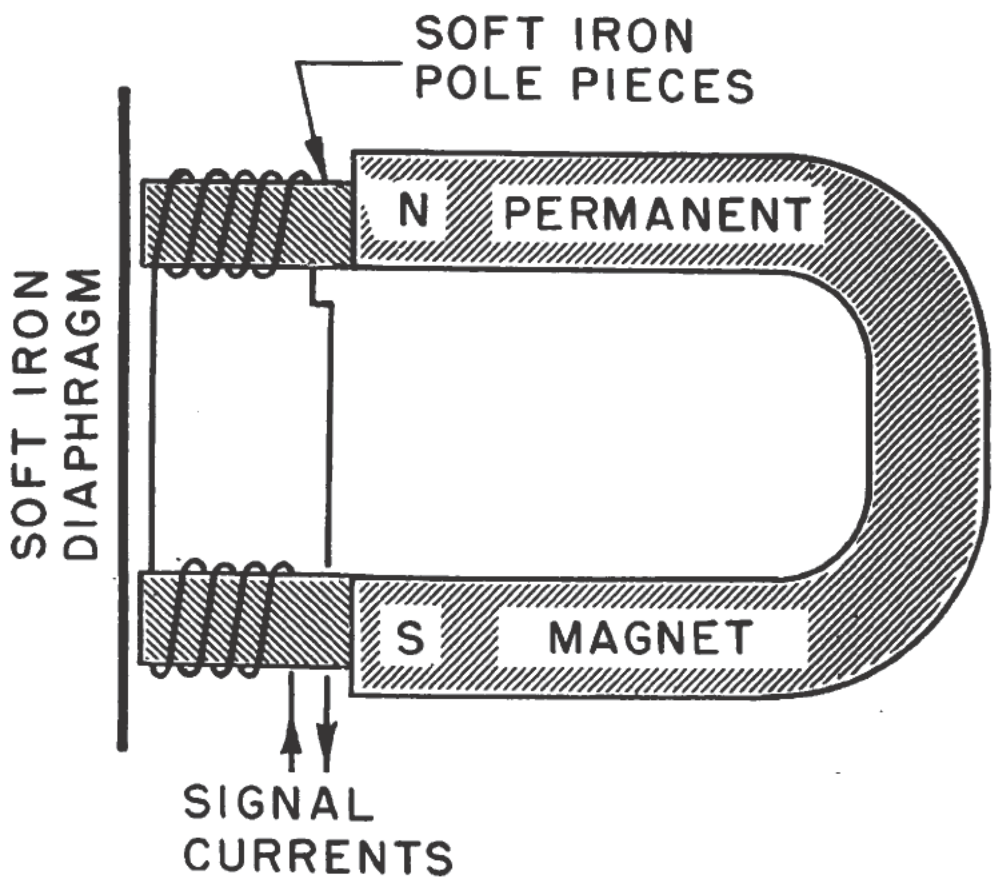


Figure 7–31.—Basic components of earphones.

phragm thus moves in or out according to the resultant field. Sound waves that have amplitudes and frequencies similar to the amplitudes and frequencies of the signal currents (within the capabilities of the reproducer) will then be produced.

QUIZ

1. What property distinguishes a magnet from a magnetic substance?
2. What are the points of maximum attraction of a magnetized steel knitting needle called?
3. Of what does a magnetic field consist?
4. From which pole of a magnet do the lines of force emanate?
5. Basically, what is a magnetic circuit?
6. How does the core material of a permanent magnet differ from the core material of an electromagnet?
7. Describe two methods of magnetizing a steel bar.
8. What is the name that identifies the magnetism that remains in a temporary magnet after the magnetizing force is removed?
9. Even though an unmagnetized iron bar consists of molecular magnets, why is no external magnetic effect produced?
10. What are so-called magnetic DOMAINS?
11. In figure 7–5, why is the curve smoother above the point of saturation?
12. Beginning at the north pole of a bar magnet, trace the path of the magnetic lines of force.
13. State the formula for flux density, B, in terms of the total number of lines of flux, Φ, and the cross-sectional area, A, of the magnetic circuit.
14. (a) Like magnetic poles ________________ each other.
    (b) Unlike magnetic poles ________________ each other.
15. If the distance between two unlike magnetic poles is increased from 3 feet to 12 feet, the resultant attractive force will be what relative magnitude compared to what it was at 3 feet?
16. If the distance between two unlike magnetic poles remains constant and the strength of both poles is doubled, the resultant attractive force will be what relative magnitude compared to what it was before the strength was increased?
17. If it is assumed that the end of the compass needle that points in a northerly direction is a north pole, what is the polarity of the earth's magnetic pole that is located near the north geographic pole?
18. If current flows in a conductor toward an observer, what is the direction (clockwise or counterclockwise) of the field around the conductor?
19. Two adjacent parallel conductors (a) ________________ each other when carrying currents in opposite directions; and (b) ________________ each other when they carry currents in the same direction.

20. What is the left-hand rule for determining the magnetic polarity of a coil in relation to the direction of the current through the turns?
21. How may the field strength of a coil (fig. 7–15) be increased?
22. What is the magnetizing force in ampere turns of a 1,000-turn coil carrying a current of 100 milliamperes?
23. How are electric instruments and meters protected from stray magnetic fields?
24. If in a magnetic circuit the magnetomotive force is doubled and the reluctance offered by the circuit remains the same, how will the flux be affected?
25. Define the maxwell.
26. Express 1,000 ampere-turns in gilberts.
27. What is the intensity of the magnetizing force in gilberts per centimeter of length of 1,000 ampere turns acting over a length of 10 centimeters?
28. How does the reluctance of a magnetic substance in a magnetic circuit vary with (a) length, (b) permeability, and (c) cross-sectional area?
29. If the ring in figure 7–18 were made of cast iron instead of cast steel (table number 9), how many ampere-turns would be necessary to produce 20,000 lines of flux?
30. Inserting an iron core into a coil increases its flux density from 2 lines per square centimeter to 3,000 lines per square centimeter. What is the permeability of the iron?
31. What is meant by the magnetic retentivity of a substance?
32. What causes the hysteretic lag in magnetization behind the force that produces the magnetization?
33. In figure 7–22, the areas within the hysteresis loops are an indication of what value?
34. Inserting an iron core into a coil the current of which is constant will ________________________ the permeability of the core.
35. In figure 7–24, what would be the effect of reversing the connections of one of the bell coils with respect to the other?
36. When the resistance of a telegraph line is great and the current is not sufficient to activate a heavy sounder, what electrical device must be used to activate the sounder?
37. In the circuit shown in figure 7–27, the speed of the generator increases. How does this affect the solenoid, the carbon pile, and the output voltage?

CHAPTER 8

INDUCTANCE AND CAPACITANCE

INTRODUCTION

INDUCTANCE is the inherent property of an electric circuit that opposes any change of CURRENT in the circuit. It is also defined as the property of a circuit whereby energy may be stored in a MAGNETIC field.

CAPACITANCE is the inherent property of an electric circuit that opposes any change of VOLTAGE in the circuit. It is also defined as the property of a circuit whereby energy may be stored in an ELECTROSTATIC field.

The properties of inductance and capacitance depend only on the circuit components; they do not depend on the type (a-c or d-c) of current that flows in the circuit. Because these two properties are of the utmost importance in both power and signal circuits it is necessary that the interested technician acquire a thorough elementary knowledge of inductance and capacitance. He should also become familiar with the effects produced by the addition of inductors or capacitors to a given circuit.

INDUCTANCE

Induced EMF

Before the property of inductance can be meaningful, it is necessary to have an elementary understanding of INDUCED emf. This subject is treated more in detail, however, in the following chapters.

If the ends of a conductor are connected to a low-reading voltmeter or galvanometer and the conductor is moved rapidly down through a strong magnetic field, as illustrated in figure 8–1, A, there is a momentary reading on the meter. When the conductor is moved up through the field (fig. 8–1, B), the meter deflects in the opposite direction. If the conductor is held sta-

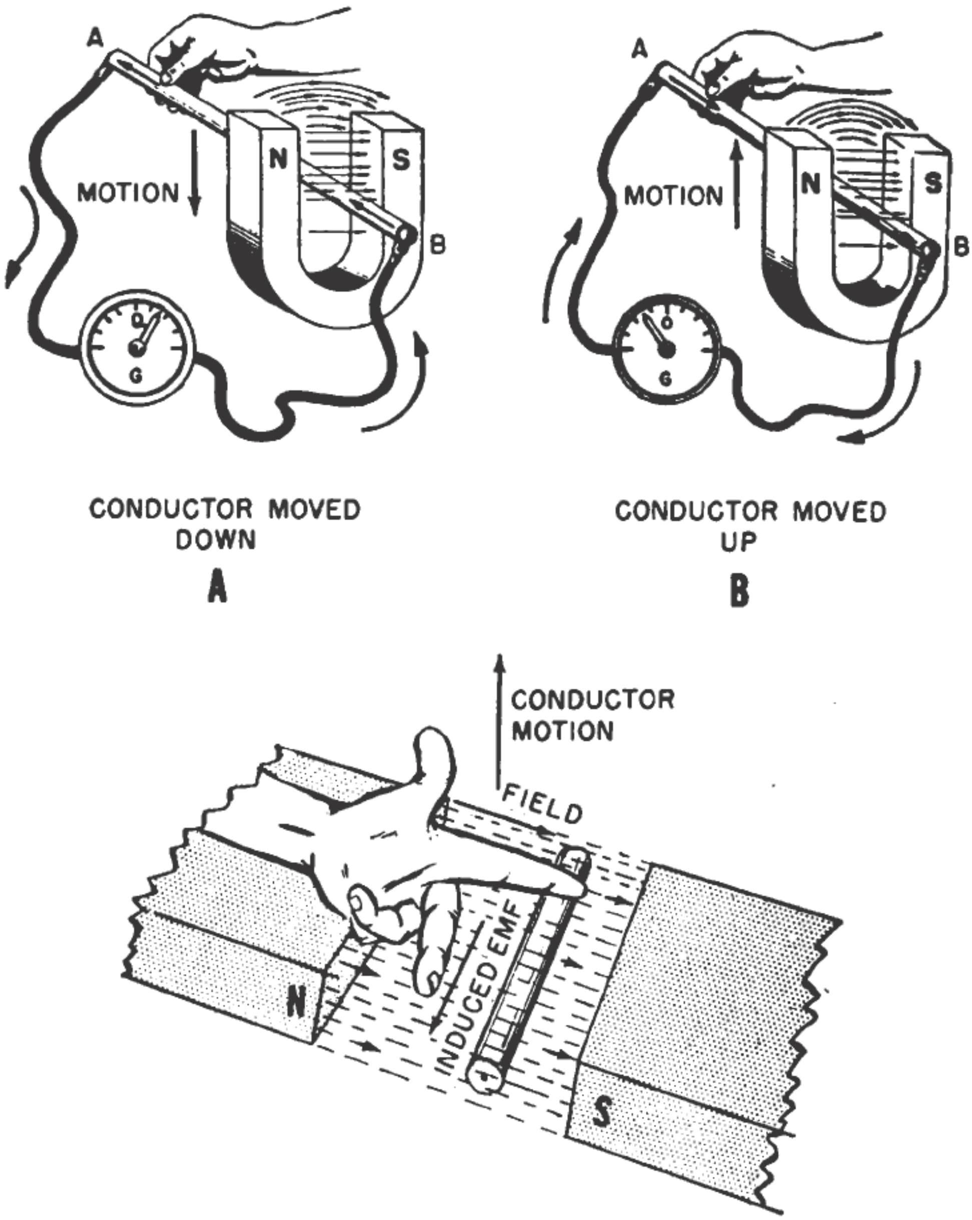


Figure 8–1.—Inducing an emf in a conductor.

tionary and the magnet is moved so that the field cuts across the conductor, the meter is again deflected in the same manner as when the conductor was moved and the field was stationary.

The voltage developed across terminals *AB* of the conductor (fig. 8–1, A and B) by electromagnetic induction is known as an

INDUCED EMF, and the current that flows as a result of this induced emf is called an INDUCED CURRENT. The induced emf exists only as long as relative motion occurs between the conductor and the field. The result is the same whether the field is stationary and the conductor moves, or the conductor is stationary (as will be the case in this chapter) and the field moves.

There is a definite relation between the direction of flux, the direction of motion of the conductor, and the direction of the induced emf. When two of these directions are known, the third can be found by applying the LEFT-HAND RULE FOR GENERATORS. To find the direction of the emf induced in a conductor, extend the thumb, first finger, and second finger of the left hand at right angles to each other, as shown in figure 8–1, C. Point the first finger in the direction of the flux (toward the south pole) and the thumb in the direction in which the conductor is moving. The second finger then points in the direction of the induced emf—that is, in the direction the induced emf will cause the electrons to flow.

Unit of Inductance

The unit for measuring inductance, L, is the HENRY, h. An inductor (coil) has an inductance of 1 henry if an emf of 1 volt is induced in the inductor when the current through the inductor is changing at the rate of 1 ampere per second. The relation between the induced voltage, inductance, and rate of change of current with respect to time is stated mathematically as

$$E=L\frac{\Delta I}{\Delta t},$$

where E is the induced emf in volts, L is the inductance in henrys, and ΔI is the change in current in amperes occurring in Δt seconds.

The henry is a large unit of inductance and is used with relatively large inductors having iron cores. The unit employed with small air-core inductors is the millihenry, mh. For still smaller air-core inductors the unit of inductance is the microhenry, μh.

The inductance of an inductor, or of any circuit element, depends on the ability of the inductor or circuit element to produce magnetic flux. It is an inherent property of a circuit and except

for circuits containing iron it is independent of the magnitude of the current flowing.

Self-Inductance

SELF-INDUCTANCE DEFINED.—As previously explained in chapter 7, current in a conductor always produces a magnetic field surrounding, or linking with, the conductor. When the current changes, the magnetic field changes and an emf is induced in the conductor. This emf is called a SELF-INDUCED EMF because it is induced in the conductor carrying the current. The direction of the induced emf has a definite relation to the manner in which the field that induces the emf is varying. When the current in a circuit is increasing, the flux linking with the circuit is increasing. This flux cut across the conductor and induces an emf in the conductor in such a direction as to oppose the increase in current and flux. Likewise, when the current is decreasing, an emf is induced in the opposite direction and opposes the decrease in current. These effects are summarized by Lenz's law, which states that THE INDUCED EMF IN ANY CIRCUIT IS ALWAYS IN SUCH A DIRECTION AS TO OPPOSE THE EFFECT THAT PRODUCES IT.

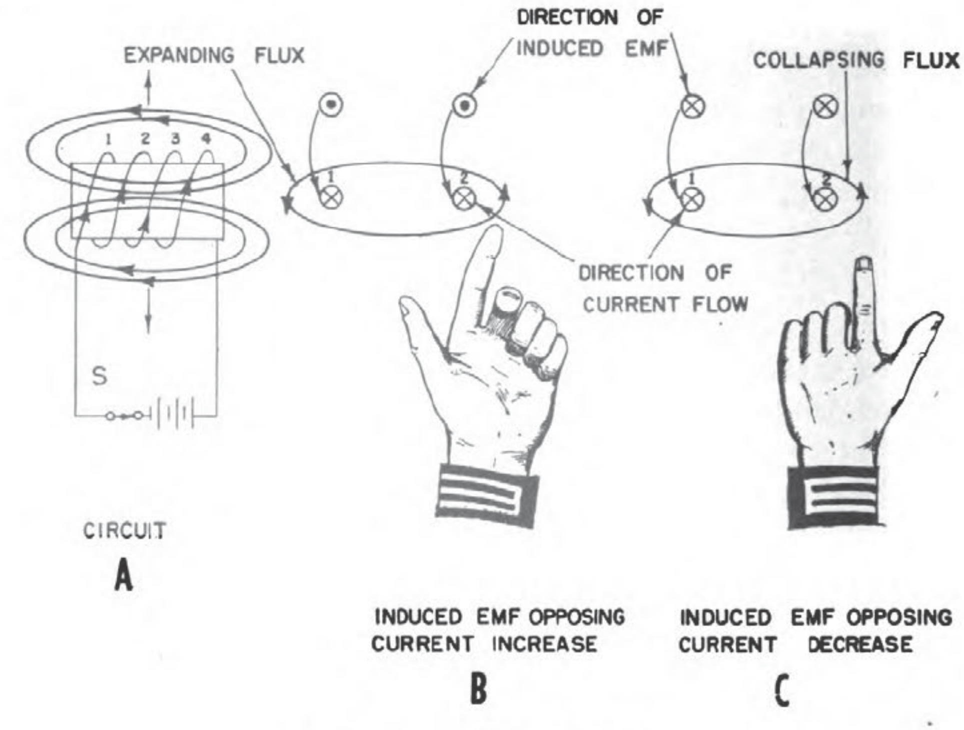


Figure 8–2.—Self-inductance.

An electric circuit in which an appreciable emf is induced while the current in it is changing has self-inductance and is called an INDUCTIVE CIRCUIT. An electric circuit consisting of two parallel conductors spaced a small distance apart has relatively low inductance because the flux linking the two conductors is small. A circuit containing a coil of many turns (fig. 8–2, A), however, has a much higher inductance because essentially all the flux links all the turns, and a much greater voltage is induced in the circuit.

When switch *S* (fig. 8–2) is closed, the current through the inductor, or solenoid, establishes a magnetic flux that links the turns of the solenoid in the direction indicated. The steady value of current is determined by the resistance of the solenoid. The corresponding value of flux is determined by the current, the number of turns on the solenoid, and the magnetic reluctance.

In the process of establishing this flux, a self-induced emf is set up in the coil in a direction opposite to the direction of current so that the current does not reach its steady value instantaneously. In the top half of turns 1, 2, 3, and 4 (fig. 8–2, A), the current is away from the observer, as indicated by the arrows and also by the tail of the arrows in figure 8–2, B. As the current increases in magnitude, the magnetic flux expands outward from the center of each turn and cuts across adjacent turns.

Consider one line of force set up by the rising current in turns 1 and 2 (fig. 8–2, B). As the flux expands it cuts across both turns and induces an emf in these turns. This emf opposes the current rise in these turns. The left-hand generator rule shows the induced emf in turns 1 and 2 to be opposite to the current in turns 1 and 2. Applying the left hand to turn number 2, the first finger points upward in the direction of the line of magnetic force, and the thumb points in the direction of the motion of the conductor with respect to the field. The field is expanding to the right and, although the conductor itself is not moving, the relative motion of the conductor with respect to the field is to the left and the thumb points in that direction. The second finger points toward the observer and the direction of the self-induced voltage is indicated as a dot in the circle directly above turn number 2. Turns 3 and 4 of the solenoid react in the same manner. A simultaneous rise of current in all the turns of the solenoid is opposed by the emf induced in these turns.

When switch S is opened, the magnetic flux around the solenoid collapses and cuts all of the turns in the opposite direction. Consider one line of force associated with turns 1 and 2 (fig. 8–2, C). As the line of force collapses, it cuts turns 1 and 2 in the opposite direction to that shown in figure 8–2, B. This action induces an emf in the reverse direction—that is, in the same direction as the current—and tends to maintain the current by opposing its decrease. A circuit containing inductance always opposes any change in current. The self-induced emf of the solenoid opposes the increase in current (fig. 8–2, B) and also opposes the decrease in current (fig. 8–2, C). Inductance has no effect on a steady current. A steady current is opposed only by the resistance of the circuit.

FACTORS AFFECTING SELF-INDUCTANCE.—The value of inductance that an inductor has depends on many things—among them, the number of turns of wire, the ratio of the length of the inductor to its diameter, and the material in the core. Therefore, various formulas have been developed for calculating the self-inductance of inductors. For an inductor consisting of a single layer of wire, and having a length 10 or more times the diameter, the following formula is essentially correct:

$$L=\frac{0.4\pi\mu N^2S}{10^8l},$$

in which L is the self-inductance in henrys, N is the number of turns, π equals 3.14, μ is the permeability of the core (if nonmagnetic, $\mu=1$); S is the cross-sectional area of the coil in cm², and l is the length of the coil in centimeters. Thus, it may be seen that the inductance is increased very rapidly as the number of turns is increased; also, the inductance increases as the coil is made shorter, the cross-sectional area is made larger, or the permeability of the core is increased.

For example, the inductance of a coil having a length of 20 cm, a diameter of 2 cm, a core permeability of 200, and 200 turns of wire is approximately

$$L=\frac{0.4\pi\times200\times200^2\times0.7854\times2^2}{10^8\times20}=0.0158 \text{ henry.}$$

The dependence of the induced voltage on the rate of change of flux with respect to time is shown by means of the simple ap-

paratus shown in figure 8–3. When the switch is closed, current builds up to a maximum, and the lamp glows with its normal brilliance. If the iron core is inserted rapidly into the coil, the flux increases rapidly (because of the increased inductance of the coil), and the induced voltage opposes, according to Lenz's law, the source voltage. Therefore, less current flows through the lamp, and it dims momentarily. If the core is withdrawn rapidly from the coil, a portion of the flux that was established around the coil collapses. The resulting induced voltage opposes the decrease (again, according to Lenz's law) by aiding the source voltage, and the coil current increases. Consequently, the lamp burns brighter momentarily. The faster the iron core is moved the greater is the flux change per unit of time and the more noticeable is the effect on the lamp.

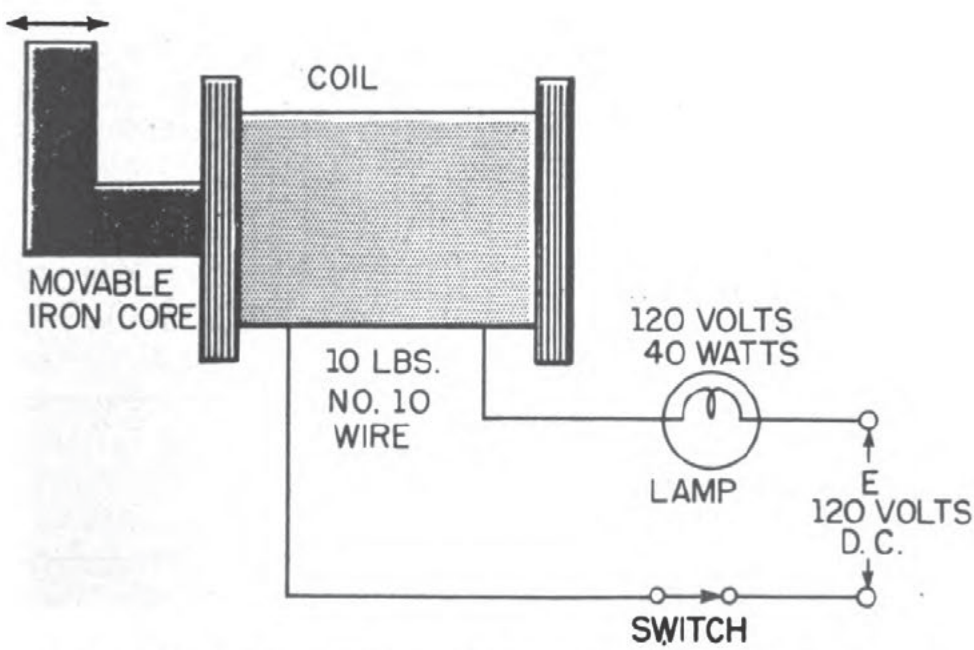


Figure 8–3.—Dependence of induced voltage on the rate of change of flux.

GROWTH AND DECAY OF CURRENT IN AN R-L SERIES CIRCUIT.—If a battery is connected across a pure inductance, the current builds up to its final value at a rate that is determined by the battery voltage and the internal resistance of the battery. The current build-up is gradual because of the counter emf generated by the self-inductance of the coil. When the current starts to flow, the magnetic lines of force move out, cut the turns of wire

on the inductor, and build up a counter emf that opposes the emf of the battery. This opposition causes a delay in the time it takes the current to build up to a steady value. When the battery is disconnected, the lines of force collapse, again cutting the turns of the inductor and building up an emf that tends to prolong the current flow.

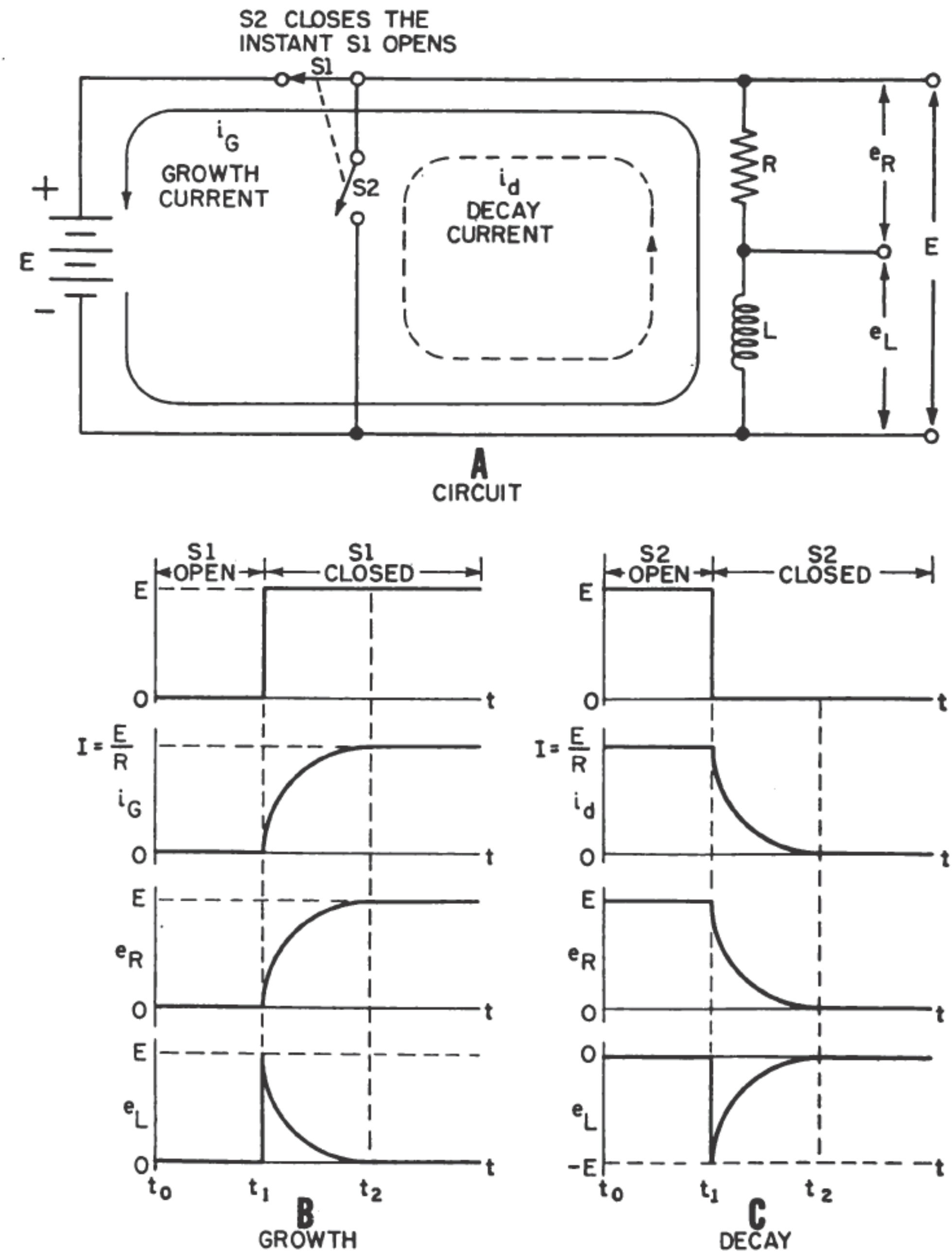


Figure 8–4.—Growth and decay of current in an *R-L* series circuit.

A voltage divider containing resistance and inductance may be connected in a circuit by means of a special switch, as shown in figure 8–4, A. Such a series arrangement is called an *R-L* series circuit.

If switch *S*1, is closed (as shown), a voltage, *E*, appears across the divider. A current attempts to flow, but the inductor opposes this current by building up a back emf that, at the initial instant, exactly equals the input voltage, *E*. Because no current can flow under this condition, there is no voltage across resistor *R*. Figure 8–4, B, shows that all of the voltage is impressed across *L* and no voltage appears across *R* at the instant switch *S*1 is closed.

As current starts to flow, a voltage, e_r, appears across *R*, and e_L is reduced by the same amount. The fact that the voltage across *L* is reduced means that the growth current, i_g, is increasing and censequently e_r is increasing. Figure 8–4, B, shows that e_L finally becomes zero when i_g stops increasing, while e_r builds up to the input voltage, *E*, as i_g reaches its maximum value. Under steady-state conditions, only the resistor limits the size of the current.

Electrical inductance is like mechanical inertia, and the growth of current in an inductive circuit can be likened to the acceleration of a boat on the surface of the water. The boat begins to move at the instant a constant force is applied to it. At this instant its rate of change of speed (acceleration) is greatest, and all the applied force is used to overcome the inertia of the boat. After a while the speed of the boat increases (its acceleration decreases) and the applied force is used up in overcoming the friction of the water against the hull. As the speed levels off and the acceleration becomes zero, the applied force equals the opposing friction force at this speed and the inertia effect disappears.

When the battery switch in the *R-L* circuit of figure 8–4, A, is closed, the rate of current increase is maximum in the inductive circuit. At this instant all the battery voltage is used in overcoming the emf of self-induction which is a maximum because the rate of change of current is maximum. Thus the battery voltage is equal to the drop across the inductor and the voltage across the resistor is zero. As time goes on more of the battery voltage appears across the resistor and less across the inductor. The rate of change of current is less and the induced emf is less. As the steady-state condition of the current flow is approached

the drop across the inductor approaches zero and all of the battery voltage is used to overcome the resistance of the circuit.

Thus the voltage across the inductor and resistor change in magnitudes during the period of growth of current the same way the force applied to the boat divides itself between the inertia and friction effects. In both examples, the force is developed first across the inertia-inductive effect and finally across the friction-resistive effect.

If switch $S2$ is closed (source voltage E removed from the circuit), the flux that has been established around L collapses through the windings and induces a voltage, e_L, in L that has a polarity opposite to E and essentially equal to it in magnitude. The induce voltage, e_L, causes current i_d to flow through R in the same direction that it was flowing when $S1$ was closed. A voltage, e_r, that is initially equal to E, is developed across R. It rapidly falls to zero as the voltage, e_L, across L, due to the collapsing flux, falls to zero.

$\frac{L}{R}$ TIME CONSTANT.—The time required for the current through an inductor to increase to 63 percent (actually, 63.2 percent) of the maximum current or to decrease to 37 percent (actually, 36.7 percent) is known as the TIME CONSTANT of the circuit. An R-L circuit and its charge and discharge graphs are shown in figure 8–5. The value of the time constant in seconds is equal to the inductance in henrys divided by the circuit resistance in ohms. One set of values is given in figure 8–5, A. $\frac{L}{R}$ is the symbol used for this time constant.

Two useful relations used in calculating $\frac{L}{R}$ time constants are as follows:

$$\frac{L\,(\text{in henrys})}{R\,(\text{in ohms})}=t\ (\text{in seconds}).$$

$$\frac{L\,(\text{in microhenrys})}{R\,(\text{in ohms})}=t\ (\text{in microseconds}).$$

The time constant may also be defined as the time required for the current through the inductor to grow or decay to its final value IF it continued to grow or decay at its initial rate. As may

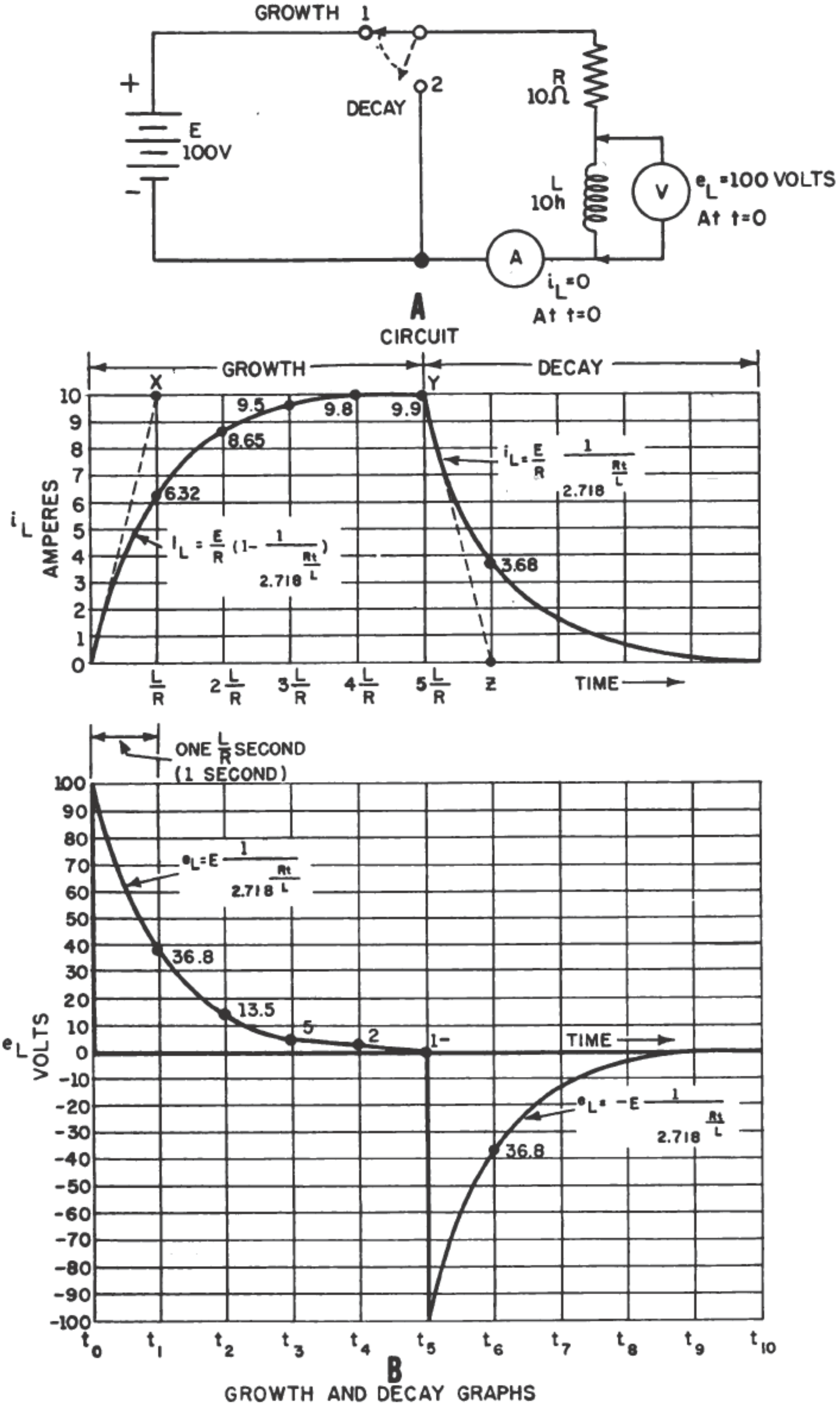


Figure 8–5.—$\frac{L}{R}$ time constant.

be seen in figure 8–5, B, the slope of the dotted tangent line, ox, indicates the initial rate of current growth with respect to time. At this rate, the current would reach its maximum value in $\frac{L}{R}$ seconds. Similarly, the slope of the dotted tangent line, YZ, indicates the initial rate of current decay with respect to time, and the decay would be completed in $\frac{L}{R}$ seconds.

The equation for the growth of current, i_L, through L is

$$i_L=\frac{E}{R}\left(1-\frac{1}{2.718^{\frac{Rt}{L}}}\right),$$

where i_L is the instantaneous current through inductor L, E is the applied voltage (100 volts in this case), R is the resistance in ohms, t is the time in seconds, and L is the inductance in henrys. Figure 8–5, B, shows a graph of this equation.

When $t=\frac{L}{R}$, the exponent $\frac{Rt}{L}$ in the preceding equation reduces to 1. Then $\frac{1}{2.718}=0.368$. Therefore,

$$i_L=\frac{E}{R}(1-0.368)=0.632\,\frac{E}{R}.$$

In other words, when $t=\frac{L}{R}$, i_L is equal to 63.2 percent of the ratio $\frac{E}{R}$, which is the maximum current. When the maximum current is 10 amperes ($E=100$ and $R=10$), the current through L grows to 6.32 amperes in $\frac{L}{R}=\frac{10}{10}$, or 1 second.

The equation for inductor voltage, e_L, on growth of current is

$$e_L=E\left(\frac{1}{2.718^{\frac{Rt}{L}}}\right).$$

The graph of this equation is also shown in figure 8–5, B. When $t=\frac{L}{R}$, $e_L=0.368E$; that is, $e_L=0.368\times 100=36.8$ volts.

Mutual Inductance

MUTUAL INDUCTANCE DEFINED.—Whenever two coils are located so that the flux from one coil links with the turns of the other a change of flux in one coil causes an emf to be induced in the other coil, and the two coils have MUTUAL INDUCTANCE. The amount of mutual inductance depends on the relative position of the two coils. If the coils are separated a considerable distance, the amount of flux common to both coils is small and the mutual inductance is low. Conversely, if the coils are close together so that nearly all the flux of one coil links the turns of the other the mutual inductance is high. The mutual inductance can be increased greatly by mounting the coils on a common iron core.

Two coils placed close together with their axes in the same plane are shown in figure 8–6. Coil *A* is connected to a battery through switch *S*, and coil *B* is connected to galvanometer *G*. When the switch is closed (fig. 8–6, A), the current that flows in coil *A* sets up a magnetic field that links coil *B*, causing an induced current and a momentary deflection of galvanometer *G*. When the current in coil *A* reaches a steady value, the galvanometer returns to zero. If the switch is opened (fig. 8–6, B), the galvanometer deflects momentarily in the opposite direction, indicating a momentary flow of current in the opposite direction in coil *B*. This flow of current in coil *B* is caused by the emf induced in coil *B*, which is produced by the collapsing flux of coil *A*.

The direction of the emf induced in coil *B* when the switch is closed can be determined by the left-hand rule. When current

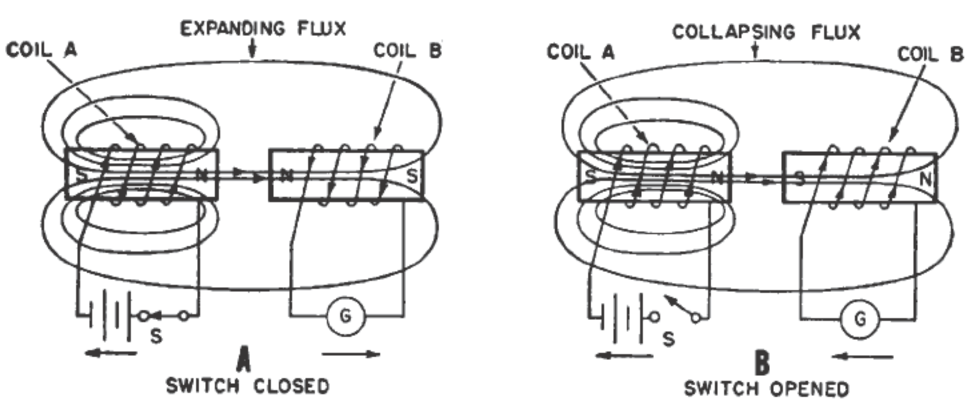


Figure 8–6.—Mutual inductance.

flows through coil A the flux expands in coil A producing a north pole nearest coil B. Some of the flux, that expands from left to right, cuts the turns of coil B. This flux induces an emf and current in coil B that opposes the growth of current and flux in coil A. Thus the current in B tries to establish a north pole nearest coil A (like poles repel).

When the switch is opened, the magnetic field produced by coil A collapses. The collapse of the flux cuts the turns of coil B in the opposite direction and produces a south pole nearest coil A (unlike poles attract). This polarity aids the magnetism of coil A, tending to prevent the collapse of its field.

FACTORS AFFECTING MUTUAL INDUCTANCE.—If the two coils are so positioned with respect to each other that all the flux of one coil cuts all the turns of the other, the coils have UNITY COEFFICIENT OF COUPLING. If all of the flux produced by one of the coils cuts only one-half the turns of the other coil, the coefficient of coupling is 0.5. Coefficient of coupling is designated by the letter, K. The coefficient of coupling is equal to the percentage of flux originating in one coil that cuts the other coil. It is never exactly equal to unity but it approaches this value in certain shell-type transformers.

The mutual inductance between two coils, $L1$ and $L2$, may be expressed in terms of the inductance of each coil and the coefficient of coupling, K, as follows,

$$M=K\sqrt{L_1L_2},$$

where M is in the same units of inductance as $L1$ and $L2$.

SERIES INDUCTORS WITHOUT MAGNETIC COUPLING.—When inductors are well shielded, or located far enough apart to make the effects of mutual inductance negligible, the inductance of the various inductors are added in the same manner that the resistances of resistors are added. For example,

$$L_T=L_1+L_2+L_3 \ . \ . \ . \ +L_N,$$

where L_T is the total inductance; L_1, L_2, L_3 are the inductances of of $L1$, $L2$, $L3$; and L_N means that any number (N) of inductors may be used.

SERIES INDUCTORS WITH MAGNETIC COUPLING.—When two in-

ductors in series are so arranged that the field of one links the other, the combined inductance is determined as

$$L_T = L_1 + L_2 \pm 2M,$$

where L_T is the total inductance, L_1 and L_2 are the self inductances of $L1$ and $L2$ respectively, and M is the mutual inductance between the two inductors. The plus sign is used with M when the magnetomotive forces of the two inductors are aiding each other. The minus sign is used with M when the mmf's of the two inductors oppose each other. The factor 2 accounts for the influence of $L1$ on $L2$ and of $L2$ on $L1$.

If the coils are arranged so that one can be rotated relative to the other to cause a variation in the coefficient of coupling, the mutual inductance between them can be varied. From a consideration of the preceding formula, the total inductance, L_T, may be varied by an amount equal to $4M$. Such an arrangement of inductors used to produce a variation in M is called a VARIOMETER.

PARALLEL INDUCTORS WITHOUT MAGNETIC COUPLING.—The total inductance, L_T, of inductors in parallel is calculated in the same manner that the total resistance of resistors in parallel is calculated, provided the coefficient of coupling between the coils is zero. For example,

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \cdots + \frac{1}{L_N},$$

where L_1, L_2, and L_3 are the respective inductances of inductors $L1$, $L2$, and $L3$; and L_N means that any number (N) of inductors may be used.

CAPACITANCE

Capacitance Defined

As has been defined previously, capacitance is the property of an electric circuit that opposes any CHANGE of voltage in the circuit. It differs from inductance, which opposes CHANGE in circuit current, and from resistance, which opposes the MOVEMENT of electrons in an electric circuit. Capacitance has also been defined as that property of a circuit in which energy may be stored in an electric field.

Capacitors, sometimes called condensers, are devices that possess the property of capacitance. In their simplest form, capacitors consist of two metal plates that are separated by an insulator called a DIELECTRIC. A capacitor stores free electrons when a voltage is impressed between the plates.

The action of a capacitor is illustrated in figure 8–7. In figure 8–7, A, the capacitor is uncharged. In figure 8–7, B, a battery is shown connected to a capacitor by means of a switch. The capacitor consists of two flat, metal plates separated by air, which is the dielectric in this capacitor. When the switch is in position 1,

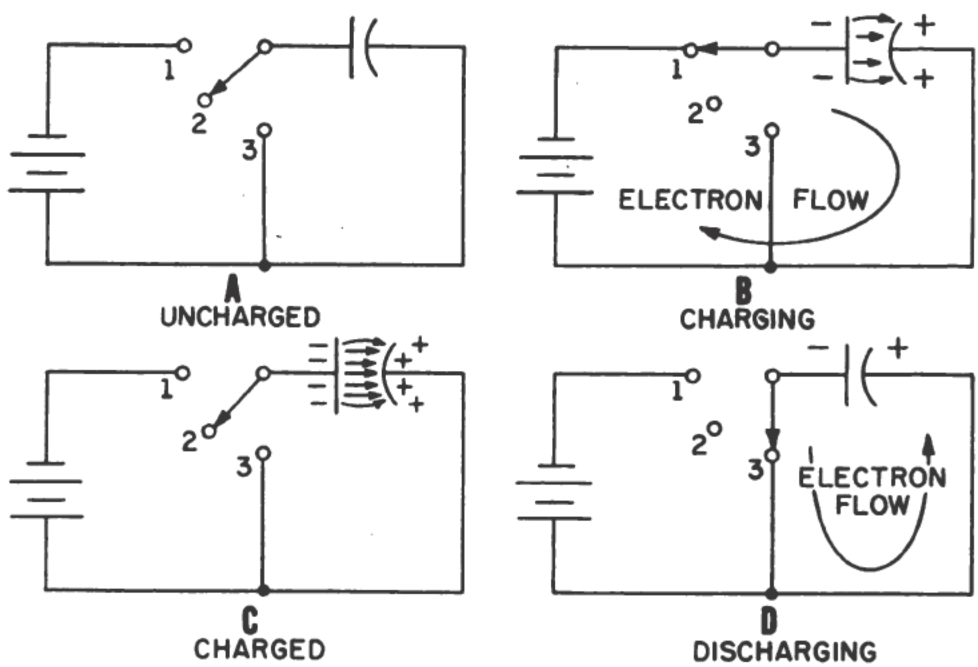


Figure 8–7.—Capacitor action.

electrons flow from the negative terminal of the battery into the left-hand plate of the capacitor. At the same time electrons are drawn from the right-hand plate into the positive terminal of the source, leaving the plate positively charged because of the deficiency of electrons there. This flow of current lasts for a very short time during which a voltage is developed between the plates of the capacitor. The voltage results from the surplus of free electrons on one plate and the deficiency of them on the other. The capacitor voltage opposes the battery voltage and is fre-

quently referred to as the counter emf of the capacitor. When the capacitor counter voltage reaches the same value as the battery voltage, all electron flow stops because the circuit then contains two equal and opposite potentials.

Two facts should be noted in connection with figure 8–7, B:

1. After a momentary surge of electrons, direct current is completely stopped by the capacitor.
2. Free electrons do not pass from one plate to the other through the dielectric because the dielectric is a nonconductor.

The electrons are distributed on the negative plate and held there by the mutual attraction of the corresponding positive charges on the positive plate. The presence of the two polarities (plus and minus) establishes the electric field through the dielectric.

The electric lines of force (flux) originate on the negatively charged plate and terminate on the positively charged plate. In other words, the lines are assumed to follow the same paths that UNIT NEGATIVE CHARGES would take if they were free to move. The lines of force would be present even if the plates were enclosed in a perfect vacuum. Electric lines of force cannot exist in a metallic conductor to any great extent but require the presence of nonconductors in order to establish a medium across which a relatively large potential may exist. Various materials differ in their ability to support electric flux or to serve as dielectric material for capacitors. This phenomenon is somewhat similar to permeability in magnetic circuits. Dielectric materials, or insulators, are rated in their ability to support electric flux in terms of a figure called the DIELECTRIC CONSTANT. The higher the value of dielectric constant (other factors being equal), the better is the dielectric material.

Dry air is the standard dielectric for purposes of reference and is assigned the value of unity (or one). The dielectric constant of a dielectric material is also defined as the ratio of the capacitance of a capacitor having that particular material as the dielectric to the capacitance of the same capacitor having air as the dielectric. By way of comparison, the dielectric constant of pure water is 81; flint glass, 9.9; and paraffin paper, 3.5. The range of dielectric constants is much more restricted than is the range

of permeabilities. Average dielectric constants for some common materials is given in the following list.

Material	*Dielectric-constant (Average values)*
Air	1
Polystyrene	2.5
Paraffin paper	3.5
Mica	6
Flint glass	9.9
Methyl alcohol	35
Glycerin	56.2
Pure water	81

The action represented in figure 8–7, B, by which the capacitor opposes the voltage of the source may be compared to the tension of a coiled spring. The battery corresponds to the tension force acting on the spring; the momentary surge of current in the circuit corresponds to the motion of the spring in response to the force; and the electric field resulting from the presence of the accumulated electrons corresponds to the energy stored in the spring under tension.

If the same spring could be clamped in its elongated position and then detached from the tension force it would be held in a state of tension compression. This condition would then be comparable to that of the capacitor shown in figure 8–7, C.

If the voltage applied to the capacitor is removed, the capacitor voltage remains, indicating that the capacitor is charged and energy is stored in the electric field accompanying the charge. The electric stress in the dielectric corresponds to the strained condition of the spring. A good capacitor, such as that used in Navy electronic equipment, will store its charge indefinitely and remains potentially dangerous until it is properly discharged.

The discharge of the capacitor is shown in figure 8–7, D. The switch, when moved to position 3, provides a conducting path between the plates. During discharge, the excess electrons quickly leave the negative plate and flow through the switch around the circuit to the positive plate until equilibrium is reached—that is, until the voltage across the plates falls to zero. The direction of electron flow on discharge is opposite to the direction of flow during charge. This corresponds to the release of tension in the coiled spring. Thus, when the restraining clamp is removed, the spring shortens to its normal position of

no strain and energy is given up as it moves in a direction opposite to that in which it received the energy.

In summary, the capacitor's basic actions are: (1) charging, during which a current flows into the plates and electrons are stored on one of the surfaces, resulting in the appearance of a voltage between the plates; (2) storage of energy in the form of an electric field in the dielectric; and (3) release of energy by discharge of electrons from one plate to another. The capacitor blocks the passage of direct current, but when alternating potentials are applied, the three actions are rapidly repeated, giving the effect of an alternating current first in one direction and then in the other.

Unit of Capacitance

The capacitance of a capacitor is proportional to the quantity of charge that can be stored in it for each volt applied across its plates. A capacitor possesses a capacitance of 1 FARAD when a quantity of charge of 1 coulomb imparted to it raises its potential 1 volt (1 coulomb is equal to 6.28×10^{18} electrons). The relation between capacitance, charge, and voltage is the basic formula of the capacitor, and may be stated as follows:

$$C = \frac{Q}{E},$$

where C is the capacitance in farads, Q the quantity of charge in coulombs, and E the difference in potential in volts.

A capacitor with a value of 1 farad would be of enormous dimensions. Hence, use of the farad as a unit is usually limited to definition and calculations. The practical units of capacitance are the microfarad (μf) and the micromicrofarad ($\mu\mu f$). A capacitor whose value is 1 microfarad is one in which a microcoulomb (6.28×10^{12} electrons) is stored when 1 volt is applied across its plates.

Factors Affecting the Value of Capacitance

The capacitance of a capacitor depends on the three following factors:

1. The area of the plates.
2. The distance between the plates.
3. The dielectric constant of the material between the plates.

These three factors are related to the capacitance of a parallel-plate capacitor consisting of two plates by the formula

$$C=0.2249\left(\frac{kA}{d}\right)$$

where C is in micromicrofarads, A is the area of one of the plates in square inches, d is the distance between the plates in inches, and k is the dielectric constant of the insulator separating the plates.

For example, the capacitance of a parallel-plate capacitor with an air dielectric and a spacing of 0.0394 inch between the plates, each of which has an area of 15.5 square inches, is approximately

$$C=0.225\left(\frac{1\times 15.5}{0.0394}\right)=88.5 \text{ micromicrofarads.}$$

From this formula it may be seen that the capacitance increases when the plates are increased in area; it decreases if the spacing of the plates is increased; and it increases if the k-value is increased.

The dielectric constant, k, expresses the relative capacitance when materials other than air are used as the insulating material between the plates. For example, if mica is substituted for air

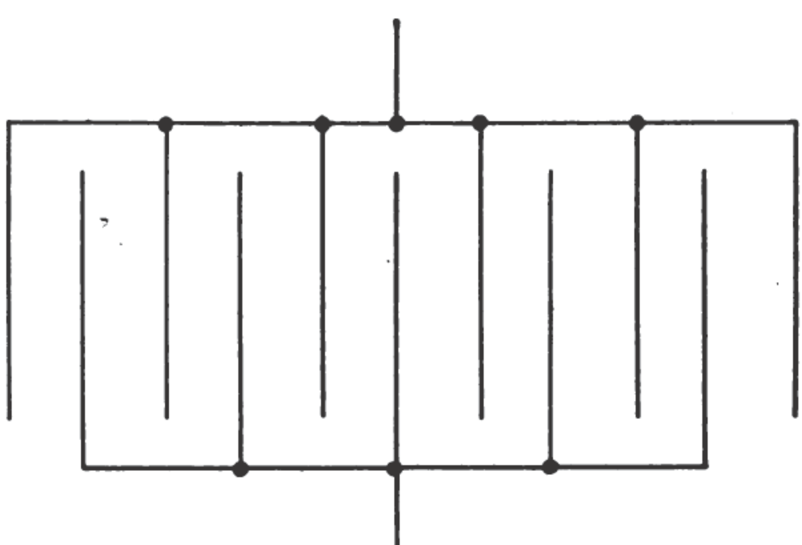


Figure 8–8.—Multiple plate construction of a capacitor.

as the dielectric the capacitance increases 6 times because the dielectric constant of mica is 6 and that of air is 1.

If the capacitor is composed of more than two parallel plates, the capacitance is calculated by multiplying the preceding formula by $N-1$, where N is the number of plates. The plates are interlaced as shown in figure 8–8, and the effect is that of increasing the capacitance of the two-plate capacitor by the factor $N-1$. In the figure there are 11 plates, and the capacitance is 10 times that of a 2-plate capacitor of the same plate area, spacing, and dielectric material.

The dependence of the current flowing in a capacitive circuit on the rate of change of voltage may be shown by means of a circuit such as that in figure 8–9. A capacitor and central-zero galvanometer are connected in series between the movable arm,

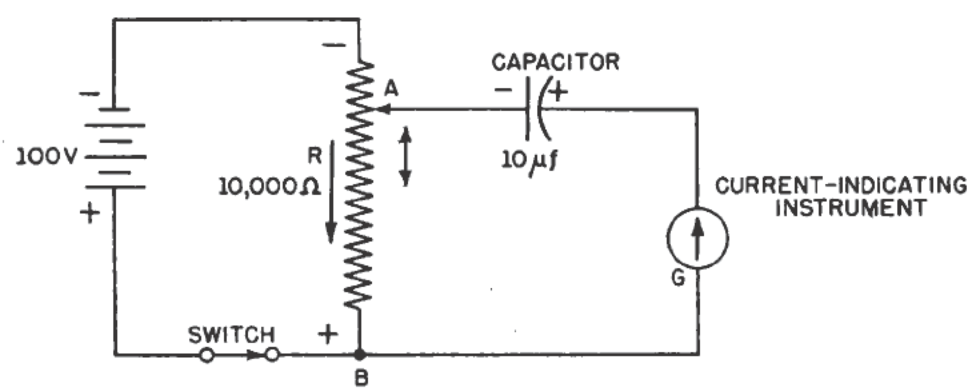


Figure 8–9.—Dependence of current on rate of change of voltage in a capacitive circuit.

A, and one side of the potentiometer, which is supplied by a 100-volt battery. When the switch is closed, current flows through potentiometer R in the direction indicated, and the capacitor charges to the potential existing between points A and B. The galvanometer deflection is zero when the voltage to which the capacitor charges is equal in magnitude to the drop across AB. If the slider is moved rapidly, the rate of change of voltage is relatively large and the galvanometer deflection indicates a relatively large transient current. If the slider is moved slowly, the rate of change of capacitor voltage is low and the galvanometer deflection is reduced accordingly.

Charge and Discharge of an *R-C* Series Circuit

Ohm's law states that the voltage across a resistance is equal to the current through it times the value of the resistance. This means that a voltage will be developed across a resistance ONLY WHEN CURRENT FLOWS through it.

A capacitor is capable of storing or holding a charge of electrons. When uncharged, both plates contain the same number of free electrons. When charged, one plate contains more free electrons than the other. The difference in the number of electrons is a measure of the charge on the capacitor. The accumulation of this charge builds up a voltage across the terminals of the capacitor, and the charge continues to increase until this voltage equals the applied voltage. The charge in a capacitor is related to the capacitance and voltage as follows:

$$Q=CE,$$

in which Q is the charge in coulombs, C the capacitance in farads, and E the difference in potential in volts. Thus, the greater the voltage, the greater the charge on the capacitor. Unless a discharge path is provided, a capacitor keeps its charge indefinitely. Any practical capacitor, however, has some leakage through the dielectric so that the charge will gradually leak off.

A voltage divider containing resistance and capacitance may be connected in a circuit by means of a switch, as shown in figure 8–10, A. Such a series arrangement is called an *R-C* series circuit.

If $S1$ is closed, electrons flow counterclockwise around the circuit containing the battery, capacitor, and resistor. This flow of electrons ceases when C is charged to the battery voltage. At the instant current begins to flow, there is no voltage on the capacitor and the drop across R is equal to the battery voltage. The initial charging current, I, is therefore equal to $\frac{E_s}{R}$. Figure 8–10, B, shows that at the instant the switch is closed, the entire input voltage, E_s, appears across R, and that the voltage across C is zero.

The current flowing in the circuit soon charges the capacitor. Because the voltage on the capacitor is proportional to its charge, a voltage, e_c, will appear across the capacitor. This voltage opposes the battery voltage—that is, these two voltages buck each

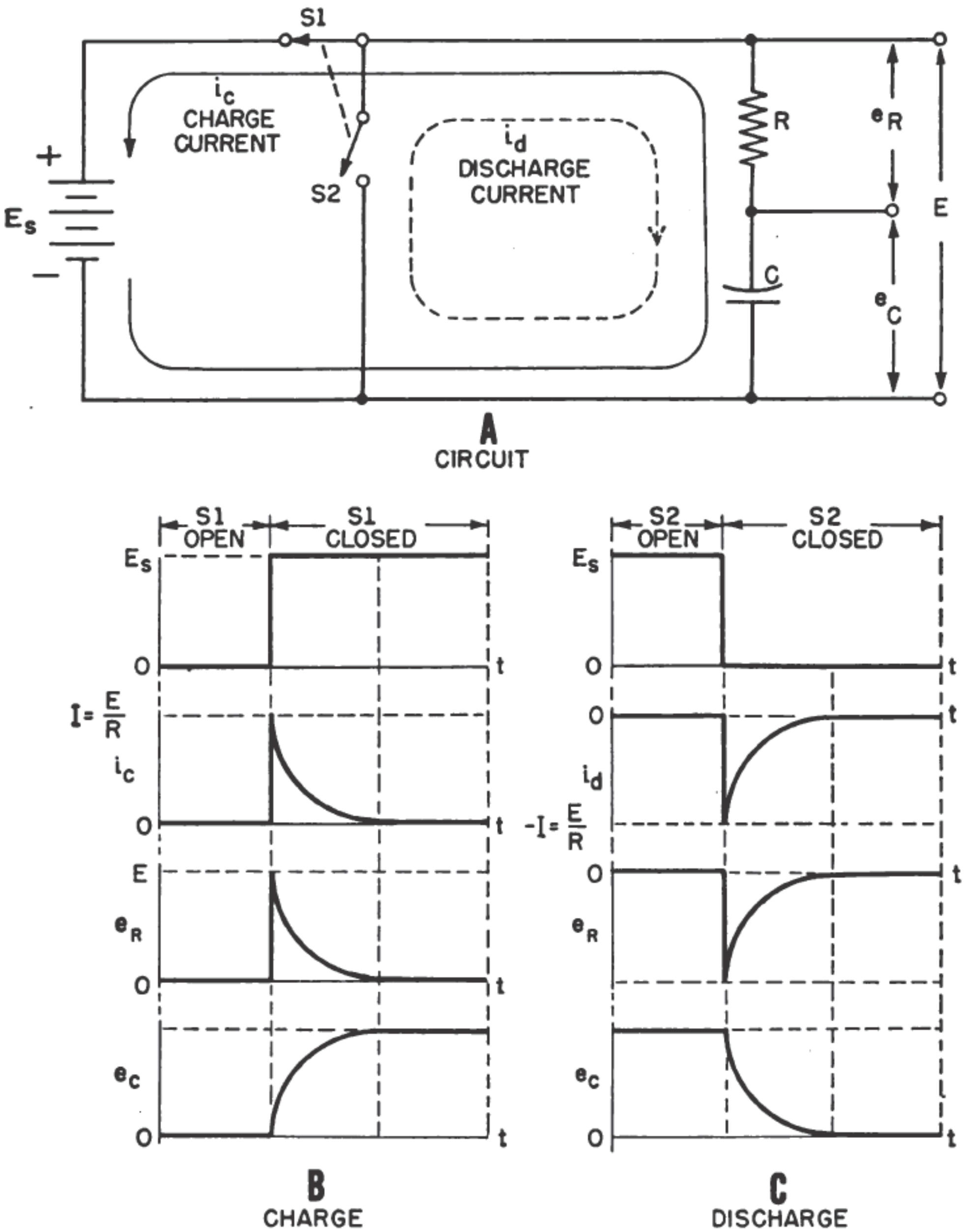


Figure 8–10.—Charge and discharge of an *R-C* series circuit.

other. As a result, the voltage e_r across the resistor is $E_s - e_c$, and this is equal to the voltage drop (i_cR) across the resistor. Because E_s is fixed, i_c decreases as e_c increases.

The charging process continues until the capacitor is fully charged and the voltage across it is equal to the battery voltage. At this instant, the voltage across R is zero and no current flows through it. Figure 8–10, B, shows the division of the battery

voltage, E_s, between the resistance and capacitance at all times during the charging process.

If $S2$ is closed ($S1$ opened) in figure 8–10, A, a discharge current, i_d, will discharge the capacitor. Because i_d is opposite in direction to i_c, the voltage across the resistor will have a polarity opposite to the polarity during the charging time. However, this voltage will have the same magnitude and will vary in the same manner. During discharge the voltage across the capacitor is equal and opposite to the drop across the resistor, as shown in figure 8–10, C. The voltage drops rapidly from its initial value and then approaches zero slowly, as indicated in the figure.

RC Time Constant

The time required to charge a capacitor to 63 percent (actually, 63.2 percent) of maximum voltage or to discharge it to 37 percent (actually, 36.8 percent) of its final voltage is known as the TIME CONSTANT of the circuit. An *R-C* circuit with its charge and discharge graphs are shown in figure 8–11. The value of the time constant in seconds is equal to the product of the circuit resistance in ohms and its capacitance in farads, one set of values of which are given in figure 8–11, A. *RC* is the symbol used for this time constant.

Some useful relations used in calculating *RC* time constants are as follows:

$$R \text{ (ohms)} \times C \text{ (farads)} = t \text{ (seconds)}.$$
$$R \text{ (megohms)} \times C \text{ (microfarads)} = t \text{ (seconds)}.$$
$$R \text{ (ohms)} \times C \text{ (microfarads)} = t \text{ (microseconds)}.$$
$$R \text{ (megohms)} \times C \text{ (micromicrofarads)} = t \text{ (microseconds)}.$$

The time constant may also be defined as the time required to charge or discharge a capacitor completely IF it continues to charge or discharge at its initial rate. As may be seen in figure 8–11, B, the slope of the dotted tangent line, *OX*, indicates the initial rate of charge. At this rate, the capacitor would be completely charged in *RC* seconds. Likewise, the slope of the dotted tangent line, *YZ*, indicates the initial rate of discharge with respect to time, and at this rate the capacitor would be completely discharged in *RC* seconds.

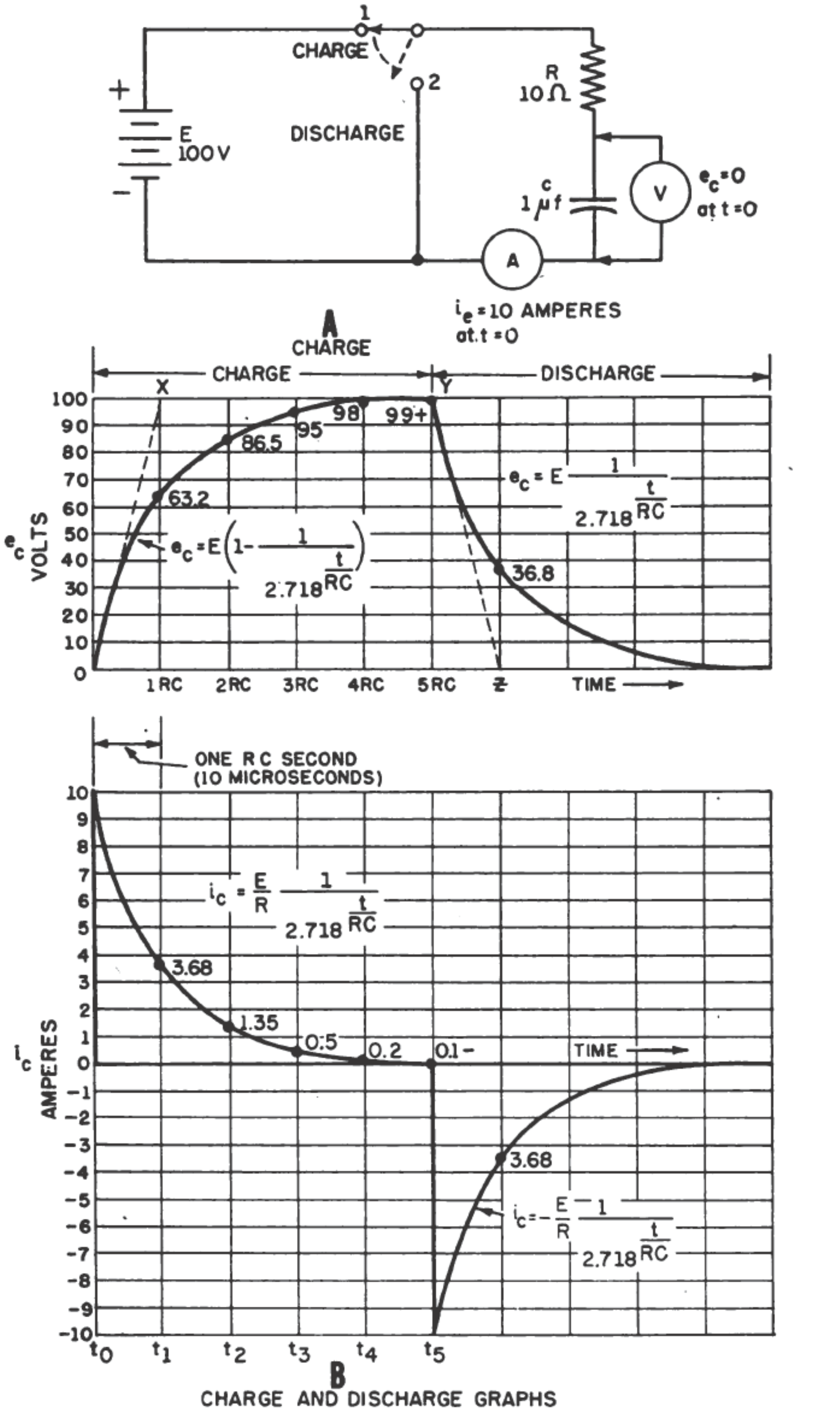


Figure 8–11.—*RC* time constant.

The equation for the rise in voltage, e_c, across the capacitor is

$$e_c = E\left(1 - \frac{1}{2.718^{\frac{t}{RC}}}\right),$$

where e_c is the instantaneous voltage across the capacitor, E the applied voltage (100 volts in this case), t the time in seconds, R the resistance in ohms, C the capacitance in farads (0.000001 farad or 1 μf in this case), and the number 2.718 the natural logarithm base. Figure 8–11, B, shows a graph of this equation.

When $t=RC$, the exponent, $\frac{t}{RC}$, reduces to 1. Therefore,

$$e_c = E\left(1 - \frac{1}{2.718}\right) = 0.632E.$$

In other words, when $t=RC$, e_c is equal to 63.2 percent of E, the maximum value. When the maximum is 100 volts, as shown, the voltage across the capacitor increases to 63.2 volts in RC seconds—that is, in 10 microseconds.

The equation for the charging current, i_c, is

$$i_c = \frac{E}{R}\left(\frac{1}{2.718^{\frac{t}{RC}}}\right).$$

The graph of this equation is shown in figure 8–11, B. When $t=RC$, $i_c = 0.368 \times \frac{E}{R}$; that is when $t=10$ microseconds, $i_c = 0.368 \times \frac{100}{10} = 3.68$ amperes.

Universal Time Constant Chart

Because the impressed voltage and the values of R and C or R and L usually will be known, a universal time constant chart (fig. 8–12) can be used. Curve A is a graph of the voltage across the capacitor on charge; it is also a graph of the inductor current and the voltage across the resistor in series with the inductor on the growth of current. Curve B is a graph of capacitor voltage on discharge, capacitor current on charge, inductor curent on decay, or the voltage across the resistor in series with the capacitor on

charge. The graphs of resistor voltage and current and inductor voltage on discharge are not shown because negative values would be involved.

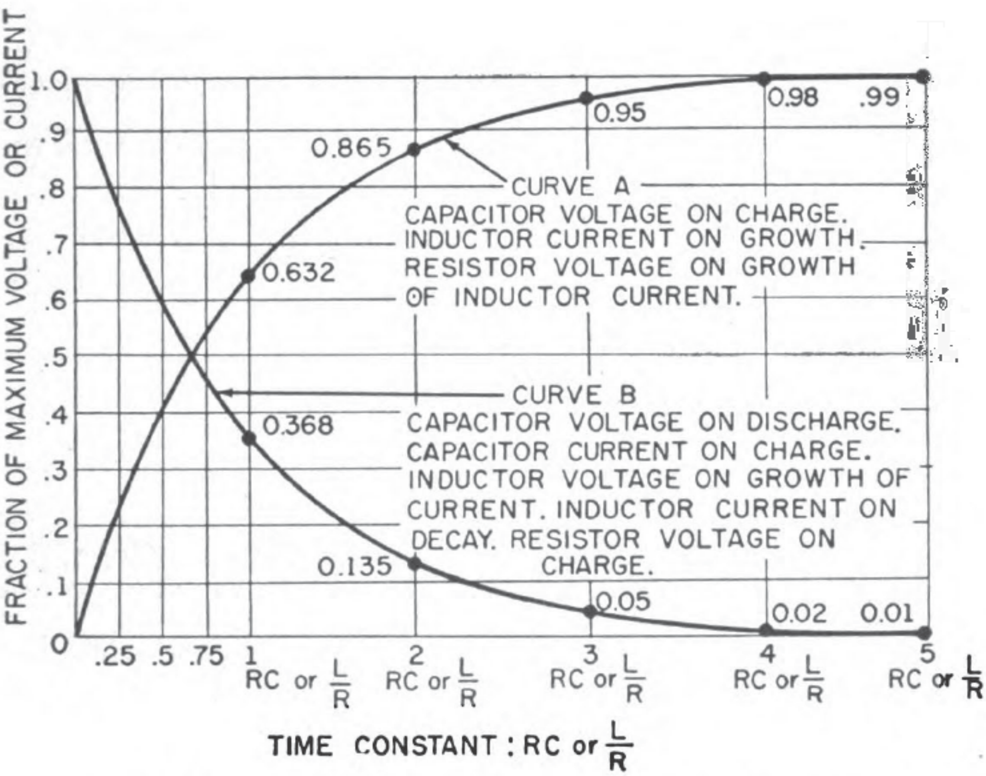


Figure 8–12.—Universal time constant chart for *R-C* and *R-L* circuits.

The time scale (horizontal scale) is graduated in terms of the RC or $\frac{L}{R}$ time constants so that the curves may be used for any value of R and C or L and R. The voltage and current scales (vertical scales) are graduated in terms of the fraction of the maximum voltage or current so that the curves may be used for any value of voltage or current. If the time constant and the initial or final voltage for the circuit in question are known, the voltages across the various parts of the circuit can be obtained from the curves for any time after the switch is closed, either on charge or discharge. The same reasoning is true of the current in the circuit.

The following problem illustrates how the universal time constant chart may be used.

A circuit is to be designed in which a capacitor must charge to one-fifth (0.2) of the maximum charging voltage in 100 micro-

seconds (0.0001 second). Because of other considerations, the resistor must have a value of 20,000 ohms. What size of capacitor is needed?

Curve *A* is first consulted to determine the *RC* time necessary to give 0.2 of the full voltage. The time is less than 0.25 *RC*, approximately 0.22 *RC*. If 0.22 *RC* must be equal to 100 microseconds, one complete *RC* must be equal to $\frac{100}{0.22}=455$ microseconds, or 0.000455 second. Therefore,

$$RC=0.000455.$$

Substituting the known value of *R* and solving for *C*,

$$C=\frac{0.000455}{20{,}000}=0.000000023 \text{ farad},$$

or 0.023 microfarad.

The graphs shown in figure 8–11 are not entirely complete—that is, the charge or discharge (or the growth or decay) is not quite completed in 5 *RC* or $5\frac{L}{R}$ seconds. However, when the values reach 0.99 of the maximum (corresponding to 5 *RC* or $5\frac{L}{R}$) the graphs may be considered accurate enough for most purposes.

Capacitors in Parallel and in Series

Capacitors may be combined in parallel or series to give equivalent values, which may be either the sum of the individual values (in parallel) or else a value less than that of the smallest capacitance (in series). Figure 8–13 shows the parallel and series connections.

In figure 8–13, A, the voltage, *E*, is the same for all the capacitors and the total charge, Q_t, is the sum of all the individual charges, Q_1, Q_2, and Q_3. From the equation of the capacitor,

$$C=\frac{Q}{E}.$$

The total charge is

$$Q_t=C_tE,$$

where C_t is the total capacitance. In parallel, the total charge is

$$Q_t = Q_1 + Q_2 + Q_3$$

$$C_tE = C_1E + C_2E + C_3E.$$

Dividing both sides of the equation by E gives

$$C_t = C_1 + C_2 + C_3.$$

The total capacitance of any number of capacitors connected in parallel is the sum of their individual values.

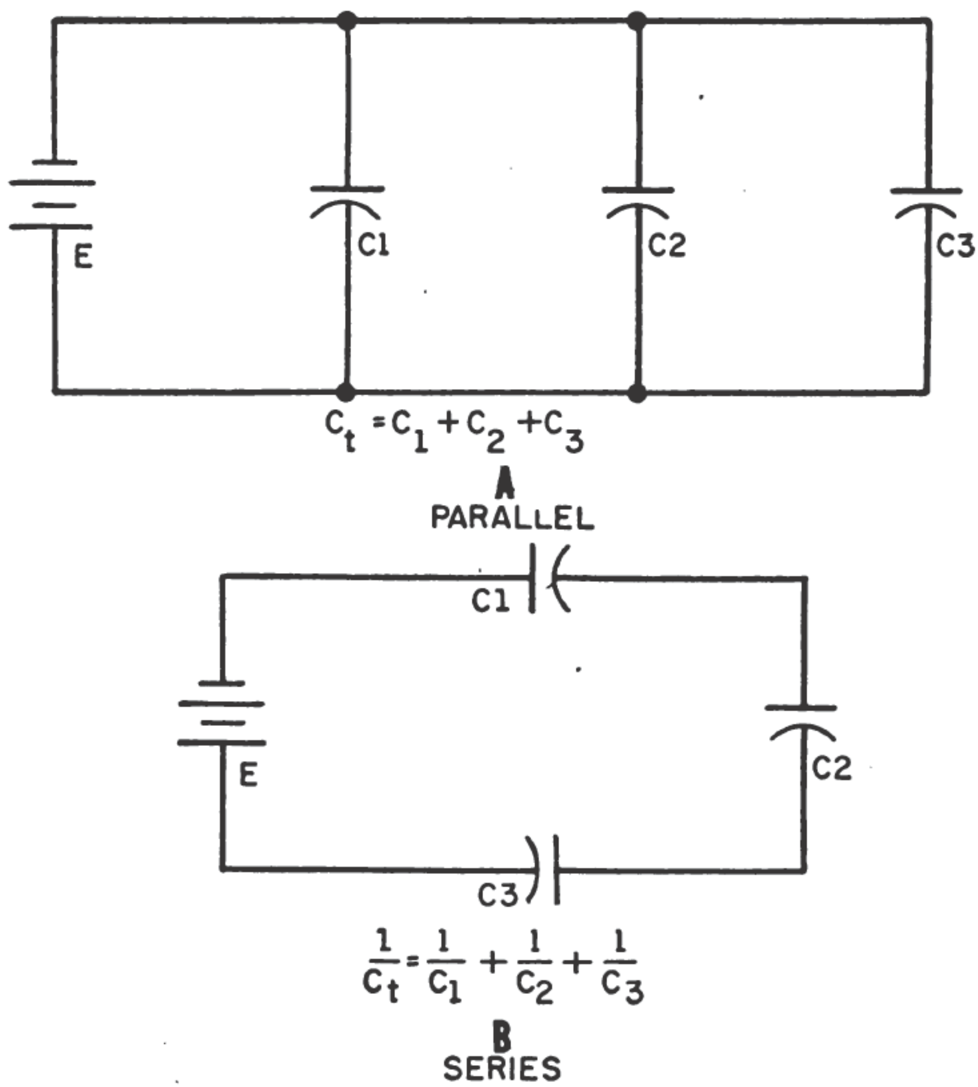


Figure 8–13.—Capacitors in parallel and in series.

In the series arrangement (fig. 8–13, B), the current is the same in all parts of the circuit. Each capacitor develops a voltage during charge, and the sum of the voltages of all the capacitors must equal the applied voltage, E. By the capacitor equation, the applied voltage, E, is equal to the total charge divided by the total capacitance, or

$$E=\frac{Q_t}{C_t}.$$

The total charge, C_t, is equal to the charge on any one of the capacitors because the same current flows in all for the same length of time, and because charge equals current times time in seconds ($Q=It$). Therefore,

$$Q_t=Q_1=Q_2=Q_3,$$

but

$$E=E_1+E_2+E_3,$$

where E_1, E_2, and E_3 are the voltages of the three capacitors. Then

$$\frac{Q_t}{C_t}=\frac{Q_t}{C_1}+\frac{Q_t}{C_2}+\frac{Q_t}{C_3}.$$

Dividing both sides of the equation by Q_t gives

$$\frac{1}{C_t}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}.$$

The reciprocal of the total capacitance of any number of capacitors in series is equal to the sum of the reciprocals of the individual values.

Parallel capacitors combine by a rule similar to that for combining resistors in series. Series capacitors combine by a rule similar to that for combining parallel resistors.

In the series arrangement of two capacitors, C_1 and C_2, the total capacitance is given by the equation:

$$C_t=\frac{C_1C_2}{C_1+C_2}.$$

Voltage Rating of Capacitors

In selecting or substituting a capacitor for use in a particular circuit, one must consider (1) the value of capacitance desired,

and (2) the amount of voltage to which the capacitor is to be subjected. If the voltage applied across the plates is too great, the dielectric will break down and arcing will occur between the plates. The capacitor is then short-circuited and the possible flow of direct current through it can cause damage to other parts of the equipment. Capacitors have a voltage rating that should not be exceeded.

The working voltage of the capacitor is the maximum voltage that can be steadily applied without danger of arc-over. The working voltage depends on (1) the type of material used as the dielectric, and (2) the thickness of the dielectric.

The voltage rating of the capacitor is a factor in determining the capacitance because capacitance decreases as the thickness of the dielectric increases. A high-voltage capacitor that has a thick dielectric must have a larger plate area in order to have the same capacitance as a similar low-voltage capacitor having a thin dielectric. The voltage rating also depends on frequency because the losses, and the resultant heating effect, increases as the frequency increases.

A capacitor that may be safely charged to 500 volts, direct voltage, cannot be safely subject to alternating or pulsating direct voltages whose effective values are 500 volts. An alternating voltage of 500 volts (rms) has a peak voltage of 707 volts, and a capacitor to which it is applied should have a working voltage of at least 750 volts. The capacitor should be selected so that its working voltage is at least 50 percent greater than the highest voltage to be applied to it. Effective (rms) voltage and the action of capacitors in a-c circuits are described in chapter 12.

Capacitor Types

Capacitors may be divided into two groups—fixed and variable. The fixed capacitors, which have approximately constant capacitance, may then be further divided, according to the type of dielectric used, into the following classes: Paper, oil, mica, and electrolytic capacitors. Ceramic capacitors are also used in electronic circuits.

When connecting electrolytic capacitors in a circuit the proper polarity MUST be observed. Paper capacitors may have one terminal marked *ground,* which means that this terminal connects to the outside foil. Polarity does not ordinarily have to be observed in connecting paper, oil, mica, and ceramic capacitors.

PAPER CAPACITORS.—The plates of paper capacitors are strips of metal foil separated by waxed paper. The capacitance of paper capacitors ranges from about 200 micromicrofarads to several microfarads. The strips of foil and paper are rolled together to form a cylindrical cartridge, which is then sealed in wax to keep out moisture and to prevent corrosion and leakage. Two metal leads are soldered to the plates, one extending from each end of the cylinder. The assembly is enclosed either in a cardboard cover or else in a hard, molded plastic covering. The cardboard type is shown in figure 8–14.

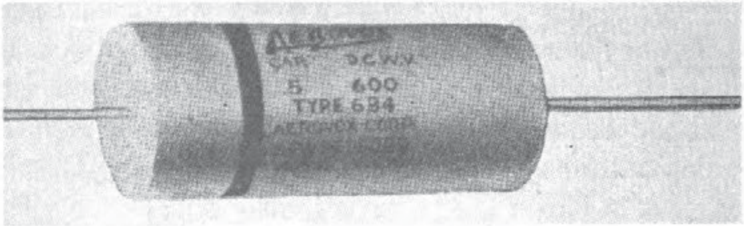


Figure 8–14.—Paper capacitor.

Bathtub-type capacitors consist of paper-capacitor cartridges hermetically sealed in metal containers. The container often serves as a common terminal for several enclosed capacitors; but when not a terminal, the cover serves as a shield against electrical interference.

OIL CAPACITORS.—In radio and radar transmitters, voltages high enough to cause arcing, or breakdown, of paper dielectrics are often employed. Consequently, in these applications capacitors that have oil, or else oil-impregnated paper, as the dielectric material are preferred. Capacitors of this type are considerably more expensive than ordinary paper capacitors and their use is generally restricted to radio and radar transmitting equipment.

MICA CAPACITORS.—The fixed mica capacitor is made of metal foil plates that are separated by sheets of mica, which form the dielectric. The whole assembly is covered in molded plastic which keeps out moisture. Mica is an excellent dielectric and will withstand higher voltages than paper without allowing arcing between the plates. Common values of mica capacitors range from approximately 50 micromicrofarads to about 0.02 microfarad. Mica capacitors are shown in figure 8–15.

ELECTROLYTIC CAPACITORS.—For capacitances greater than a few microfarads, the plate areas of paper or mica capacitors must

become very large, and electrolytic capacitors are usually employed instead. These units provide large capacitance in small physical sizes. Their values range from 1 to about 1,500 microfarads. Unlike the other types, electrolytic capacitors are generally polarized, and should be subjected to direct voltage, or pulsating direct voltage only; however, a special type of electrolytic capacitor is made for use in alternating-current motors.

The electrolytic capacitor is widely used in electronic circuits,

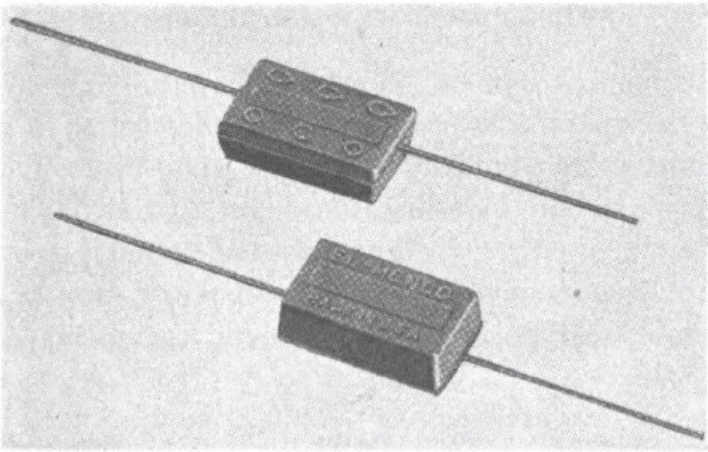


Figure 8–15.—Mica capacitors.

and consists of two metal plates separated by an electrolyte. The electrolyte in contact with the negative terminal, either in paste or liquid form, comprises the negative electrode. The dielectric is an exceedingly thin film of oxide deposited on the positive electrode of the capacitor. The positive electrode is an aluminum sheet, and is folded so as to achieve maximum area. The capacitor is subjected to a forming process during manufacture, in which current is passed through it. The flow of current results in the deposit of the thin coating of oxide on the aluminum plate. The close spacing of the negative and positive electrodes gives rise to the comparatively high capacitance value, but allows greater possibility of voltage breakdown and leakage of electrons from one electrode to the other.

Two kinds of electrolytic capacitors are in use—wet-electrolytic and dry-electrolytic capacitors. In the former, the electrolyte is a liquid and the container must be leakproof. This type should always be mounted in a vertical position.

The electrolyte of the dry-electrolytic unit is a paste contained

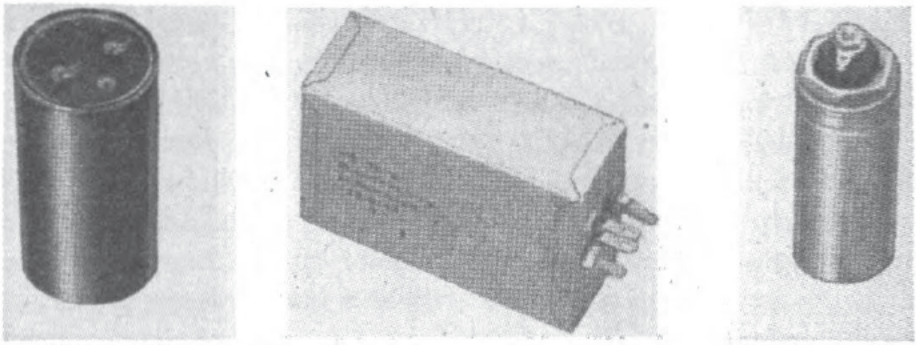


Figure 8–16.—Electrolytic capacitors.

in a separator made of an absorbent material such as gauze or paper. The separator serves to hold the electrolyte in place, and also prevents short-circuiting the plates. Dry-electrolytic capacitors are made in both cylindrical and rectangular-block form and may be contained either within cardboard or metal covers. Since the electrolyte cannot spill, the dry capacitor may be mounted in any convenient position. Electrolytic capacitors are shown in figure 8–16.

QUIZ

1. Define inductance.
2. Define capacitance.
3. If the conductor and magnet in figure 8–1, A, move in the same direction and at the same rate, what is the relative magnitude of the induced emf?
4. Describe the left-hand rule for determining the direction of the induced emf in a conductor as it moves through a magnetic field.
5. An inductance of 0.05 henry is equivalent to how many (a) millihenries? (b) microhenries?
6. State Lenz's law.
7. What inherent electrical property opposes a steady current in a circuit?
8. What is the inductance in millihenries of a coil that has a length of 200 cm, a diameter of 5 cm, a core permeability of 1, and 2,400 turns of wire?
9. In figure 8–4, A:
   (a) At the instant $S1$ is closed, what is the relative magnitude of e_L?
   (b) At the instant $S1$ is closed, what is the relative magnitude of e_r?
   (c) If L is 10 henries and R is 5 ohms, how many seconds after $S1$ is closed will it take for the current to rise to 63.2 percent of its final value?

10. Two coils possess the property of mutual inductance when the ____________ of one coil links the ____________ of the other coil.

11. What is the coefficient of coupling between two coils if 30 percent of the flux produced by the first coil links the second coil?

12. Express by formula the mutual inductance between two coils in terms of the inductance of each coil and the coefficient of coupling between the coils.

13. What is the total inductance in henries of a series circuit containing three 10-henry inductors each of which is well shielded from the other two?

14. What is the inductance in henries of a parallel circuit containing three 15-henry inductors each of which is well shielded from the other two?

15. What is the capacitance of a capacitor, the voltage of which is increased from 0 to 50 volts when the capacitor receives a charge of 0.1 coulomb?

16. (a) A capacitance of 0.00002 farad is equivalent to how many microfarads?

    (b) A capacitance of 0.002 microfarad is equivalent to how many micromicrofarads?

17. Upon what three factors does the capacitance of a parallel-plate capacitor depend?

18. What is the capacitance in micromicrofarads of a parallel-plate capacitor that has a plate area of 7.42 square inches, mica dielectric (dielectric constant=6), and a spacing between the plates of 0.01 inch?

19. In figure 8–10, A:

    (a) If C is 1 microfarad and R is 2 megohms, how many seconds after $S1$ is closed will it take for the voltage across C to rise to 63.2 percent of its maximum value?

    (b) At the instant $S1$ is closed, what is the relative value of e_c?

    (c) At the instant $S1$ is closed, what is the relative value of e_r?

    (d) If R is 0.5 megohm, what value of C (in micromicrofarads) will permit e_c to increase from zero volts to approximately full value in 1,000 microseconds?

20. What is the total capacitance when a 100-micromicrofarad capacitor is connected in series with a 200-micromicrofarad capacitor?

21. What is the total capacitance when a 100-micromicrofarad capacitor is connected in parallel with a 200-micromicrofarad capacitor?

22. The working voltage of a capacitor should be ________ percent greater than the highest voltage to be applied to it.

23. The working voltage of a capacitor depends on the ______________ and ____________ of the dielectric.
24. Name four types of fixed capacitors according to the type of dielectric used.
25. What feature of an electrolytic capacitor permits the relatively large capacitance rating?

CHAPTER 9

BASIC ELECTRICAL INDICATING INSTRUMENTS

INTRODUCTION

In the field of electricity, as in all the other physical sciences, accurate quantitative measurements are essential. This involves two important items—numbers and units. Simple arithmetic is used in most cases, and the units are well-defined and easily understood. The standard units of current, voltage, and resistance as well as other units are defined by the National Bureau of Standards. At the factory, various instruments are calibrated by comparing them with established standards.

The technician commonly works with ammeters, voltmeters, ohmmeters, and electron-tube analyzers; but he may also have many occasions to use wattmeters, watt-hour meters, power-factor meters, synchroscopes, frequency meters, and capacitance-resistance-inductance bridges.

Electrical equipments are designed to operate at certain efficiency levels. To aid the technician in maintaining the equipment, technical instruction books and sheets containing optimum preformance data, such as voltages and resistances, are prepared for each Navy equipment.

One of the duties of a technician is to keep the equipments pertaining to his rating specialty operating at peak performance levels in order that they may be utilized most efficiently. To this end the Navy supplies good test equipment, which, when properly understood and used, greatly simplifies maintenance. The importance to the technician of a good understanding of the functional design and operation of electrical measuring instruments cannot be overemphasized. In electrical service work one or more of the following methods are commonly used to determine if the circuits of an equipment are operating properly.

1. Use an ammeter to measure the amount of current flowing in the circuit.

2. Use a voltmeter to determine the voltage existing between two points in a circuit.
3. Use an ohmmeter or megger (megohmmeter) to measure circuit continuity and total or partial circuit resistance.

The technician may also find it necessary to employ a wattmeter to determine the total POWER being consumed by certain equipments. If he wishes to measure the ENERGY consumed by certain equipments or certain circuits, a watt-hour or kilowatt-hour meter is used.

For measuring other quantities such as power factor and frequency, which will be treated in later chapters, the technician employs the appropriate instruments. In each case the instrument indicates the value of the quantity measured, and the technician interprets the information in a manner that will help him understand the way the circuit is operating. Occasionally the technician will need to determine the value of a capacitor or an inductor. Inductance or capacitance bridges may be employed for this purpose.

Although in this chapter the discussion is confined largely to d-c instruments, some of the instruments discussed may be used with alternating as well as direct current. Alternating-current instruments are treated in chapter 16.

A thorough understanding of the construction, operation, and limitations of the basic types of electrical measuring instruments, coupled with the theory of circuit operation, is most essential in servicing and maintaining electrical equipment.

D'ARSONVAL METER

Construction

The stationary-magnet moving-coil meter is the basic movement used in most measuring instruments for servicing electrical, especially d-c, equipment. This type of movement is commonly called the D'Arsonval movement because it was first employed by the Frenchman D'Arsonval in making electrical measurements.

The basic D'Arsonval movement consists of a stationary permanent magnet and a movable coil with attached mirror or pointer. The use of a pointer permits over-all simplicity in that the use of a light source and a system of mirrors is avoided. However, the use of a pointer introduces the problem of balance, especially if the

pointer is long. The use of mirrors and a beam of light simplifies the problem of coil balance and gives approximately the same effective deflection that could be achieved by the use of a pointer about 3 feet long.

A simplified diagram of one type of permanent-magnet moving-coil instrument employing a mirror is shown in figure 9–1. Such an instrument is commonly called a GALVANOMETER. The galvanometer indicates very small amounts (or the relative amounts) of current or voltage, and is distinguished from other instruments used for the same purpose in that the movable coil is suspended by means of metal ribbon instead of by means of a shaft and jewel bearings.

The movable coil of the galvanometer in figure 9–1 is suspended between the poles of the magnet by means of thin flat

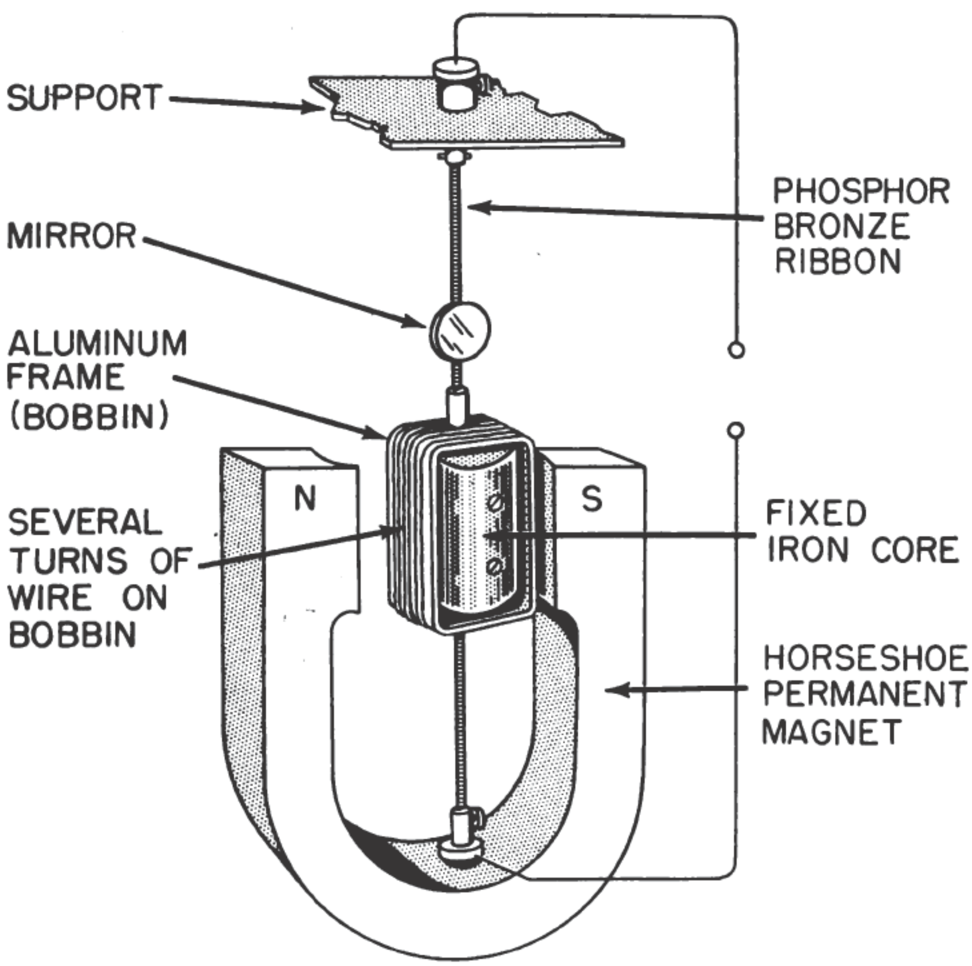


Figure 9–1.—Simplified diagram of a galvanometer.

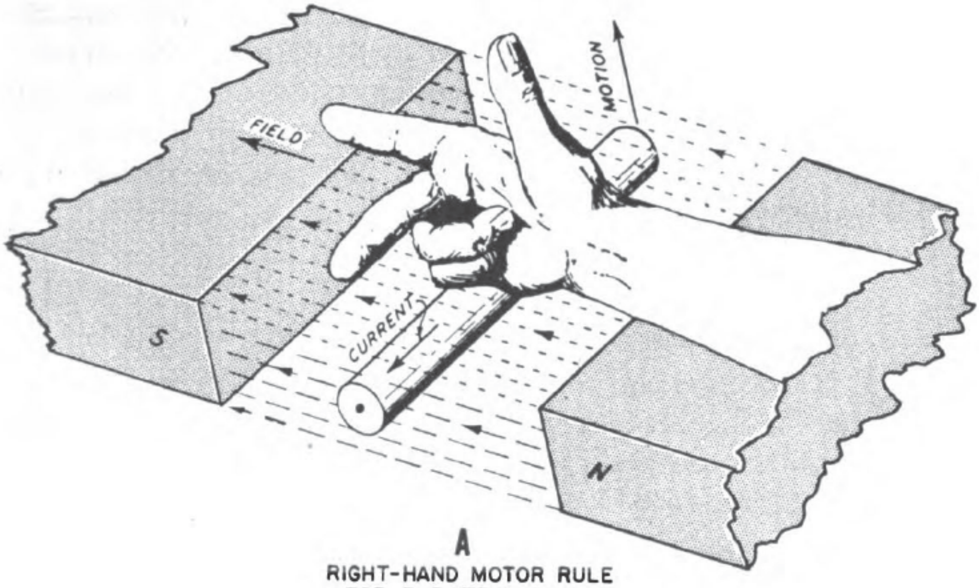
ribbons of phosphor bronze. These ribbons provide the conducting path for the current between the circuit under test and the movable coil. They also provide the restoring force for the coil. The restoring force, exerted by the twist in the ribbons, is the force against which the driving force of the coil current (to be described later) is balanced in order to obtain a measurement of the current strength. The ribbons thus tend to oppose the motion of the coil, and will twist through an angle that is proportional to the force applied to the coil by the action of the coil current in the magnetic field. The ribbons thus restrain or provide a counter force, for the magnetic force acting on the coil. When the driving force of the coil current is removed, the restoring force returns the coil to its zero position.

If a beam of light and mirrors are used, the beam of light is swept to the right or left across a central-zero translucent screen (scale) having uniform divisions. If a pointer is used, the pointer is moved in a horizontal plane to the right or left across a central-zero scale having uniform divisions. The direction in which the beam of light or the pointer moves depends on the direction of current through the coil.

This instrument is used to measure minute currents as, for example, in bridge circuits. In modified form, the basic D'Arsonval movement has the highest sensitivity of any of the various types of meters in use today.

Operating Principle

In order to understand the operating principle of the D'Arsonval meter it is first necessary to consider the force acting on a current-carrying conductor placed in a magnetic field. The magnitude of the force is proportional to the product of the magnitudes of the current and the field strength. The field is established between the poles of a U-shaped permanent magnet and is concentrated through the conductor by means of a soft-iron stationary member mounted between the poles to complete the magnetic circuit. The conductor is made movable by shaping it in the form of a closed loop and mounting it between fixed pivots so that it is free to swing about the fixed iron member between the poles of the magnets. A convenient method of determining the direction of motion of the conductor is by the use of the RIGHT-HAND MOTOR RULE FOR ELECTRON FLOW (fig. 9-2, A).



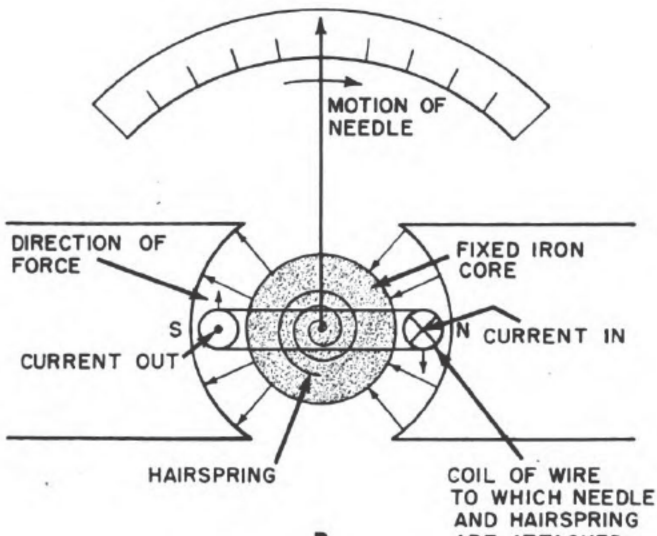


Figure 9–2.—Right-hand motor rule and application to the D'Arsonval movement.

To find the direction of motion of a conductor, the thumb, first finger, and second finger of the right hand are extended at right angles to each other, as shown. The first finger is pointed in the direction of the flux (toward the south pole) and the second finger is pointed in the direction of electron flow in the conductor. The thumb then points in the direction of motion

of the conductor with respect to the field. The conductor, the field, and the force are mutually perpendicular to each other.

The force acting on a current-carrying conductor in a magnetic field is directly proportional to the field strength of the magnet, the active length of the conductor, and the intensity of the electron flow through it. Thus,

$$F = \frac{8.85 \times BLI}{10^8},$$

where F is the force in pounds, B the flux density in lines per square inch, L the active length of the conductor in inches, and I the current in amperes.

In the D'Arsonval-type meter, the length of the conductor is fixed and the strength of the field between the poles of the magnet is fixed. Therefore, any change in I causes a proportionate change in the force acting on the coil.

The principle of the D'Arsonval movement may be more clearly shown by the use of the simplified diagram (fig. 9–2, B) of the D'Arsonval movement commonly used in d-c instruments. In the diagram, only one turn of wire is shown; however, in an actual meter movement many turns of fine wire would be used, each turn adding more effective length to the coil. The coil is wound on an aluminum frame or bobbin, to which the pointer is attached. Oppositely wound hairsprings (one of which is shown in fig. 9–2, B), are also attached to the bobbin, one at either end. The circuit to the coil is completed through the hairsprings. In addition to serving as conductors, the hairsprings serve as the restoring force that returns the pointer to the zero position when no current flows.

As has been stated, the deflecting force is proportional to the current flowing in the coil. The deflecting force tends to rotate the coil against the restraining force of the hairspring. The angle of rotation is proportional to the force that the spring exerts against the moving coil (within the elastic limit of the spring). When the deflecting force and the restraining force are equal, the coil and the pointer cease to move. Because the restoring force is proportional to the angle of deflection, it follows that the driving force, and the current in the coil, are proportional to the angle of deflection. When current ceases to flow in the coil, the driving

force ceases, and the restoring force of the springs returns the pointer to the zero position.

If the current through the single turn of wire is in the direction indicated (away from the observer on the right-hand side and toward the observer on the left-hand side), the direction of force, by the application of the right-hand motor rule, is upward on the left-hand side and downward on the right-hand side. The direction of motion of the coil and pointer is clockwise. If the current is reversed in the wire, the direction of motion of the coil and pointer is reversed.

A detailed view of the basic D'Arsonval movement, as commonly employed in ammeters and voltmeters, is shown in figure 9–3. This instrument is essentially a MICROAMMETER because the current necessary to activate it is of the order of 1 microampere. The principle of operation is the same as that of the simplified versions discussed previously. The iron core is rigidly supported between the pole pieces and serves to concentrate the flux in the narrow space between the iron core and the pole piece—in other words, in the space through which the coil and the bobbin moves. Current flows into one hairspring, through the coil, and out of the other hairspring. The restoring force of the spiral springs returns the pointer to the normal, or zero, position when the current through the coil is interrupted. Conductors connect the hairsprings with the outside terminals of the meter.

If the instrument is not damped—that is, if viscous friction or some other type of loss is not introduced to absorb the energy of the moving element—the pointer will go beyond its final position, then fall below it and oscillate for a long time about its final position before coming to rest. This action makes it nearly impossible to obtain a reading and some form of damping is necessary to make the meter practicable. Damping is accomplished in many D'Arsonval movements by means of the motion of the aluminum bobbin upon which the coil is wound. As the bobbin oscillates in the magnetic field, an emf is induced in it because it cuts through the lines of force. Therefore, according to Lenz's law, induced currents flow in the bobbin in such a direction as to oppose the motion, and the bobbin quickly comes to rest in the final position after going beyond it only once.

In addition to factors such as increasing the flux density in the

air gap, the over-all sensitivity of the meter can be increased by the use of a light-weight rotating assembly (bobbin, coil, and pointer) and by the use of jewel bearings as shown.

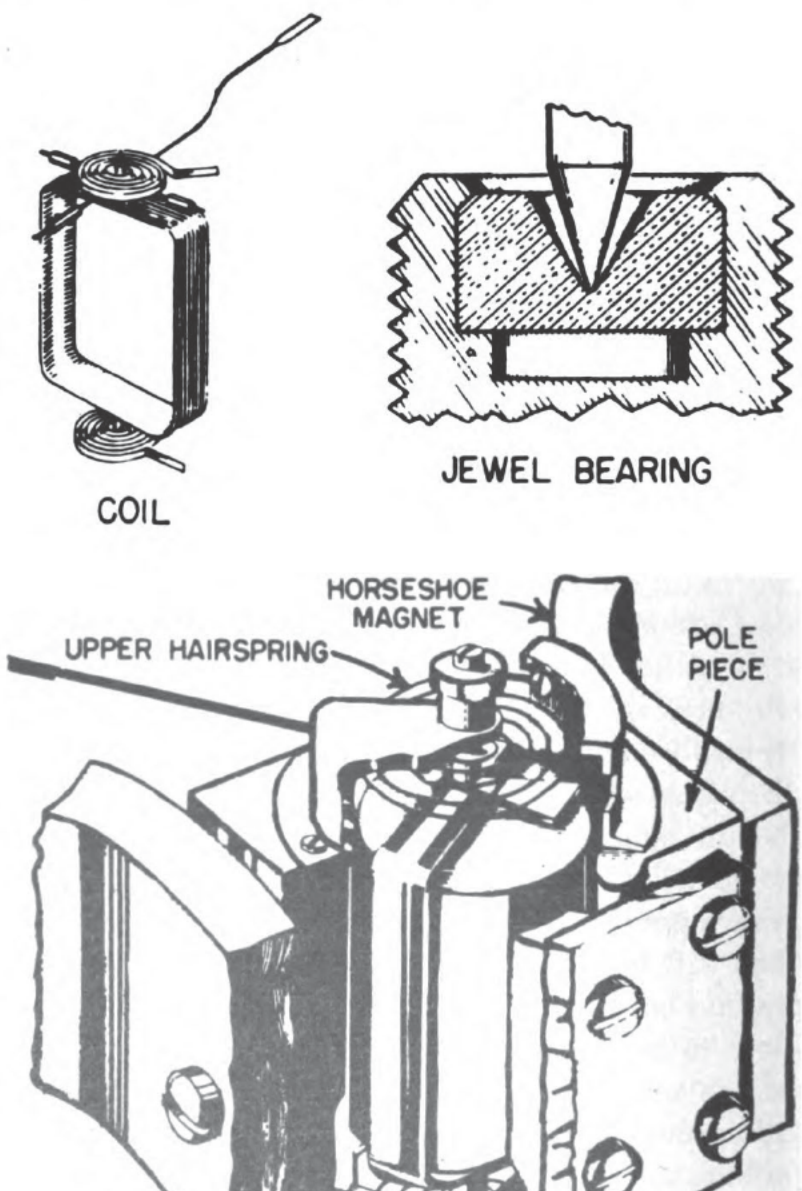


Figure 9–3.—Detailed view of basic D'Arsonval movement.

AMMETER

Construction

The basic D'Arsonval movement discussed thus far may be used to indicate or measure only very small currents—for example, microamperes (10^{-6} amperes) or milliamperes (10^{-3} amperes), depending on how sensitive the meter movement is. The small wire with which the movable coil is wound limits the current that may be permitted to flow through it to a relatively low value.

To measure larger currents, shunts (parallel resistors) are connected across the movable coil. The shunt carries that part of the circuit current that is in excess of the movable coil current necessary for full-scale deflection of the pointer. If the shunt is of such a value that the meter indicates milliamperes, the instrument is called a MILLIAMMETER. If the shunt is of such a value that the meter indicates amperes, it is called an AMMETER.

A simplified diagram of an ammeter composed of the basic D'Arsonval movement and an external shunt is shown in figure 9–4, A. The resistance of the shunt is equal to the voltage drop for full-scale deflection divided by the rated current of the shunt.

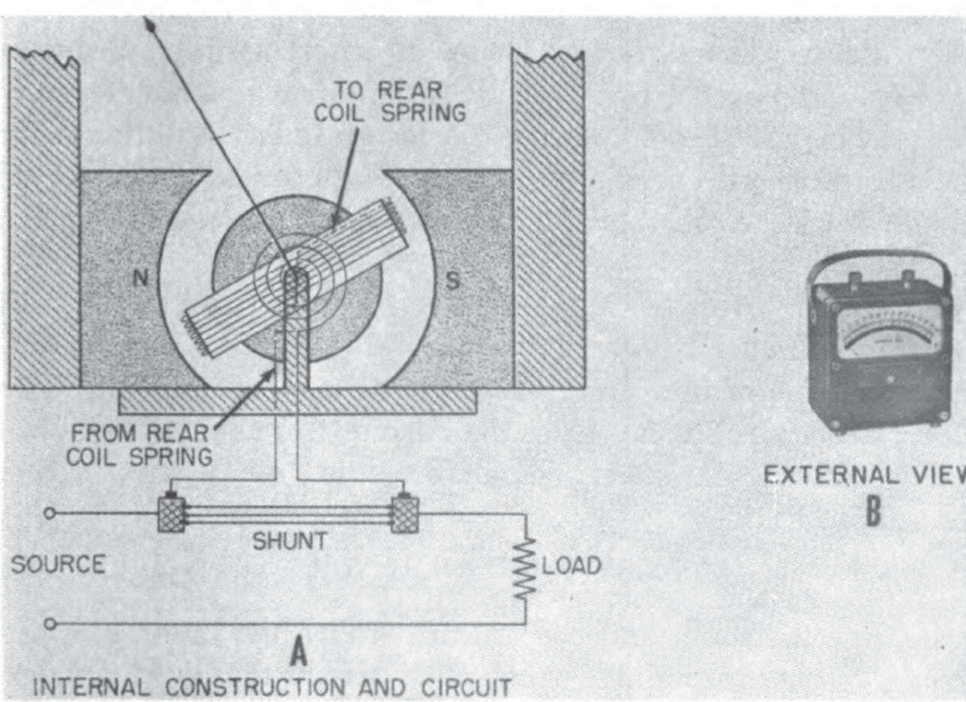


Figure 9–4.—Ammeter composed of D'Arsonval meter and external shunt.

Two values are therefore stamped on each shunt—the maximum current-carrying capacity, and the voltage drop across the shunt for the maximum current rating.

The shunt is carefully designed so that accurate readings may be made on the meter. The resistance strips are usually made of manganin, an alloy having almost zero temperature coefficient of resistance and a low thermoelectric effect (to be treated later) when used with copper. The ends of the resistance strips are embedded in heavy coper blocks to which are attached the meter lead terminals and the line terminals. To ensure accurate readings, the meter leads for a particular shunt should not be used interchangeably with those of other shunts. Slight changes in lead length and size may vary the resistance of the meter circuit appreciably and therefore may cause an incorrect meter reading.

Meter movements are designed to be used interchangeably with suitable shunts. For example, the same 50-millivolt meter movement will operate satisfactorily with either a 1-ampere shunt or a 50,000-ampere shunt depending on the magnitude of the load current to be measured. The external view of an ammeter is shown in figure 9–4, B.

Extending the Range by the Use of Internal Shunts

For limited current ranges (below 50 amperes) internal shunts are often employed. In this manner the range of the meter may be easily extended by employing the correct internal shunt having the necessary current rating. Before the required resistance of the shunt can be calculated, the resistance of the meter movement must be known.

For example, suppose it is desired to convert a 100-microampere D'Arsonval meter having a resistance of 100 ohms into an ammeter capable of measuring line currents up to 1 ampere. The meter deflects full scale when the current through the 100-ohm coil is 100 microamperes. Therefore, the voltage drop across the coil is IR, or

$$0.0001 \times 100 = 0.01 \text{ volt.}$$

Because the shunt and coil are in parallel, the shunt will also have a voltage drop of 0.01 volt. The current that flows through the shunt is the difference between the full-scale meter current

and the line current. In this case, the meter current is 100×10^{-6}, or 0.0001 ampere. This current is negligible compared with the line current, and the shunt current is approximately 1 ampere. The resistance, R_s, of the shunt is therefore

$$R_s = \frac{E}{I} = \frac{0.01}{1} = 0.01 \text{ ohm (approx.)},$$

and the range of the 100-microampere meter may be increased to 1 ampere by paralleling it with the 0.01-ohm shunt.

The 100-microampere instrument may be converted to a 10-ampere meter by the use of a proper shunt. For full-scale deflection of the meter, the voltage drop, E, across the shunt (and across the meter) must be 0.01 volt. The meter current is again negligible and the shunt current is approximately 10 amperes. The resistance, R_s, of the shunt is therefore

$$R_s = \frac{E}{I} = \frac{0.01}{10} = 0.001 \text{ ohm.}$$

The same instrument may likewise be converted to a 50-ampere meter by the use of the proper type of shunt. The current, I_s, through the shunt is approximately 50 amperes and the resistance, R_s, of the shunt is

$$R_s = \frac{E}{I_s} = \frac{0.01}{50} = 0.0002 \text{ ohm.}$$

It is important to select a suitable shunt when using an external shunt ammeter so that the scale indication is easily read. For example, if the scale has 150 divisions and the load current to be measured is known to be between 50 and 100 amperes, a 150-ampere shunt is suitable. If the scale deflection is 75 divisions, the load current is 75 amperes and the needle deflects half-scale.

A shunt having exactly the same current rating as the estimated load current should never be selected because any increase in load would drive the pointer off scale and might damage the movement. A good choice would bring the needle somewhere near the midscale indication.

Various values of shunt resistance may be used, by means of a suitable switching arrangement, to increase the number of current ranges that may be covered by the meter. Two switching arrange-

ments are shown in figure 9–5. Figure 9–5, A, is the simpler of the two arrangements from the point of view of calculating the value of the shunt resistors when a number of shunts are used. It has two disadvantages, however:

1. When the switch is moved from one shunt resistor to another the shunt is momentarily removed from the meter and the line current then flows through the meter coil. Even a momentary surge of current could easily damage the coil.
2. The contact resistance—that is, the resistance between the blades of the switch when they are in contact—is in series with the shunt but not with the meter coil. In shunts that must pass high currents the contact resistance becomes an appreciable part of the total shunt resistance. Because the contact resistance is of a variable nature, the ammeter indication may not be accurate.

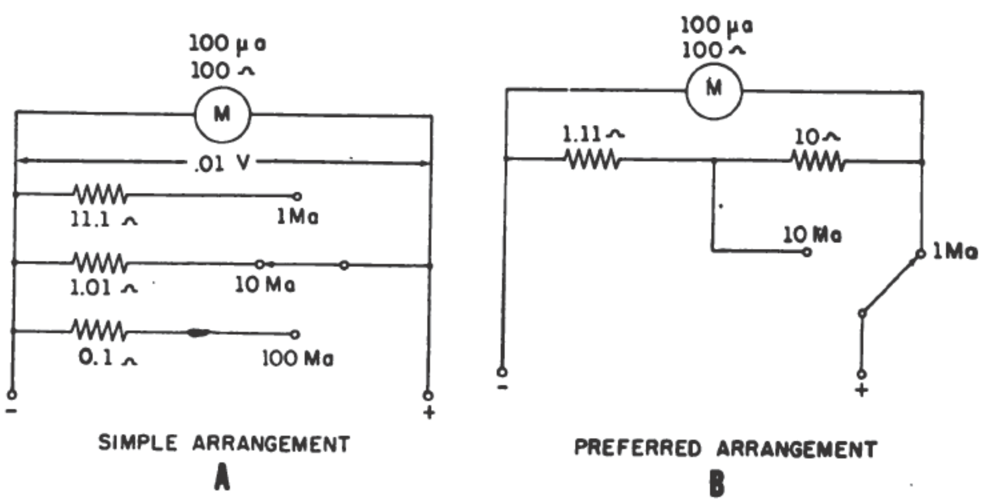


Figure 9–5.—Ammeter shunts.

A more generally accepted method of range switching is shown in figure 9–5, B. Although only two ranges are shown, as many ranges as needed can be used. In this type of circuit the range selector switch contact resistance is external to the shunt and meter in each range position, and therefore has no effect on the accuracy of the current measurement.

CURRENT-MEASURING INSTRUMENTS MUST ALWAYS BE CONNECTED IN SERIES WITH A CIRCUIT AND NEVER IN PARALLEL WITH IT. If an ammeter were connected across a constant-potential

source of appreciable voltage it would be the equivalent of a short circuit, and the meter would burn out. If the approximate value of current in a circuit is not known, it is best to start with the highest range of the ammeter and to progressively lower the range until a suitable reading is obtained.

Most ammeters indicate the magnitude of the current by being deflected from left to right. If the meter is connected with reversed polarity, it will be deflected backwards, and this action may damage the movement. Hence the proper polarity should be observed in connecting the meter in the circuit. That is, the meter should always be connected so that the electron flow will be into the negative terminal and out of the positive terminal.

VOLTMETER

Construction

The 100-microampere D'Arsonval meter used as the basic meter for the ammeter may also be used to measure voltage if a high resistance is placed in series with the moving coil of the meter.

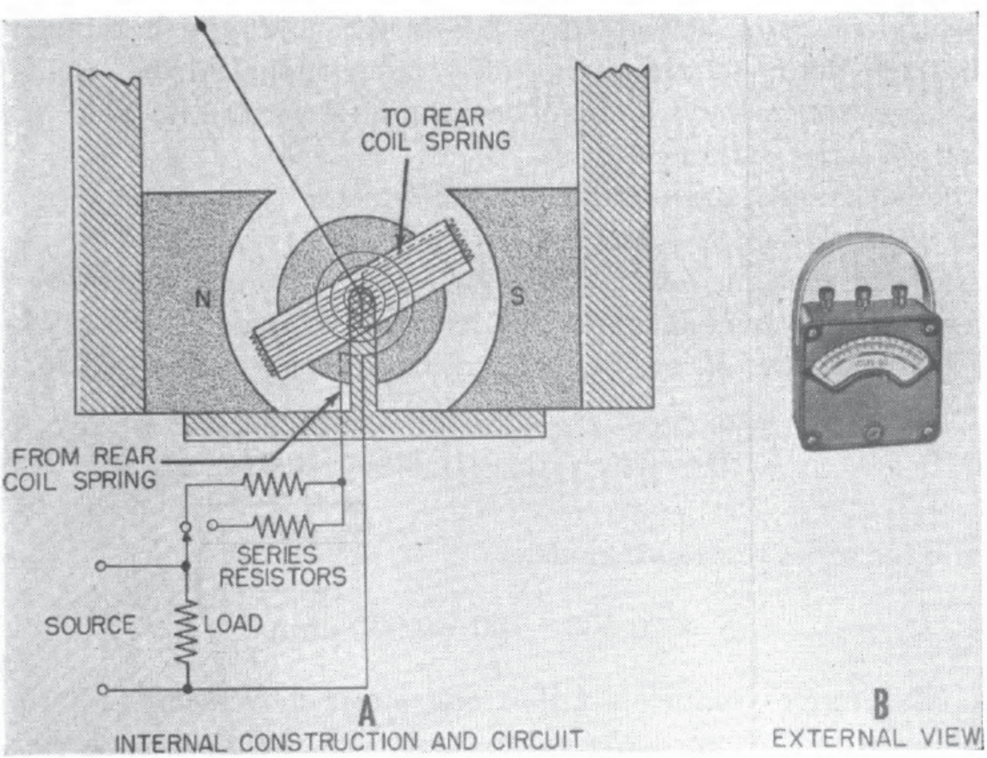


Figure 9–6.—Simplified voltmeter circuit.

For low-range instruments, this resistance is mounted inside the case with the D'Arsonval movement and typically consists of resistance wire having a low temperature coefficient and wound either on spools or card frames. For higher voltage ranges, the series resistance may be connected externally. When this is done the unit containing the resistance is commonly called a MULTIPLIER.

A simplified diagram of a voltmeter is shown in figure 9–6, A. The resistance coils are treated in such a way that a minimum amount of moisture will be absorbed by the insulation. Moisture reduces the insulation resistance and increases leakage currents, which cause incorrect readings. Leakage currents through the insulation increase with length of resistance wire and become a factor that limits the magnitude of voltage that may be measured. An external view of a voltmeter is shown in figure 9–6, B.

Extending the Range

The value of the necessary series resistance is determined by the current required for full-scale deflection of the meter and by the range of voltage to be measured. Because the current through the meter circuit is directly proportional to the applied voltage, the meter scale can be calibrated directly in volts for a fixed series resistance.

For example, assume that the basic meter (microammeter) is to be made into a voltmeter with a full-scale reading of 1 volt. The coil resistance of the basic meter is 100 ohms, and 0.0001 ampere (100 microamperes) causes a full-scale deflection. The total resistance, R, of the meter coil and the series resistance is

$$R=\frac{E}{I}=\frac{1}{0.0001}=10{,}000 \text{ ohms},$$

and the series resistance alone is

$$R_s=10{,}000-100=9{,}900 \text{ ohms}.$$

Multirange voltmeters utilize one meter movement with the required resistances connected in series with the meter by a convenient switching arrangement. A multirange voltmeter with

three ranges is shown in figure 9–7. The total circuit resistance for each of the three ranges beginning with the 1-volt range is:

$$R=\frac{E}{I}=\frac{1}{100}=0.01 \text{ megohm},$$

$$\frac{100}{100}=1 \text{ megohm},$$

and

$$\frac{1{,}000}{100}=10 \text{ megohms.}$$

VOLTAGE-MEASURING INSTRUMENTS ARE CONNECTED ACROSS (IN PARALLEL WITH) A CIRCUIT. If the approximate value of the voltage to be measured is not known, it is best to start with the highest range of the voltmeter and progressively lower the range until a suitable reading is obtained.

In many cases, the voltmeter is not a central-zero indicating instrument. Thus, it is necessary to observe the proper polarity when connecting the instrument to the circuit, as is the case in connecting the d-c ammeter. The positive terminal of the voltmeter is always connected to the positive terminal of the source,

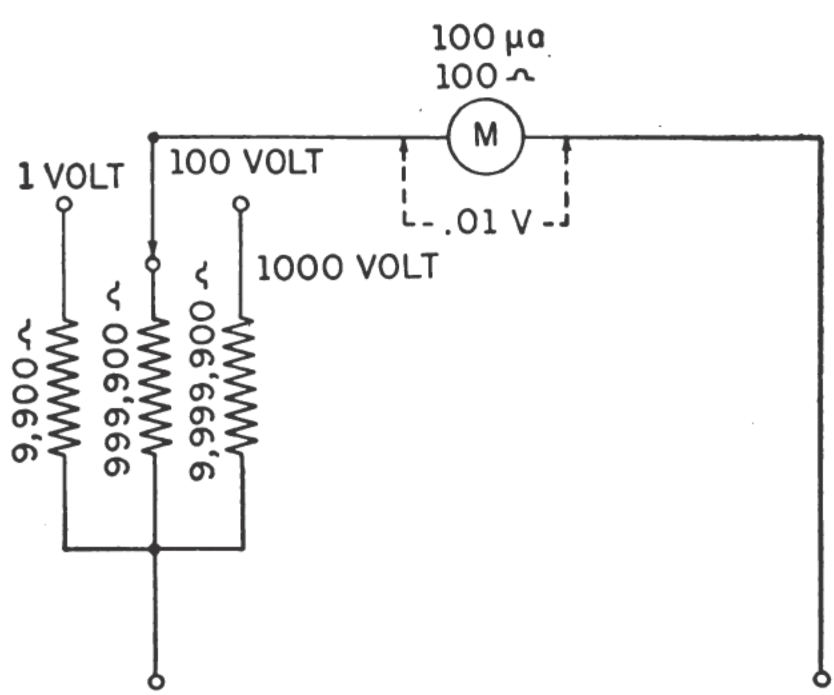


Figure 9–7.—Multirange voltmeter.

and the negative terminal to the negative terminal of the source when the source voltage is being measured. In any case, the voltmeter is connected so that electrons will flow into the negative terminal and out of the positive terminal of the meter.

Influence in a Circuit

The function of a voltmeter is to indicate the potential difference between two points in a circuit. When the voltmeter is connected across a circuit, it shunts the circuit. If the voltmeter has low resistance it will draw an appreciable amount of current. The effective resistance of the circuit will be lowered and the voltage reading will consequently be lowered.

When voltage measurements are made in high-resistance circuits, it is necessary to use a high-resistance voltmeter to prevent the shunting action of the meter. The effect is less noticeable in low-resistance circuits because the shunting effect is less.

Sensitivity

The sensitivity of a voltmeter is given in ohms per volt, (Ω/E), and may be determined by dividing the resistance R_m, of the meter plus the series resistance, R_s, by the full-scale reading in volts. Thus,

$$\text{sensitivity}=\frac{R_m+R_s}{E}.$$

This is the same as saying that the sensitivity is equal to the reciprocal of the current (in amperes)—that is,

$$\text{sensitivity}=\frac{\text{ohms}}{\text{volts}}=\frac{1}{\dfrac{\text{volts}}{\text{ohms}}}=\frac{1}{\text{amperes}}.$$

Thus, the sensitivity of a 100-microampere movement is the reciprocal of 0.0001 ampere, or 10,000 ohms per volt.

Table 10 shows how the sensitivity of permanent-magnet movable-coil voltmeters has been increased over many years of manufacture. The table indicates the sensitivity of a 0 to 150-volt voltmeter, its resistance, and the current required to produce a full-scale deflection.

Accuracy

The accuracy of a meter is generally expressed in percent. For example, a meter that has an accuracy of 1 percent will indicate a value that is within 1 percent of the correct value. The statement means that if the correct value is 100 units, the meter indication may be anywhere within the range of 99 to 101 units.

TABLE 10.—*Increase in sensitivity in voltmeters*

Time of manufacture	Sensitivity	Resistance	Current to deflect full scale
Before 1924	10 ohms per volt	1.5 k-ohms	100 ma
After 1924	100 ohms per volt	15 k-ohms	10 ma
Today	1,000 ohms per volt	150 k-ohms	1 ma
	20,000 ohms per volt	3 megohms	50 μa
	200,000 ohms per volt	30 megohms	5 μa

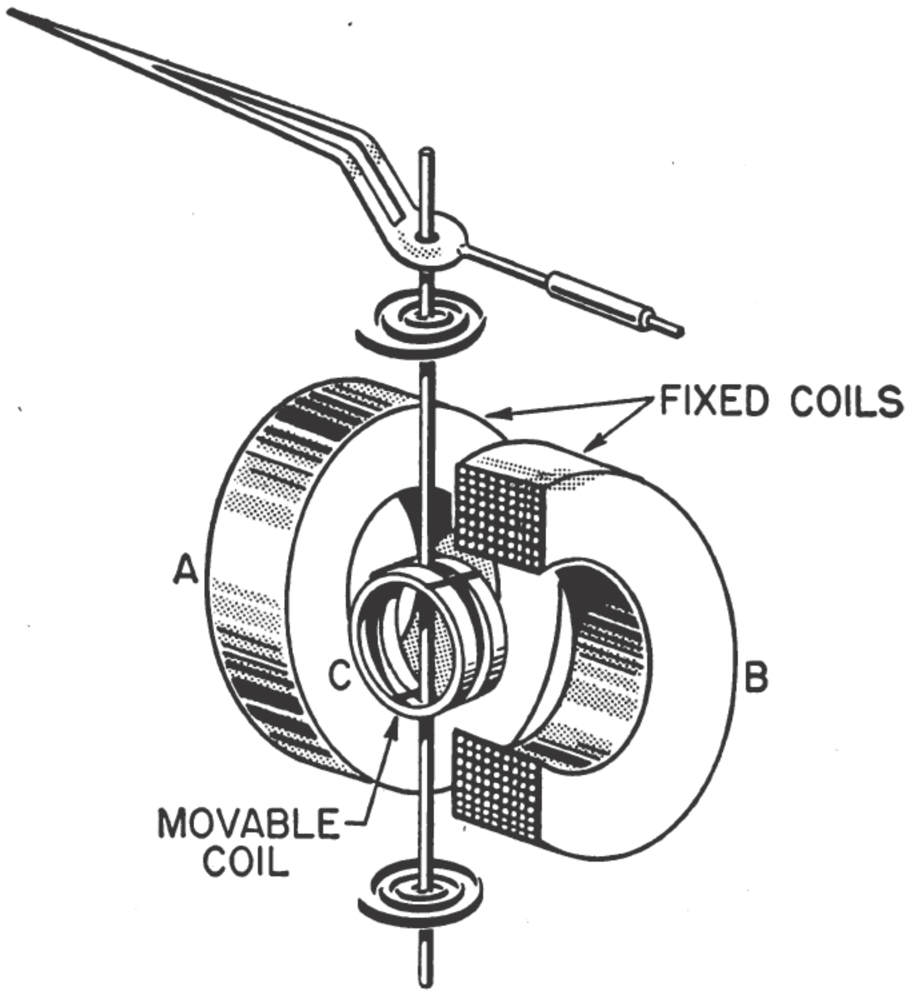
The sensitivity has been increased by increasing the strength of the permanent magnet, by using lighter weight materials for the moving element (consistent with increased number of turns on the coil), and by using sapphire jewel bearings to support the moving coil.

ELECTRODYNAMOMETER-TYPE METER

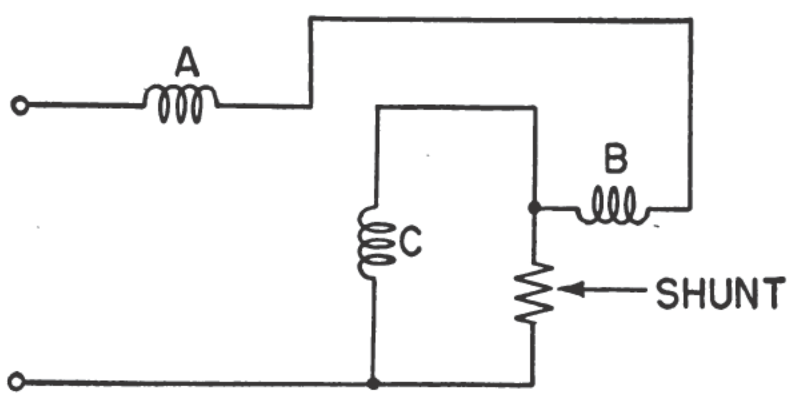
The electrodynamometer-type meter operates on the same principle as the permanent-magnet moving-coil meter—except that the permanent magnet is replaced by an air-core electromagnet the field of which is developed by the same current that flows through the moving coil.

In the electrodynamometer-type of meter, two stationary field coils are connected in series with the movable coil. The movable coil is attached to the central shaft and rotates inside the two stationary field coils, as shown in figure 9–8, A. The spiral springs provide the restoring force for the meter and also a means of introducing current to the movable coil.

When current flows through field coils *A* and *B* and movable coil *C*, coil *C* tends to rotate in opposition to the springs and to



A SIMPLIFIED METER



B SCHEMATIC DIAGRAM (AMMETER)

Figure 9–8.—Simplified diagram of an electrodynamometer movement.

place itself parallel to the field coils. The more current that flows through the coils the more the moving coil overcomes the opposition of the springs and the farther the pointer moves across the scale. If the scale is properly calibrated and the proper shunts or multipliers are used, the dynamometer movement will indicate current or voltage. A simplified schematic diagram of the meter movement is shown in figure 9–8, B.

In the electrodynamometer-type AMMETER the coils are made of relatively large low-resistance wire and connected in series between the source and the load so that only a minmum amount of voltage will be developed across the ammeter. A shunt is connected across the movable coil so that only a small fraction of the line current will flow through this coil. The scale is calibrated to indicate the circuit current.

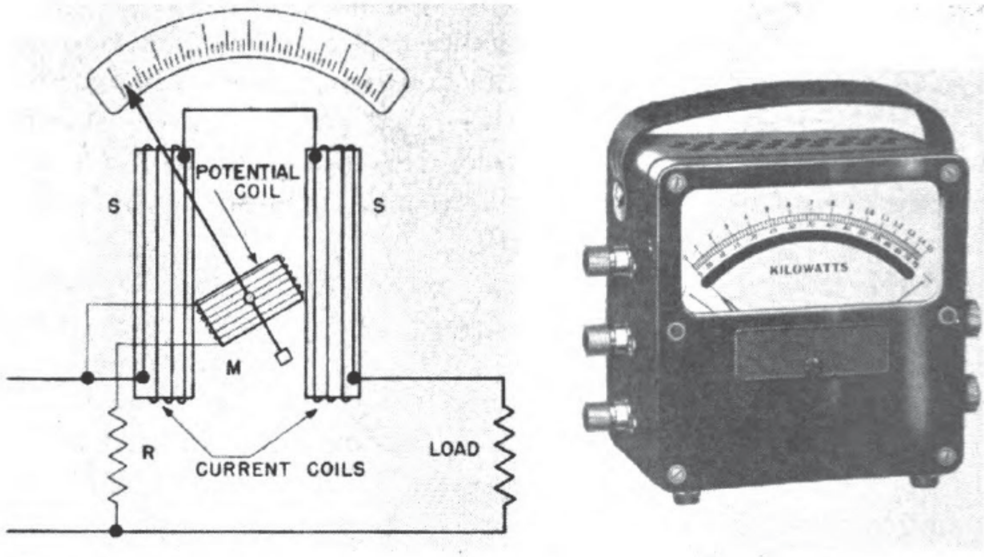
When the electrodynamometer mechanism is designed to be used as a VOLTMETER, the fixed coils are wound with fine wire and are connected in series with the movable coil. A high resistance is also connected in series with the dynamometer movement to limit the current when the meter is connected across the voltage source. The scale is calibrated to indicate the voltage.

The polarity of the fields produced by the flow of current through the stationary and movable coils is reversed by a reversal of line current, and therefore the deflection of the movable element is always in the same direction. In other words, the meter reads up scale regardless of the direction of current flow through the coils. Therefore, the electrodynamometer meter can be used for either alternating or direct current and voltage measurements.

The meter is mechanically damped by means of aluminum vanes that move in enclosed air chambers. Although electrodynamometer-type meters are very accurate they do not have the sensitivity of the D'Arsonval-type meters and for this reason are not widely used outside the laboratory.

WATTMETER

Electric power is measured by means of a wattmeter. Because electric power is the product of current and voltage, a wattmeter must have two elements, one for current and the other for voltage, as indicated in figure 9–9, A. For this reason, wattmeters are usually of the electrodynamometer type.



CIRCUIT
A

EXTERNAL VIEW
B

Figure 9–9.—Simplified electrodynamometer wattmeter circuit.

The movable coil with a series resistance forms the voltage element, and the stationary coils constitute the current element. The strength of the field around the potential coil depends on the amount of current that flows through it. The current, in turn, depends on the load voltage applied across the coil and the high resistance in series with it. The strength of the field around the current coils depends on the amount of current flowing through the load. Thus, the meter deflection is proportional to the product of the voltage across the potential coil and the current through the current coils. The effect is almost the same (if the scale is properly calibrated) as if the voltage applied across the load and the current through the load were multiplied together.

If the current in the line is reversed, the direction of current in both the current coils and the potential coil is reversed, and the net result is that the pointer continues to read up scale. Therefore, this type of wattmeter can be used to measure either a-c or d-c power. An external view of a kilowattmeter is shown in figure 9–9, B. Wattmeters are seldom used in d-c circuits because power is the product of the voltmeter and ammeter readings. In a-c circuits the power is seldom equal to the product

of the voltmeter and ammeter readings, and the wattmeter gives an accurate indication of power. Wattmeters are treated more in detail under a-c indicating instruments in chapter 15.

METERS USED FOR MEASURING OF RESISTANCE

Two instruments are commonly used to check the continuity or to measure the resistance of a circuit or circuit element. These instruments are the OHMMETER and the MEGGER, or megohmmeter. The ohmmeter is widely used to measure resistance and to check the continuity of electrical circuits and devices. Its range usually extends to a few megohms. The megger is widely used for measuring insulation resistance, such as the resistance between the windings and the frame of electric machinery, and the insulation resistance of cables, insulators, and bushings. Its range may extend to more than 1,000 megohms. When measuring very high resistances of this nature it is not necessary to find the exact value of resistance, but rather to know that the insulation is either above or below a certain standard. When precision measurements are required, some type of bridge circuit is used.

Ohmmeter

The SERIES-TYPE ohmmeter consists essentially of a sensitive milliammeter, a voltage source, and a fixed and a variable resistor all connected in series between the two terminals of the instrument, as shown in figure 9–10, A. Before the unknown resistance is measured, the test leads are shorted together and the variable resistance is adjusted for full-scale deflection. The point on the meter scale corresponding to full-scale deflection is marked "zero resistance".

If a 1-milliampere movement (1 milliampere causes a full-scale deflection) is used with a 3-volt battery, the series resistance together with the meter resistance is $\frac{3}{0.001}$, or 3,000 ohms. Part of the series resistance is made variable to compensate for changes in battery voltage.

When the unknown resistance, R_x, is inserted between the test leads, the meter reading decreases. For example, a 6,000-ohm resistor (fig. 9–10, A) inserted between the probes decreases the

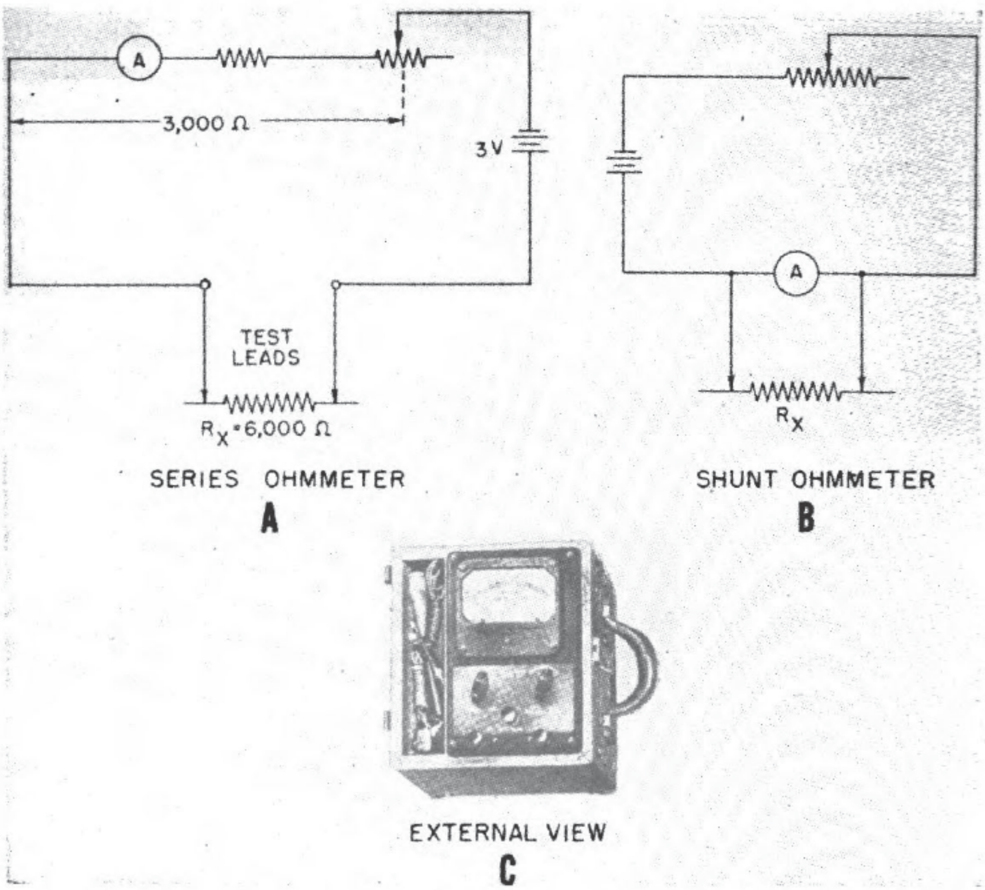


Figure 9–10.—Simplified ohmmeter circuits.

meter current to $\frac{3}{3+6}$, or 0.33 milliamperes. The corresponding point on the scale is marked "6,000 ohms." Other points may be similarly determined. Unlike the other meters considered thus far that have increasing values from left to right, the series ohmmeter has increasing values from right to left.

A SHUNT-TYPE ohmmeter circuit is shown in figure 9–10, B. As in the series type, there is a variable resistor in series with the milliameter. In this case, however, the test leads are brought out from the meter terminals, thus placing the unknown resistance in parallel with the meter instead of in series with it. In this case the lower the value of the unknown resistor the lower is the value of the current that flows through the milliammeter. Conversely, the higher the value of the unknown resistance the higher is the value of the current through the milliammeter. Thus, the

shunt-type ohmmeter reads up scale from left to right like a voltmeter or ammeter.

In general, the series type of ohmmeter is used to measure high resistance, and the shunt type, low resistance. High resistance measurements are limited by the magnitude of the voltage applied to the meter circuit. An external view of an ohmmeter is shown in figure 9–10, C.

Megger

The megger (fig. 9–11, A) consists of two primary elements both of which are provided with individual magnetic fields from a common permanent magnet—(1) a hand-driven d-c generator, *G,* which supplies the necessary current for making the measurement, and (2) the instrument portion, which indicates the value of the resistance being measured. The instrument portion is of the opposed-coil type. Coils *A* and *B* are mounted on the movable member with a fixed angular relationship to each other and are free to turn as a unit in a magnetic field. Coil *B* tends to move the pointer counterclockwise, and coil *A,* clockwise. The coils are mounted on a light movable frame that is pivoted in jewel bearings and that is free to move about axis *O.*

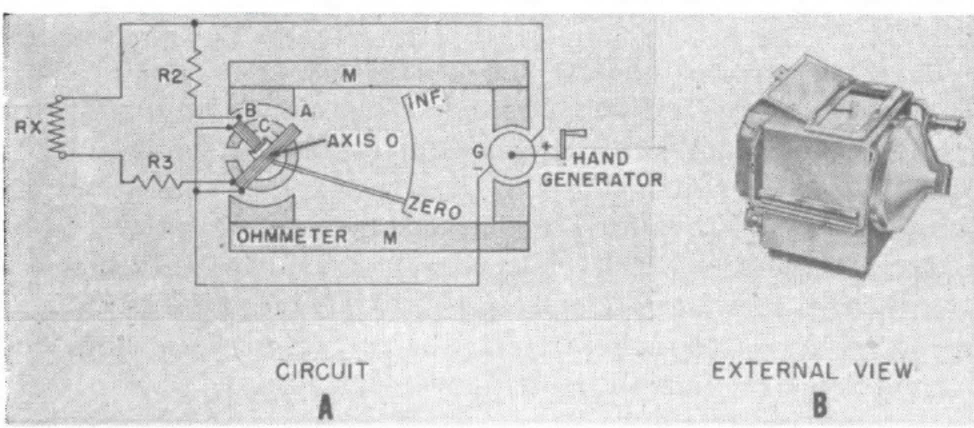


Figure 9–11.—Simplified megger circuit.

Coil *A* is connected in series with *R*3 and the unknown resistance, R_x, to be measured. The series combination of coil *A,* *R*3, and R_x is connected between the + and − brushes of the d-c generator. Coil *B* is connected in series with *R*2 and this combination is also connected across the generator. There are no restraining springs on the movable member of the instrument

portion of the megger and when the generator is not operated the pointer floats freely and may come to rest at any position on the scale.

If the terminals are open-circuited, no current flows in coil *A*, and the current in coil *B* alone controls the movement of the moving system. Coil *B* takes a position opposite the gap in the core (since the core cannot move and coil *B* can), and the pointer indicates infinity on the scale. When a resistance is connected between the terminals, current flows in coil *A*, tending to move the pointer clockwise. At the same time, coil *B* tends to move the pointer counterclockwise. Therefore, the moving element, composed of both coils and the pointer, comes to rest at a position at which the two forces are balanced. This position depends upon the value of the external resistance, which controls the relative magnitude of current in coil *A*. Because changes in voltage affect both coil *A* and coil *B* in the same proportion, the position of the moving system is independent of the voltage. If the terminals are short-circuited, the pointer rests at zero because the current in *A* is relatively large. The instrument is not injured under these circumstances because the current is limited by *R*3.

The external view of one type of megger is shown in figure 9–11, B.

Meggers provided aboard ship usually are rated at 500 volts. To avoid excessive test voltages, most meggers are equipped with friction clutches. When the generator is cranked faster than its rated speed, the clutch slips and the generator speed and output voltage are not permitted to exceed their rated values. For extended ranges, a 1,000-volt generator is available. When extremely high resistances—for example, 10,000 megohms or more—are to be measured, a high voltage is needed to cause sufficient current flow to actuate the meter movement.

MOVING IRON-VANE METER

The moving iron-vane meter is another basic type of meter. Unlike the D'Arsonval-type meter, which employs permanent magnets, the moving iron-vane meter depends on induced magnetism for its operation. It employs the principle of repulsion between two concentric iron vanes, one fixed and one movable, placed inside a solenoid, as shown in figure 9–12, A. A pointer is attached to the movable vane.

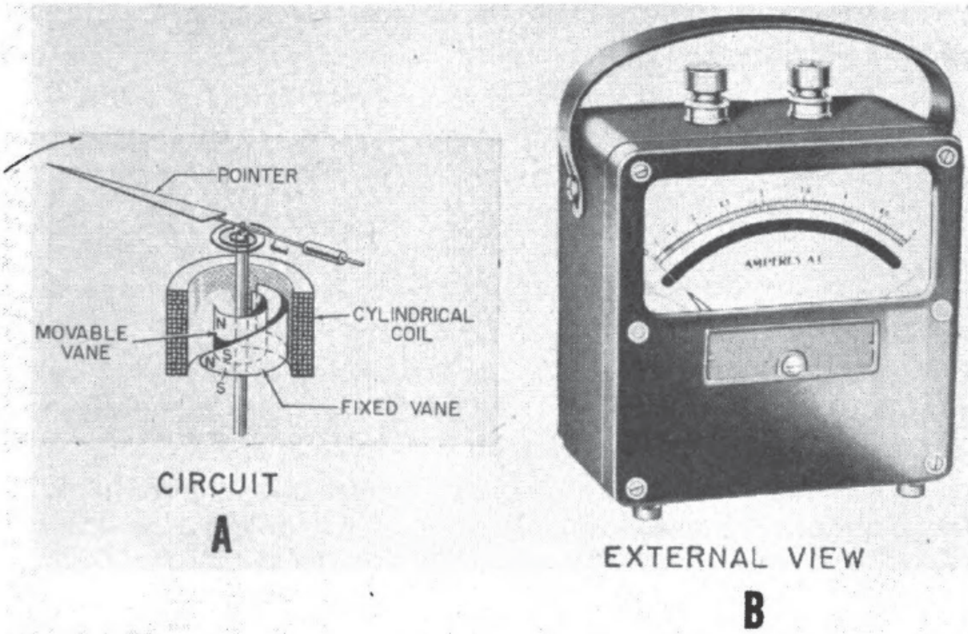


Figure 9–12.—Simplified diagram of a moving iron-vane meter.

When current flows through the coil, the two iron vanes become magnetized with north poles at their upper ends and south poles at their lower ends for one direction of current through the coil, as shown in the figure. Because like poles repel, the unbalanced component of force tangent to the movable element causes it to turn against the force exerted by the springs.

The movable vane is rectangular in shape, and the fixed vane is tapered. This design permits the use of a relatively uniform scale.

When no current flows through the coil, the movable vane is positioned so that it is opposite the larger portion of the tapered fixed vane, and the scale reading is zero. The amount of magnetization of the vanes depends on the strength of the field, which, in turn, depends on the amount of current flowing through the coil. The force of repulsion is greater opposite the larger end of the fixed vane than it is nearer the smaller end. Therefore, the movable vane moves toward the smaller end through an angle that is proportional to the magnitude of the coil current. The movement ceases when the force of repulsion is balanced by the restoring force of the spring.

Because the repulsion is always in the same direction (toward the smaller end of the fixed vane) regardless of the direction of

current flow through the coil, the moving iron-vane instrument operates on either d-c or a-c circuits.

Mechanical damping in this type of instrument is obtained by the use of an aluminum vane attached to the shaft (not shown in the figure) in such a way that, as the shaft moves, the vane moves in a restricted air space.

When the moving iron-vane meter is designed to be used as an ammeter, the coil is wound with relatively few turns of large wire in order to carry the rated current.

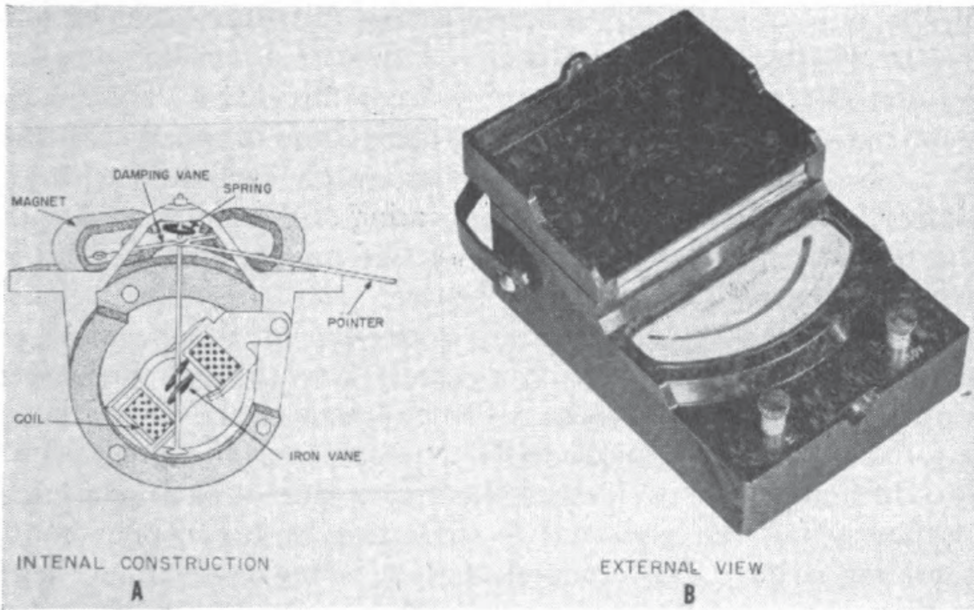
When the moving iron-vane meter is designed to be used as a voltmeter the solenoid is wound with many turns of small wire. Portable voltmeters are made with self-contained series resistance for ranges up to 750 volts. Higher ranges are obtained by the use of additional external multipliers. An external view of a moving iron-vane meter is shown in figure 9–12, B.

The moving iron-vane instrument may be used to measure direct current, but has an error due to residual magnetism in the vanes. The error may be minimized by reversing the meter connections and averaging the readings. When used on a-c circuits the instrument has an accuracy of 0.5 percent. Because of its simplicity, its relatively low cost, and the fact that no current is conducted to the moving element, this type of movement is used extensively to measure current and voltage in a-c power circuits.

However, because the reluctance of the magnetic circuit is high, the moving iron-vane meter requires much more power to produce full-scale deflection than is required by a D'Arsonval meter of the same range. Therefore, the moving iron-vane meter is seldom used in high-resistance low-power circuits.

INCLINED-COIL IRON-VANE METER

The principle of the moving iron-vane mechanism is applied to the inclined-coil type of meter shown in figure 9–13, A. The inclined-coil iron-vane meter has a coil mounted at an angle to the shaft. Attached obliquely to the shaft, and located inside the coil, are two soft-iron vanes. When no current flows through the coil, a control spring holds the pointer at zero and the iron vanes lie in planes parallel to the plane of the coil. When current flows through the coil, the vanes tend to line up with magnetic lines passing through the center of the coil at right angles to the plane of the coil. Thus the vanes rotate against the spring action to move the pointer over the scale.



Courtesy General Electric Company

Figure 9–13.—Inclined-coil iron-vane meter.

The iron vanes tend to line up with the magnetic lines regardless of the direcion of current flow through the coil. Therefore, the inclined-coil iron-vane meter can be used to measure either alternating current or direct current. The aluminum disk and the drag magnets provide electromagnetic damping, which is treated later in the text.

Like the moving iron-vane meter, the inclined-coil type requires a relatively large amount of current for full-scale deflection and hence is seldom used in high-resistance low-power circuits.

As in the moving iron-vane instrument, the inclined-coil instrument is wound with few turns of relatively large wire when used as an ammeter and with many turns of small wire when used as a voltmeter. An external view of an inclined-coil iron-vane meter is shown in figure 9–13, B.

THERMOCOUPLE-TYPE METER

If two of the ends of two dissimilar metals are welded together and this junction is heated, a d-c voltage is developed across the two open ends. The voltage developed depends on the material of which the wires are made and on the difference in temperature between the heated junction and the open ends.

In one type of instrument, the junction is heated electrically

by the flow of current through a heater element. It does not matter whether the current is alternating or direct because the heating effect is independent of current direction. The maximum current that may be measured depends on the current rating of the heater, the heat that the thermocouple can stand without being damaged, and on the current rating of the meter used with the thermocouple. Voltage may also be measured if a suitable resistor is placed in series with the heater.

A simplified schematic diagram of one type of thermocouple is shown in figure 9–14. The input current flows through the heater strip via the terminal blocks. The function of the heater strip is to heat the thermocouple, which is composed of a junction of two dissimilar wires welded to the heater strip. The open ends of these wires are connected to the center of two copper compensating strips. The function of these strips is to radiate heat so that the open ends of the wires will be much cooler than the junction end of the wires; thus permitting a higher voltage to be developed across the open ends of the thermocouple. The compensating strips are thermally and electrically insulated from the terminal blocks.

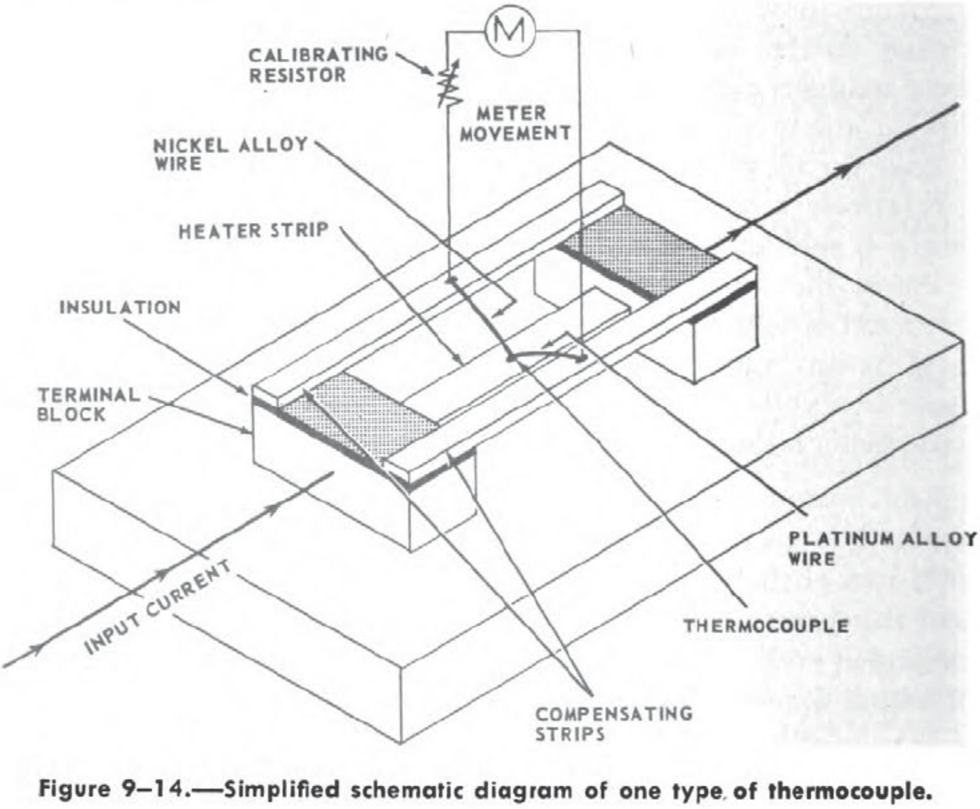


Figure 9–14.—Simplified schematic diagram of one type of thermocouple.

The heat produced by the flow of line current through the heater strip is proportional to the square of the heating current ($P=I^2R$). Because the voltage appearing across the two open terminals is proportional to the temperature, the movement of the meter element connected across these terminals is proportional to the square of the current flowing through the heater element. The scale of the meter is crowded near the zero end, and is progressively less crowded near the maximum end of the scale. Because the lower portion of the scale is crowded the reading is necessarily less accurate. For the sake of accuracy in making a given measurement, it is desirable to choose a meter in which the deflection will extend at least to the more open portion of the scale.

The meter used with the thermocouple should have low resistance to match the low resistance of the thermocouple, and it must deflect full scale when rated current flows through the heater. Because the resistance must be low and the sentitivity high, the moving element must be light.

A more nearly uniform meter scale may be obtained if the permanent magnet of the meter is constructed so that as the coil rotates (needle moves up scale), it moves into a magnetic field of less and less density. The torque then increases approximately as the first power of the current instead of as the square of the current, and a more linear scale is achieved.

If the thermocouple is burned out by excessive current through the heater strip, it may be replaced and the meter recalibrated by means of the calibrating variable resistor.

SUMMARY

In this chapter the operating principles of the more common d-c instruments (some of which may also be used with a-c) have been presented. Other indicating instruments are treated in chapter 15. Electronic test equipment is treated in chapter 13 of *Basic Electronics,* NavPers 10087.

The D'Arsonval movement was presented first because this is the basic movement (with slight modifications) used in most portable test equipment. In this movement, the field in which the movable coil is suspended is established by means of a powerful permanent magnet. The flow of current through the movable coil is accompanied by a force that moves the coil through an

angle that is proportional to the force. This action is made possible by the use of restoring springs that are not stretched beyond their elastic limit. If a pointer is attached to the coil, and a suitable scale is used, an indication of the magnitude of the current flowing in the coil may be obtained by the final position of the pointer along the scale.

As was pointed out, only a small amount of current may be permitted to flow through the coil. If too much current flows through the coil, it may be damaged. An instrument that indicates minute currents is called a GALVANOMETER.

Currents higher than those permitted to flow through the coil of the O'Arsonval movement may be measured if the coil is shunted by a suitable resistor. Such an arrangement is called an AMMETER. For example, the shunt may have a value of resistance low enough to carry nine-tenths of the line current, leaving only one-tenth for the meter. The meter scale is calibrated to indicate the total line current. Ammeters are always connected in series with the circuit whose current is to be measured.

A high resistance in series with the meter movement permits voltage measurements. Such an instrument is called a VOLTMETER. The high resistance limits the current through the coil to a safe value. The higher the voltage applied across the coil and its series resistor the greater is the current flow through the coil and the greater is the deflection of the meter. The meter is calibrated to read in volts.

The ELECTRODYNAMOMETER movement operates on essentially the same principle as the permanent-magnet moving-coil meter. However, the electrodynamometer movement employs an air-core electromagnet instead of a permanent magnet. The electromagnet is energized by the same current that flows through the movable coil. Shunts are used to extend the range of this instrument when it is used as an ammeter, and series resistors are used to extend the range when it is used as a voltmeter.

The electrodynamometer movement may be employed in a WATTMETER. The wattmeter is an instrument used to measure power. Because power in watts is equal to the product of the volts and the amperes in a circuit, the wattmeter must in effect perform this multiplication. The electrodynamometer movement is well suited to perform this function. The current that flows through a device whose power consumption is to be meas-

ured also flows through the fixed coils of the meter; and their magnetic field strength depends on the amount of the current flowing through them and through the device. Likewise, the voltage applied across the device is also applied across the movable coil of the meter. The higher the voltage, the greater will be the current flowing through the movable coil, and the greater will be the tendency of the coil to move about its axis. Thus, both the current through the fixed coil and the voltage across the movable coil determine the amount of movement of the movable coil. A pointer attached to the movable coil indicates on an appropriate scale the power in watts consumed in a device or circuit.

Care must be exercised in connecting a wattmeter. The current coils (the fixed coils) are connected in series with the line, and the potential coil (the movable coil) is connected across the line.

The OHMMETER is commonly used to give an indication of the resistance of a circuit or a circuit component. Ohmmeters are also used to indicate circuit continuity. The simple series ohmmeter is composed of a permanent-magnet moving-coil microammeter or milliammeter in series with a variable resistor, a fixed resistor, and a voltage source. If the circuit is completed, a certain amount of current will flow through the meter. The variable resistor is adjusted until the pointer moves to the extreme right-hand side of the scale to indicate zero ohms. If a circuit or component having an unknown resistance is connected in series with the ohmmeter circuit less current will flow through the meter and the pointer will move to some intermediate position, depending on the unknown resistance. The higher the resistance, the more the pointer moves to the left.

The scale of the ohmmeter is calibrated in ohms or megohms. When higher resistances are measured, a higher source voltage or a more sensitive meter must be used in order to obtain the necessary indication.

The MEGGER is used for measuring high resistance—for example, insulation resistance of the order of 10,000 megohms. It is composed of a hand-propelled d-c generator with voltage ranges up to 2,000 volts or more, and a suitable indicating instrument together with the necessary resistors. The indicating instrument has no restraining springs and therefore the pointer may come to rest at any point on the dial when the instrument is not being

used. The moving part of the meter is composed of two coils mounted on a common assembly at right angles to each other. One coil, called the POTENTIAL coil, is connected in series with a suitable resistor directly across the generator terminals. The other coil, called the CURRENT coil, is connected in series with a suitable resistor between one terminal of the generator and one output terminal. The other generator terminal is connected to the remaining output terminal.

When a resistor is connected between the output terminals, and the hand crank is turned, current flows in the potential and current coils. The coils are so positioned that they tend to move in opposite directions, and therefore the position at which the pointer comes to rest on the scale is determined by the relative amount of current flowing in the current coil. If no current flows through the current coil, the pointer rests at infinity. If the resistance being tested has some value below infinity, a current inversely proportional to the resistance flows through the current coil, and the pointer comes to rest at a dial reading below infinity.

The basic mechanism used in d-c measurements is the D'Arsonval movement—in other words, the permanent-magnet moving-coil movement. The basic mechanism used in a-c measurements is the MOVING IRON-VANE METER. It may also be used for measuring d-c values. However, in practice, a-c measurements in industrial applications are usually made with some form of the moving-iron meter. In service work suitable rectifiers or thermocouples are often employed with the D'Arsonval-type movement to measure a-c current and voltage.

In the moving-iron type of meter, current flows through a fixed coil. Within the coil are mounted two iron vanes, one fixed and one movable. In one type of instrument the fixed vane is tapered and curved into a cylindrical form inside the coil. Within this fixed vane a rectangular vane is pivoted and balanced by springs in such a way that when no current flows through the coil the moving vane is opposite the long edge of the taper. When current flows through the coil both vanes become magnetized with the same polarity and therefore the moving vane is repelled toward the narrow edge of the tapered fixed coil by an amount that depends on the magnitude of the current.

There are many types of moving iron-vane meters—for example, the radial-vane type of instrument or the magnetic-vane

attraction-type of instrument—all of which are different applications of the principle that like poles repel.

The INCLINED-COIL IRON-VANE METER is well suited for current measurements because it has a long evenly distributed scale and considerable torque; and current does not flow through the moving parts.

It operates on the principle that if a magnetic substance that is free to move is placed in a magnetic field it will tend to align itself with the field—that is, parallel with the flux—so that the flux path will be through the greatest length of the substance. The rectangular vane is attached at a 45° angle to the shaft and the coil is likewise mounted at a 45° angle to the shaft. The pointer is attached to the shaft and moves across a properly calibrated scale. When no current flows through the coil the restraining springs hold the shaft in such a position that the plane of the vane is perpendicular to the axis of the coil. When current flows through the coil the vane lines up so that, at maximum current, it is parallel with the axis of the coil and parallel with the flux. At intermediate values of current the vane will assume intermediate positions in the coil, and the pointer will indicate intermediate values of current.

The THERMOCOUPLE METER is well suited and widely used for radio-frequency current measurements because the torque is unidirectional and proportional to the effective r-f current. It employs the very sensitive type of d-c D'Arsonval meter movement with modifications in the permanent magnet field to provide a more uniform scale for radio-frequency measurements.

QUIZ

1. What instrument is used to measure the power consumed in an electric circuit or equipment?
2. What instrument is used to measure the energy consumed in an electric circuit or equipment?
3. What is the name of the basic meter movement used in most d-c voltmeters, ammeters, and ohmmeters?
4. What three functions are performed by the phosphor bronze ribbon in a D'Arsonval galvanometer?
5. Give the formula for the force in pounds acting on a conductor in terms of conductor current in amperes, conductor length in inches, and the strength of the field in which the conductor is located in lines per square inch.

6. In figure 9–2, A:
   (a) If the direction of the conductor current is reversed, what will be the direction of the force acting on the conductor?
   (b) If the direction of the conductor current is reversed, and the magnetic polarities are also reversed, what will be the direction of the force acting on the conductor?
7. What is the function of the soft-iron core in the D'Arsonval meter movement?
8. How is damping accomplished in the D'Arsonval meter movement?
9. What three design factors have contributed to the increase in sensitivity of the D'Arsonval meter?
10. What is the function of a shunt when used with a d-c ammeter?
11. What is the effect on the resistance of an ammeter shunt when its temperature is increased?
12. What is the voltage drop across a shunt when the meter to which it is connected has a resistance of 50 ohms and the meter coil current for full-scale deflection is 1 milliampere?
13. Find the resistance of a 500-ampere shunt to be used with a 100-microampere, 100-ohm meter movement.
14. Find the resistance of a 10-ampere shunt to be used with a 50-ohm, 1-milliampere meter movement.
15. Why is an ammeter never connected in shunt with the load, but always in series with it?
16. What is the relative resistance of a 100-volt voltmeter compared to that of a 100-ampere ammeter?
17. Approximately how much resistance in K-ohms should be connected in series with a 1-milliampere, 50-ohm meter movement to permit a full-scale deflection on a 150-volt circuit?
18. Approximately how much resistance in megohms should be connected in series with a 100-ohm, 50-microampere movement to permit a full-scale deflection on a 250-volt circuit?
19. How much resistance in ohms should be connected in series with a 50-ohm, 1-milliampere movement to permit a full-scale deflection on 15 volts?
20. When voltage measurements are made in high resistance circuits, (a) what should be the relative resistance of the voltmeter? (b) why?
21. What is the sensitivity of a voltmeter having a full-scale deflection of 300 volts when the meter current is 50 microamperes?
22. What is the sensitivity of a 10-microampere meter movement in ohms per volt?

23. What is the relative sensitivity of an electrodynamometer-type voltmeter compared to a D'Arsonval-type voltmeter?
24. How are the current coils of a wattmeter connected with respect to the source and load?
25. How are the potential coils of a wattmeter connected with respect to the source and the load?
26. In figure 9–10, A, what is the effect on the ohmmeter reading of a weak battery?
27. In figure 9–10, B, what is the function of the variable resistor?
28. What device is used to measure the insulation resistance of cables, electrical machinery, bushings, and other types of equipment?
29. What voltage source is used in most meggers?
30. What effect does a small variation in voltage have on the indication of the resistance being measured by a megger?
31. (a) What is the voltage rating of most meggers provided aboard ship?
    (b) To extend the range, what change in voltage is necessary?
32. What is the function of the stationary coil of a moving-vane meter?
33. What is the relative sensitivity of a moving iron-vane type voltmeter compared to the D'Arsonval-type voltmeter that has the same range of voltage?
34. What type of meter is used to measure radio-frequency currents?

CHAPTER 10

DIRECT-CURRENT GENERATORS

INTRODUCTION

Electric circuits that require larger amounts of power than can be supplied by batteries derive their power from rotating electrical machines called GENERATORS. For example, multiple electric lights and heavy motors require larger voltages and currents than can be supplied by batteries of practical size. As a result, electric generators comprise the principal source of electric power for all industry and also the principal source of electric power aboard ship.

Generators can be designed to supply small amounts of power or they can be designed to supply many thousands of kilowatts of power. Also generators may be designed to supply direct current or alternating current. Direct-current generators are described in this chapter. Alternating-current generators are described in chapter 14.

A d-c generator is a rotating machine that converts mechanical energy into electrical energy. This conversion is accomplished by rotating an armature, which carries conductors, in a magnetic field, thus inducing an emf in the conductors. As stated before, in order for an emf to be induced in the conductors, a relative motion must always exist between the conductors and the magnetic field in such a manner that the conductors cut through the field. In most d-c generators the armature is the rotating member and the field is the stationary member. A mechanical force is applied to the shaft of the rotating member to cause the relative motion. Thus when mechanical energy is put into the machine in the form of a mechanical force or twist on the shaft, causing the shaft to turn at a certain speed, electrical energy in the form of voltage and current is delivered to the external circuit.

It should be understood that mechanical power must be applied to the shaft constantly so long as the generator is supplying electrical energy to the external electric circuit.

The power source used to turn the armature is commonly called a PRIME MOVER. Many forms of prime movers are in use, such as steam turbines, diesel engines, gasoline engines, and steam engines. However, in the Navy, the principal sources of power aboard ship are steam turbines and diesel engines. Also, auxiliary power units containing d-c generators driven by gasoline engines are widely used in naval maintenance operations.

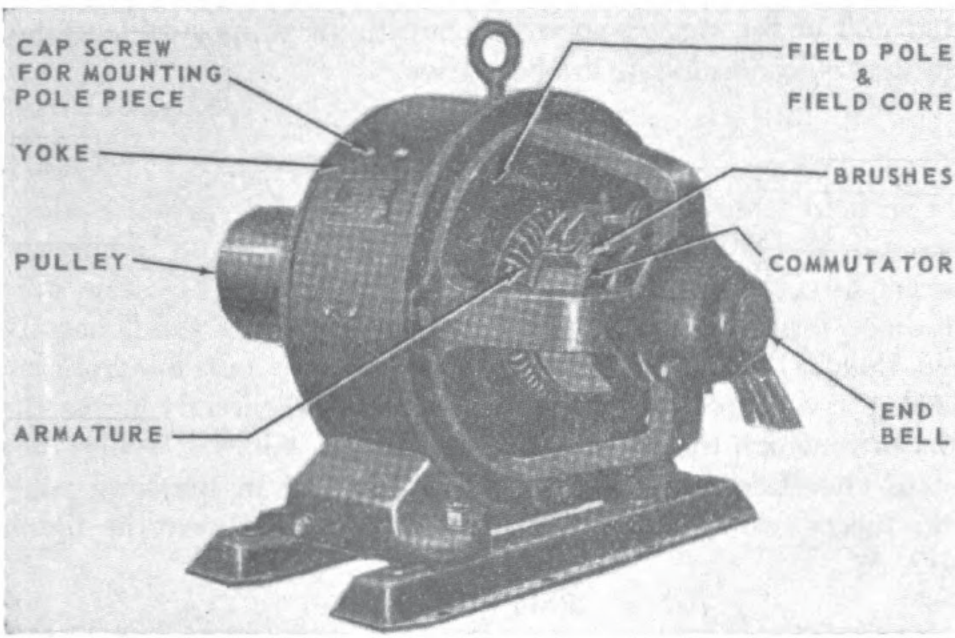


Figure 10–1.—D-c generator.

In naval aviation, aircraft engines function as prime movers for both a-c and d-c generating equipment.

A d-c generator (fig. 10–1) consists essentially of (1) a steel frame or yoke containing the pole pieces and field windings, (2) an armature consisting of a group of copper conductors mounted in a slotted cylindrical core, made up of thin steel disks called LAMINATIONS, (3) a commutator for maintaining the current in one direction through the external circuit, and (4) brushes with brush holders to carry the current from the commutator to the external circuit.

CONSTRUCTION

Frame

The d-c generator field frame, or yoke, is usually made of steel. The use of steel reduces the reluctance of the magnetic circuit to a low value and therefore reduces the necessary size of the field windings. The frame provides a mechanical support for the pole pieces and serves as a portion of the magnetic circuit to provide the necessary flux across the air gap.

The end bells are bolted to the frame structure and support the armature shaft bearings. One end bell supports the brush rigging and extends over the commutator. The pulley on the other end of the shaft is mounted outside its supporting end bell in order to accommodate the belt drive.

Field Windings

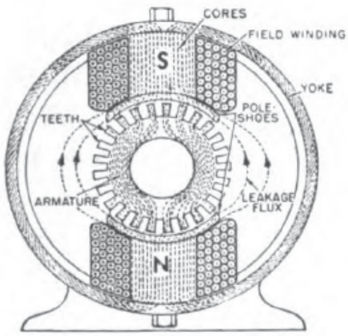
The field windings are connected so that they produce alternate north and south poles (fig. 10–2, A and F) to obtain the correct direction of emf in the armature conductors. The field windings form an electromagnet which establishes the generator field flux. These field windings may receive current from an external d-c source or they may be connected directly across the armature, which then becomes the source of voltage. When they are so energized they establish magnetic flux in the field yoke, pole pieces, air gap, and armature core, as shown in figure 10–2, A.

Pole Pieces

The pole pieces, which support the field windings, are mounted on the inside circumference of the yoke, with cap screws that extend through the frame (fig. 10–1). These pole pieces are usually built of sheet steel laminations riveted together. The pole faces are shaped to fit the curvature of the armature, as shown in figure 10–2, A. The preformed field coil extending shown in figure 10–2, B is mounted on the laminated core from the back. The coil is held securely in place between the frame and the flanged end of the pole.

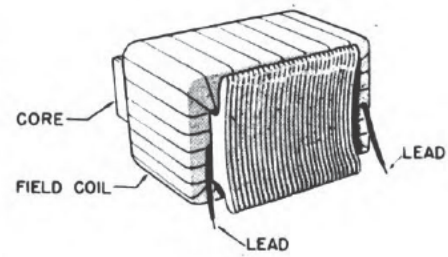
Armature

The armature (fig. 10–2, C) is mounted on a shaft and rotates through the field. If the output of the armature is connected



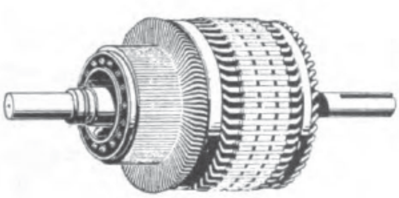
MAGNETIC CIRCUIT OF
A 2-POLE GENERATOR

A



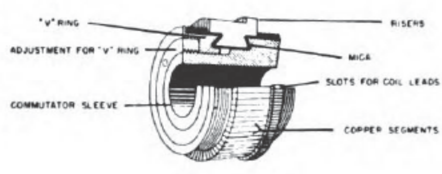
FIELD COIL ON POLE PIECE

B



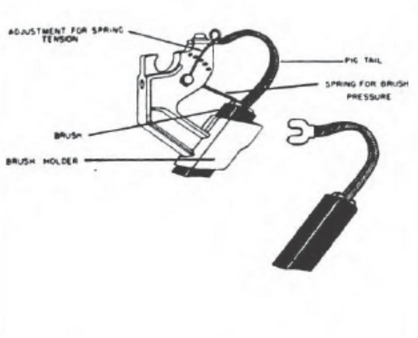
A COMPLETED ARMATURE

C



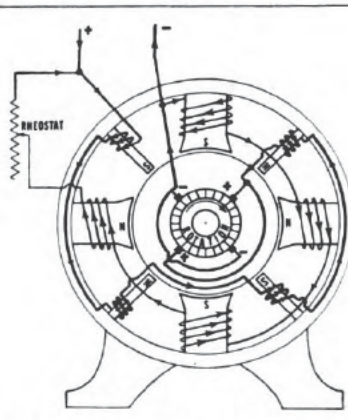
COMMUTATOR CONSTRUCTION

D



TYPICAL PIGTAIL BRUSH
AND HOLDER

E



SCHEMATIC WIRING DIAGRAM
OF SHUNT GENERATOR

F

Figure 10–2.—D-c generator parts.

across the field windings, the voltage and the field current at start, will be small because of the small residual flux in the field poles. However, as the generator continues to run, the small voltage across the armature will circulate a small current through the field coils and the field will become stronger. This action causes the generator voltage to rise to the proper value and the machine is said to "build up" its voltage.

The armature core is made of sheet steel laminations. In small machines these laminations are keyed directly to the shaft. In large machines the laminations are assembled on a spider which is keyed to the shaft. The outer surface of this cylindrical core is slotted to provide a means of securing the armature coils. Radial ventilating ducts are provided in the core by inserting spacers between the laminations at definite intervals. These ducts permit air to circulate through the core and to carry off heat produced in the armature winding and core. The armature coils on most generators are form-wound to the correct size and shape. Additional insulation between the core and windings is obtained by placing sheet insulation in the slots. The windings are secured by fiber wedges driven into the tops of the slots. The free ends of the armature coils are connected to the commutator, as indicated in figure 10–2, C. The entire armature winding forms a closed circuit.

Commutator

The commutator consists of a number of wedge-shaped segments, or bars, of hard drawn copper that are assembled into a cylinder and held together by flanges, as shown in figure 10–2, D. The commutator segments are insulated from each other by sheet mica and the entire commutator is insulated from the supporting rings on the shaft by mica collars. Because the brushes bear on the outside surface of the commutator, better brush contact, less sparking, and less noise, are obtained by undercutting the mica to about $\frac{1}{64}$ inch below the level of the commutator surface. The voltage and the number of poles in the generator determine the number of commutator segments. To prevent flashover between segments, generators are designed to have an average voltage not to exceed 15 volts between adjacent segments. Therefore a high-voltage generator requires more commutator segments than a low-voltage generator.

Brushes and Brush Holders

The brushes carry the current from the commutator to the external circuit. They are usually made of a mixture of carbon and graphite. For low-voltage machines the brushes are made of a mixture of graphite and a metallic powder. The brushes must be free within limits to slide in their holders (fig. 10–2, **E**) so that they may follow any small irregularities in the curvature of the commutator. However, excessive play not only would encourage brush vibration but also might cause misalignment of the brush with the axis of commutation. This would result in excessive sparking.

The proper pressure of the brushes against the commutator is maintained by means of springs and should be from 1½ to 2 pounds per square inch. A low resistance connection between the brushes and brush holders is maintained by means of braided copper wires, or pigtails, that are attached to each brush holder and to each brush.

The brush holders, which are attached to brush studs, hold the brushes in their proper positions on the commutator. The brush studs are fastened to a rocker arm, or brush holder yoke, that is attached to the frame. Multipolar generators usually have as many brush studs as there are main poles. The brush studs are of alternate positive and negative polarity and those of like polarity are connected together as indicated in figure 10–2, **F**.

ARMATURE WINDINGS

Simple Coil Armature

The simplest generator armature winding is a loop or single coil. Rotating this loop in a magnetic field will induce an emf whose strength is dependent upon the strength of the magnetic field and the speed of rotation of the conductor.

A single-coil generator with each coil terminal connected to a bar of a 2-segment metal ring is shown in figure 10–3. The two segments of the split ring are insulated from each other and the shaft, thus forming a simple commutator. The commutator mechanically reverses the armature coil connections to the external circuit at the same instant that the direction of the generated voltage reverses in the armature coil. This action involves the

process known as commutation, which is described later in the chapter.

When the coil rotates clockwise from the position shown in figure 10–3, A, to the position shown in figure 10–3, B, an emf is generated in the coil in the direction (indicated by the heavy arrows) that deflects the galvanometer to the right. Current flows out of the negative brush, through the galvanometer, and back to the positive brush to complete the circuit through the

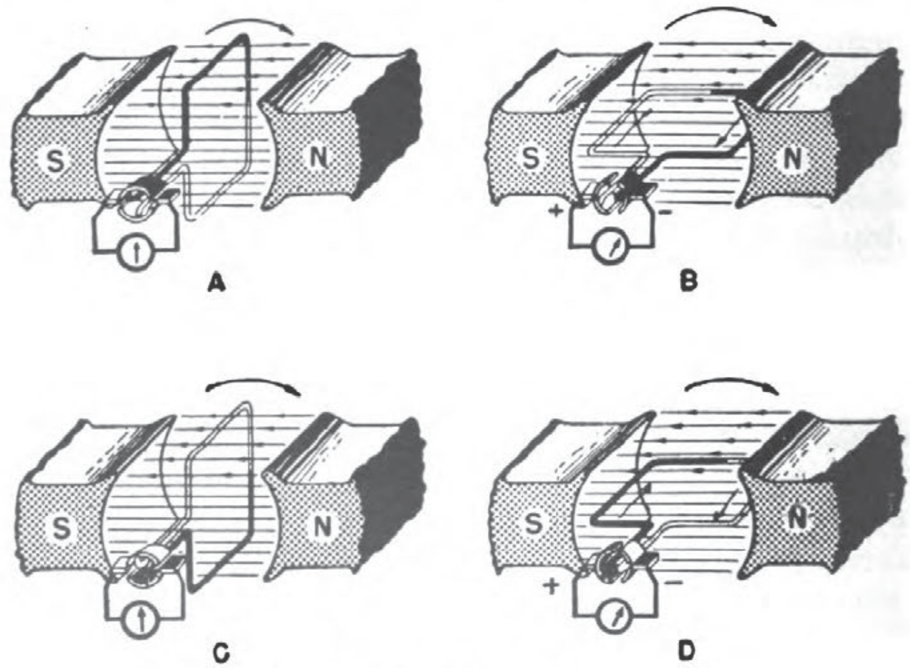


Figure 10–3.—Single-coil generator with commutator.

armature coil. If the coil is rotated to the position shown in figure 10–3, C, the generated voltage and the current fall to zero, as in figure 10–3, A. At this instant the brushes make contact with both bars of the commutator and short-circuit the coil. As the coil moves to the position shown in figure 10–3, D, an emf is generated again in the coil but of opposite polarity.

The emf's generated in the two sides of the coil shown in figure 10–3, D, are in the reverse direction to that of the emf's shown in figure 10–3, B. Since the bars of the commutator have rotated with the coil and are connected to opposite brushes, the direction of the flow of current through the galvanometer remains the same. The emf developed across the brushes is pul-

sating and unidirectional, varying twice during each revolution between zero and maximum.

Figure 10–4 is a graph of the pulsating direct emf for one revolution of a single-loop 2-pole armature. A pulsating direct voltage of this characteristic (called RIPPLE) is unsuitable for most applications. Therefore, in practical generators more coils and more commutator bars are used to produce an output voltage waveform with less ripple.

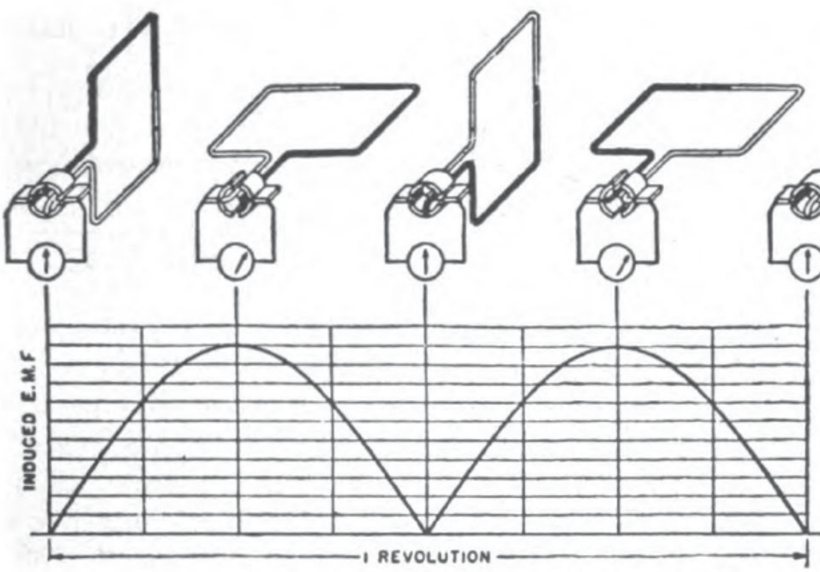


Figure 10–4.—Voltage from a single-coil armature.

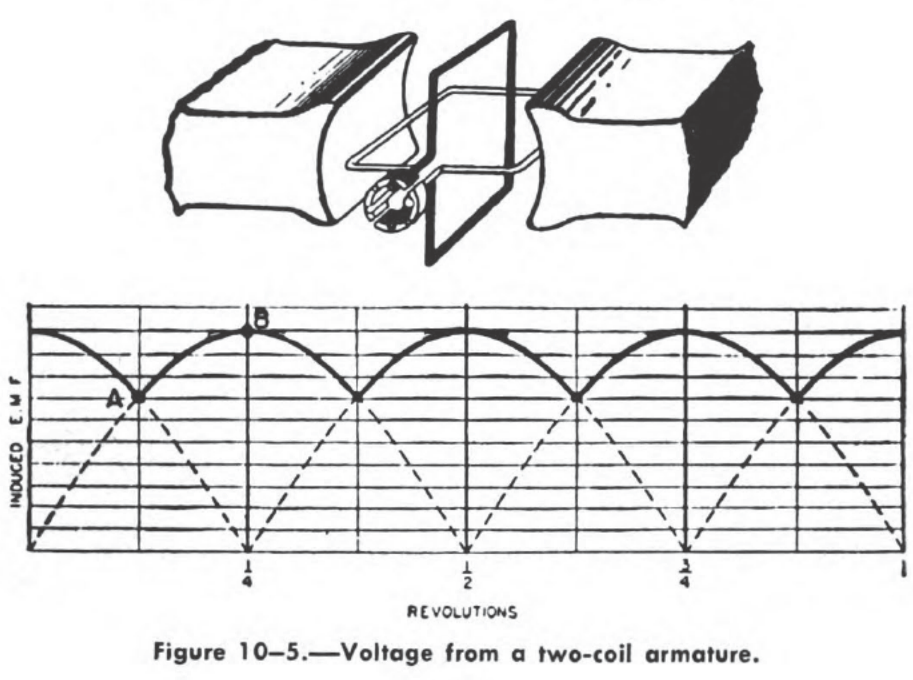


Figure 10–5.—Voltage from a two-coil armature.

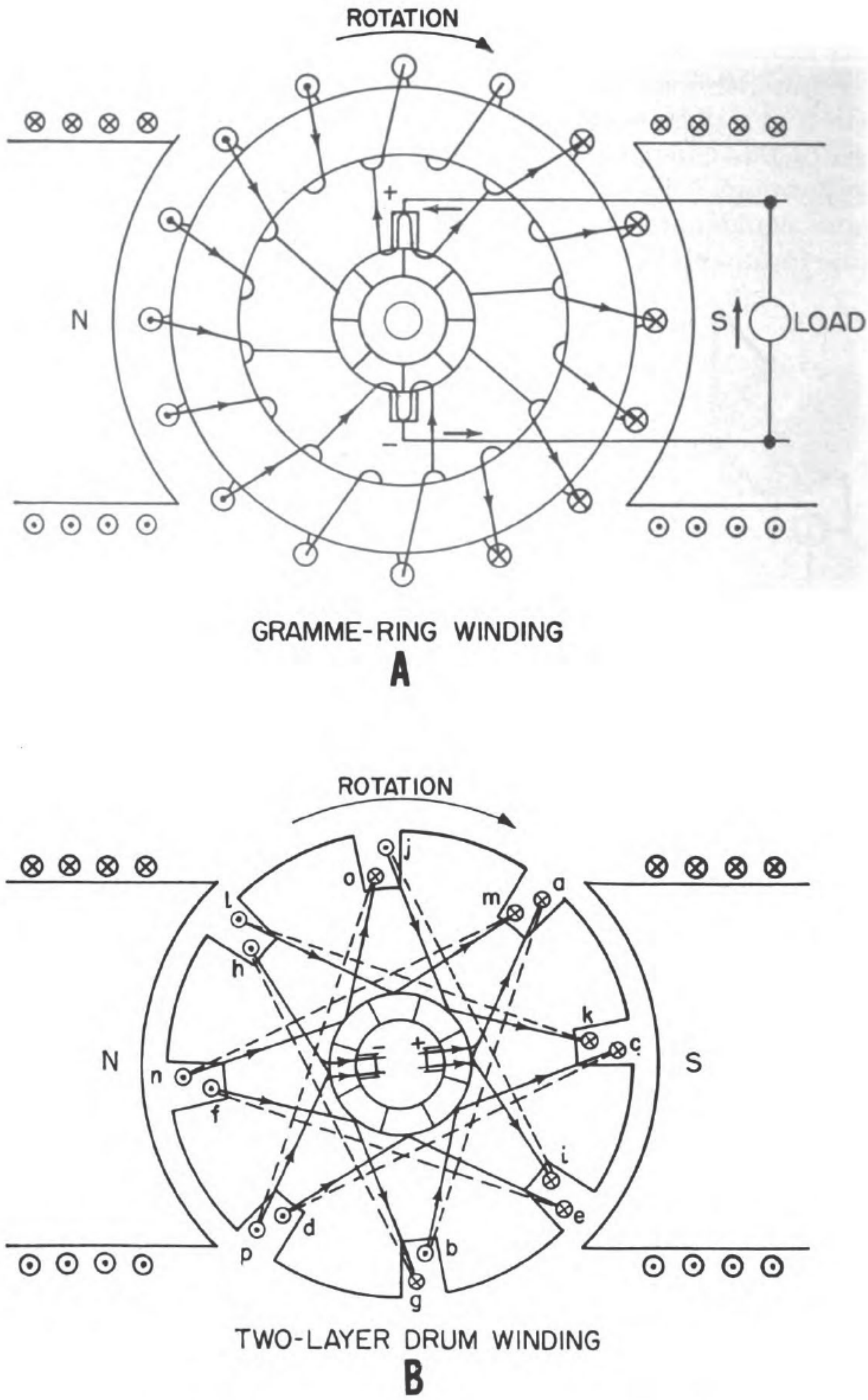


Figure 10–6.—Basic d-c generator armature windings.

Effect of More Coils

Figure 10–5 shows the reduction in ripple component of the voltage obtained by the use of two coils instead of one. Since there are now four commutator segments but only two brushes in the commutator, the voltage cannot fall any lower than point A; therefore, the ripple is limited to the rise and fall between points A and B. By adding still more armature coils, the ripple voltage can be reduced still more.

Gramme-Ring Winding

A Gramme-ring winding (fig. 10–6) is formed by winding insulated wire around a hollow iron ring and tapping it at regular intervals to the commutator bars. It is a basic winding and can be adapted to any number of poles. Voltage limitations, however, should not be exceeded. The portions of the windings on the inside of the ring cut practically no flux and act as connectors for the active portions of the conductors which lie on the outer surface of the ring. Because only a small fraction of the conductor is used to generate voltage in such a winding, a relatively large amount of copper is required to produce a given voltage. The schematic wiring diagram of a Gramme-ring winding is frequently used as a simplified equivalent circuit for the practical but more complex drum armature wiring diagrams. The Gramme-ring winding is used in this discussion to simplify the drum winding circuits.

In the 2-pole winding of figure 10–6, A, there are two parallel paths for armature current between the brushes. One path is through the coils on the left side and the other is through the coils on the right side. The voltage between the brushes is the vector sum of the voltages generated in all the coils of each path. No circulating current flows between the two paths because the generated voltages in the two paths are equal and in opposition. The polarity of the brushes may be determined by the left-hand rule for generator action. The negative terminal is the one from which electrons flow out to the load; the positive terminal is the one to which the electrons return from the load.

Drum Armature

In the basic drum winding shown in figure 10–6, B, all the conductors lie in slots near the surface of the armature. The arma-

ture conductors are indicated in the figure as circles. Those on the left contain a dot to indicate that the direction of generated voltage is toward the observer; those on the right contain an x to show that the direction of the generated voltage is away from the observer. The dotted lines connecting the circles represent end connections on the back of the armature between the two halves of each coil. The solid lines between the commutator and the armature conductors represent the end connections on the front of the armature between the coils and commutator segments.

There are 8 coils, 8 slots, and 8 commutator segments in this simplified basic drum winding. One half of a coil lies in the upper portion of a slot and the other half of the coil lies in the lower portion of a slot that is approximately halfway around the armature. This arrangement permits the generated voltages in the two halves of a coil to be a maximum at almost the same instant, because the armature is rotating in a 2-pole field. The polarity of the brushes may be determined by the left-hand rule for generator action. The brushes are shown on the inside of the commutator surface to simplify the drawing. Normally, the brushes contact the outside surface of the commutator.

In drum armatures, with the exception of the coil-end connections, all of the copper is used to generate the emf. The distance between the two sides of a coil is known as the COIL PITCH. This distance should be about the same as the distance between the centers of adjacent poles as mentioned previously.

The following analysis is made for no-external-load condition. There are two paths for armature current through the drum winding—one through conductors *a, b, c, d, e, f, g,* and *h;* the other through conductors *i, j, k, l, m, n, o,* and *p.* The generated voltages in all conductors in the first path are in the same direction—from the positive brush to the negative brush. The generated voltages in all conductors in the second path are also all in the same direction—from the positive brush to the negative brush. The two paths form a closed circuit and the voltages of each path are equal and in opposition with respect to each other. Therefore no circulating current will flow.

Certain similarities exist in the Gramme-ring winding (fig. 10–6, A) and the drum winding (fig. 10–6, B). The generated voltage distribution is the same. The number of paths for load current is the same. In both types, when a brush contacts two

segments, a coil is short-circuited. In both types, the brushes are positioned so that only coils that are moving approximately parallel to the field are periodically short-circuited. In both types, there are 8 commutator bars and 2 brushes and the armatures are wound for 2 poles. Also, in both types there are single-turn coils and 16 active conductors in which the voltage of the machine is generated.

The differences that exist in the two types of armatures are in the position of the brush axis, the method of mounting the coils, and the path for the magnetic field flux. In the Gramme-ring winding the brush axis is perpendicular to the field axis. In the drum winding the brush axis coincides with the field axis. In the Gramme-ring armature the coils are wound on a ring. In the drum type they are preformed and inserted in slots. In the Gramme-ring type the magnetic field crosses the air gap and is confined to the ring. In the drum type the field crosses the air gap and permeates the entire cross section of the armature including the outer slotted portion as well as the inner section. Finally, the end connections form a much higher percentage of the total winding in the Gramme-ring type than in the drum type. It is principally for this reason the Gramme-ring armatures have been replaced by the drum-type armature.

Simplex-Lap Winding

As mentioned previously, direct-current armatures are generally wound with preformed coils, as shown in figure 10–7. The term "span" is the distance from one winding element of a coil to the other winding element of the same coil and is usually given in terms of the number of slots included between them. A winding element is a coil side consisting of one or more active (face) conductors in series and taped together to form that part of the coil that is inserted in the armature slot. The span of the armature coil should be about equal to the peripheral distance between the centers of adjacent poles so that the voltages generated in the two sides of the coil will be in series addition. This distance is called POLE PITCH. When the span of a coil is less than the pole pitch the winding is called a FRACTIONAL-PITCH WINDING. A fractional pitch coil can be as low as 0.8 pole pitch. Fractional pitch coils have reduced emf because the coil side voltages do not

reach their maximum values at the same time. Ordinarily, the reduction in emf is small but the savings in copper, which results by using shorter end-connections, warrants the loss.

Direct-current armature windings are generally 2-layer windings. In this arrangement, the coils are placed on the armature

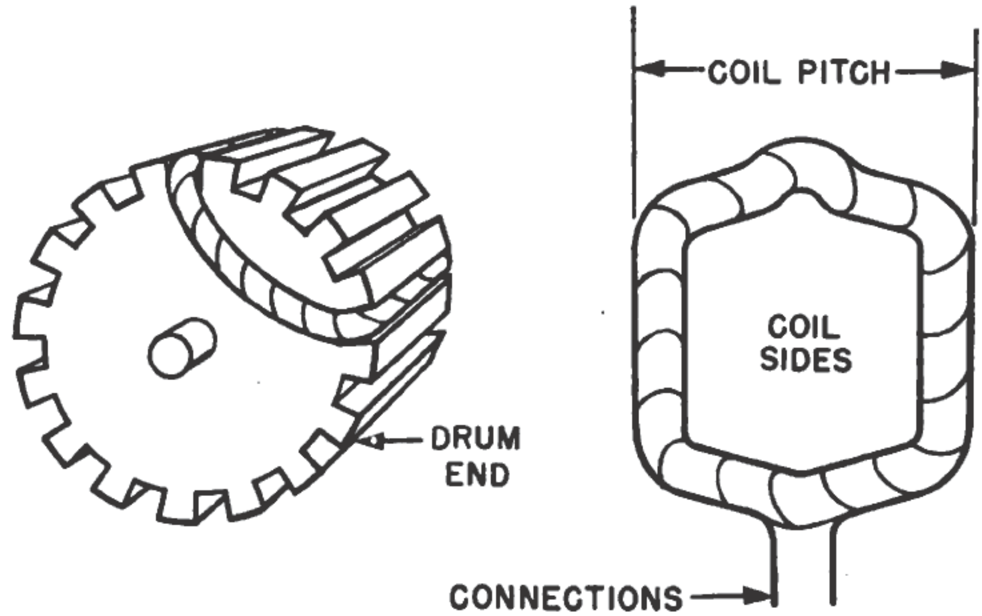


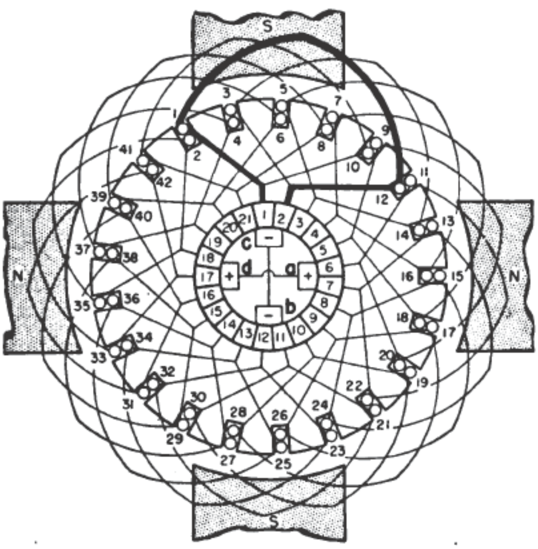
Figure 10–7.—Formed armature coil for 4-pole armature.

with one side of each coil occupying the top of one slot and the other side occupying the bottom of another slot. The distance between the slots is approximately 1 pole span. Such an arrangement allows the windings to fit readily on the armature with an equal number of coils and slots. Thus each slot contains two layers of conductors in which voltages are generated as the armature rotates through the field. This arrangement is shown in the 2-layer drum winding of figure 10–6, B.

In simplex-lap windings, groups of coils under similar pairs of poles at any instant generating equal voltages are connected in parallel. A 4-pole simplex-lap winding is shown in figure 10–8. Starting with commutator bar 1, the circuit may be traced through the heavy black coil to the adjacent commutator bar 2. The trace may be continued through successive coils until the entire armature circuit has been traced from one end to the other. Upon

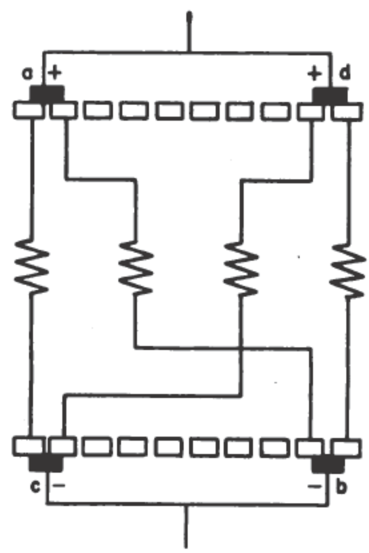
reentering the starting commutator bar 1, the trace is completed. Thus the circuit is seen to be a closed-circuit winding.

There are four groups of coils generating the same voltage between brushes of opposite polarity in the example shown in figure 10–8. One group consists of coils occupying winding spaces



ARMATURE WINDING

A



SIMPLIFIED CIRCUIT

B

Figure 10–8.—Simplex-lap 4-pole armature winding.

1, 12, 3, 14, 5, 16, 7, 18, 9, and 20. A second group consists of coils occupying winding spaces 11, 22, 13, 24, 15, 26, 17, 28, 19, and 30. A third group consists of coils occupying winding spaces 21, 32, 23, 34, 25, 36, 27, 38, 29, and 40. The fourth group consists of coils occupying winding spaces 31, 42, 33, 2, 35, 4, 37, 6, 39, 8, 41, and 10. These four groups of coils are placed in parallel by connecting the two positive brushes together and the two negative brushes together as indicated in the schematic diagram of figure 10–8, B. No circulating current flows between these four parallel paths when no load is connected between the positive and negative brushes, because the voltage of each of the groups is equal and of opposite polarity to the voltages of the other groups.

A simplified schematic of the 4-pole simplex-lap armature winding is shown in figure 10–9. Connecting a load between the positive and negative brushes causes current to flow through the

armature in these four paths. The current is distributed equally in the four paths. Thus if the total output current of the armature is 400 amperes, each path through the armature will carry 100 amperes. In this winding there are as many parallel paths for current through the armature as there are poles in the field.

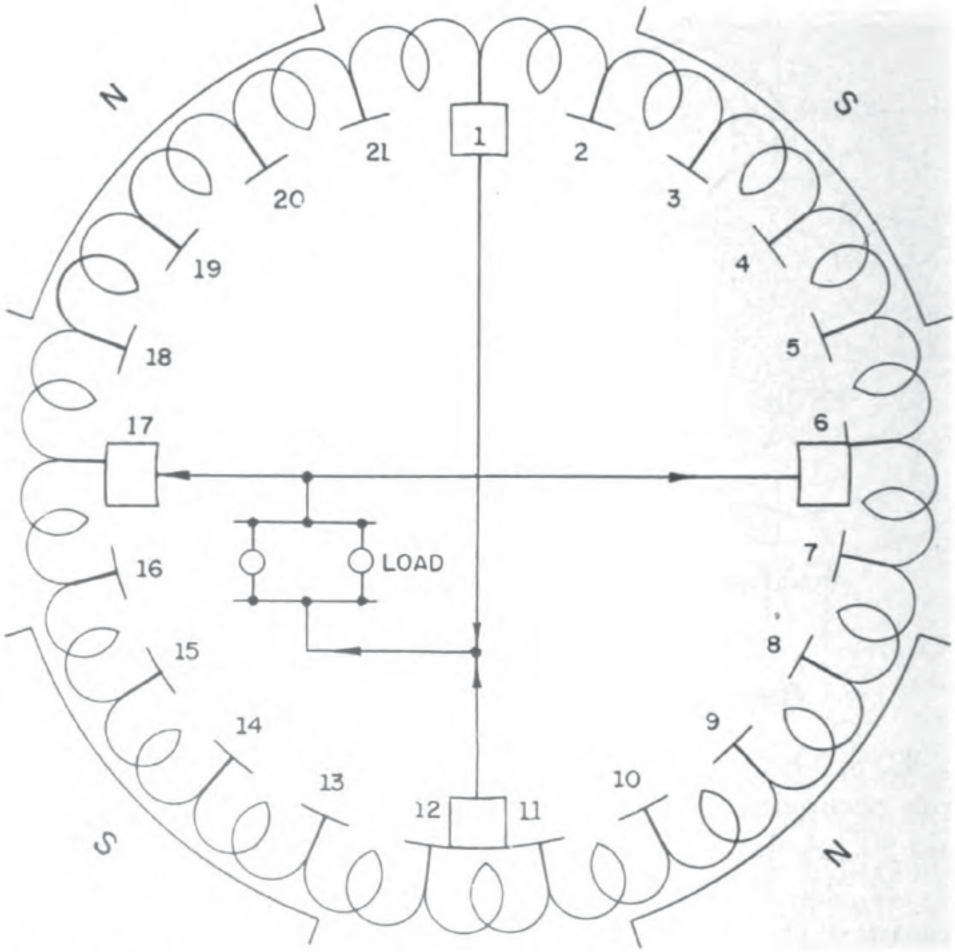


Figure 10–9.—Simplified schematic of simplex-lap 4-pole armature winding.

The simplex-lap winding described in this chapter is the simplest type of lap winding. It is identified as a single winding to distinguish it from more complex double and triple lap windings. It is single-reentrant—that is, the winding closes on itself at the end of one complete turn around the armature. This arrangement distinguishes the winding from more complex types that reenter upon themselves after one, two, or three complete turns around the armature. The more complex lap windings are described in the rating texts.

The characteristics of the (single) simplex-lap winding may be summarized as follows:

1. There are as many paths for current as there are field poles.
2. There are as many brush positions on the commutator as there are field poles.
3. The two ends of an armature coil connect to adjacent commutator segments.
4. The armature winding is close-circuited.
5. The armature winding is used for relatively high-current low-voltage loads.
6. The voltage generated between positive and negative brushes is

$$E=\frac{\Phi ZN}{10^8},$$

where E is the generated emf in volts, Z the number of armature face conductors, Φ the lines of magnetic flux per pole, and N the armature speed in revolutions per second.

A face conductor lies in a slot and generates a voltage as the armature rotates. An armature coil includes one or more turns, each of which consists of two face conductors and their respective end connections. For example, a 4-pole d-c generator with a simplex-lap armature winding having 10^6 lines of magnetic flux per pole, 440 armature conductors, and a speed of 50 revolutions per second (3,000 rpm) has a generated voltage of

$$E=\frac{10^6\times 440\times 50}{10^8}=220 \text{ volts.}$$

If the load current is 1,000 amperes, each armature coil will carry $\frac{1,000}{4}$, or 250 amperes.

Simplex-Wave Winding

In single simplex-wave winding (fig. 10–10) groups of coils under similar pairs of poles at any instant are generating equal voltages and are connected in series. This winding is also called a SERIES WINDING. Each coil has its ends connected to the commutator bars that are two pole spans apart. This distance is measured around the armature circumference from the center of

one field pole to the center of the next pole of like sign. In this example, two pole spans are halfway around the commutator; in a 6-pole machine, two pole spans are one-third the way around; and in an 8-pole machine, one-fourth the way around. In the single simplex-wave winding there are as many coils connected in series between adjacent commutator bars as there are pairs of poles in the field. In this example there are two pairs of poles, so there are two coils in series between adjacent commutator bars. This arrangement may be seen by starting at segment 1 and tracing through the circuit of the two coils occupying winding spaces 1, 10, 17, and 26, and ending at commutator segment 2.

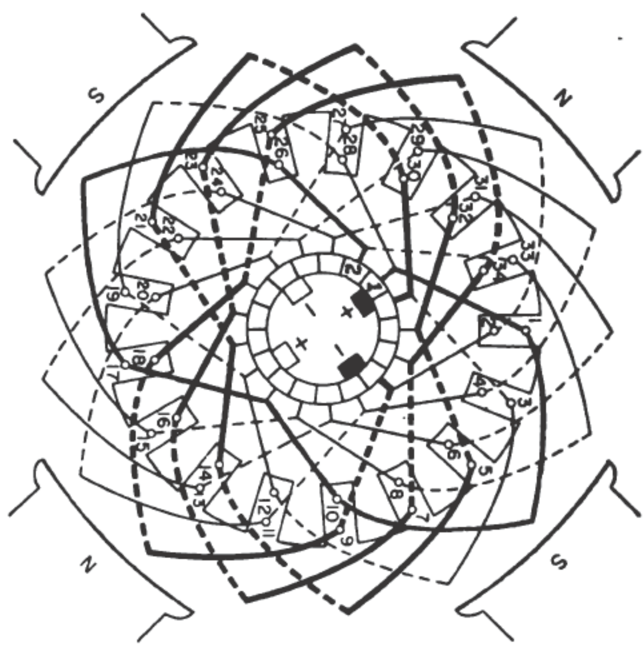


Figure 10–10.—Single simplex-wave winding.

The single simplex-wave winding is a closed-circuit winding as may be seen by starting with one coil and tracing through the entire armature circuit to complete the trace by ending at the starting position. In the single simplex wave-wound armature there are two paths for current regardless of the number of poles for which it is wound and regardless of the number of brushes. Starting with the upper positive brush, one path includes the armature conductors that occupy winding spaces 1, 10, 17, 26, 33, 8, 15, 24, 31, 6, 13, 22, 29, 4, 11, 20, 27, and 2, returning to the upper negative brush. The other path from the upper positive brush includes the armature conductors that occupy winding

spaces 28, 19, 12, 3; 30, 21, 14, 5, 32, 23, 16, 7, 34, 25, 18, and 9, returning to the upper negative brush.

Although four brushes are shown, only two are required. The full voltage of the winding is developed across the upper brushes, and the full voltage is also developed across the lower brushes. The load current divides equally between the two pairs of brushes when they are connected in parallel because the resistance of the two paths through the armature is the same. For example, if the armature has a 20-ampere load at 220 volts the upper brushes alone will carry 20 amperes at 220 volts. Adding the lower brushes will reduce the current to 10 amperes per brush. If the commutator is designed for four brushes, the removal of one pair will overload the remaining brushes unless the load is halved. However, where commutators are inaccessible, wave windings permit the use of two brush positions irrespective of the number of poles, so that servicing the brushes and commutator is facilitated.

The characteristics of the simplex-wave winding may be summarized as follows:

1. There are two parallel paths irrespective of the number of field poles.
2. There is a minimum of two brush positions irrespective of the number of field poles.
3. The two ends of a coil connect to commutator segments that are two pole spans apart (fig. 10–10).
4. The winding has relatively high voltage and low current.
5. The winding is closed-circuited.
6. There are as many coils in series between adjacent commutator segments as there are poles in the field.
7. The voltage generated between positive and negative brushes is

$$E=\frac{\Phi ZNP}{10^8},$$

where P is the number of pairs of field poles and the other symbols are the same as those used in the equation for generated voltage in a simplex-lap winding.

For example, a 6-pole d-c generator having a simplex-wave armature winding, 1.05×10^6 lines of flux per pole, 3,500 arma-

ture conductors, and a speed of 10 revolutions per second (600 rpm) has a generated voltage of

$$E=\frac{\Phi ZNP}{10^8}=\frac{1.05\times10^6\times3{,}500\times10\times3}{10^8}=1{,}100 \text{ volts.}$$

If the armature supplies 200 amperes to a load, each armature coil will carry $\frac{200}{2}=100$ amperes.

ARMATURE LOSSES

There are three losses in every d-c generator armature. These are (1) the I^2R, or copper loss in the winding, (2) the eddy current loss in the core, and (3) the hysteresis loss due to the friction of the revolving magnetic particles in the core.

Copper Losses

The copper loss is the power lost in heat in the windings due to the flow of current through the copper coils. This loss varies directly with the armature resistance and as the square of the armature current. The armature resistance varies with the length of the armature conductors and inversely with their cross-sectional area.

Armature conductor size is based on an allowance of from 300 to 1,200 circular mils per ampere. For example, a 2-pole armature that is required to supply 100 amperes may use a wire size based on 800 circular mils per ampere, or $\frac{100}{2}\times800=40{,}000$ circular mils. This value coresponds to a No. 4 wire. Very small armature windings may use only 300 circular mils per ampere with a resulting high current density. Large generators (5,000 kw) require an allowance of 1,200 circular mils per ampere with a resulting low current density in the windings. These variations are the result of the variable nature of the heat-radiating ability of the armature conductors.

Very small round conductors have a much higher ratio of surface to volume than do large round conductors. For example,

a 0.1-inch diameter round conductor of a given length has a surface-to-volume ratio of

$$\frac{4\pi D}{\pi D^2}, \text{ or } \frac{4}{0.1}=40.$$

A 1.0-inch diameter conductor of the same length has a surface-to-volume ratio of $\frac{4}{1}$, or 4. Since the heat-radiating ability of a round conductor varies as the ratio of its surface to volume, the 0.1-inch diameter conductor has $\frac{40}{4}$, or 10 times the heat radiating ability of the 1-inch diameter conductor, other factors being equal.

High-speed generators use a lower circular-mil-per-ampere allowance than low-speed generators because of better cooling. The temperature rise is limited by ventilating ducts, and in some cases by the use of forced ventilation with blowers.

The hot resistance of an armature winding is higher than its cold resistance. A 2.5° centigrade increase in temperature of a copper conductor corresponds to an increase in resistance of approximately 1 percent. For example, if the no-load temperature of an armature winding is 20° C. and its full-load temperature is 70° C., the increase in resistance is $\frac{70-20}{2.5}$, or 20 percent. Thus, if the no-load resistance is 0.05 ohm between brushes, the hot resistance will be 1.2×0.05, or 0.06 ohm. If the full-load armature current is 100 amperes, the full-load armature copper loss will be $(100)^2 \times 0.06$, or 600 watts. The armature copper loss varies more widely with the variation of electrical load on the generator than any other loss occurring in the machine. This is because most generators are constant-potential machines supplying a current output that varies with the electrical load across the brushes. The limiting factor in load on a generator is the allowable current rating of the generator armature.

The armature circuit resistance includes the resistance of the windings between brushes of opposite polarity, the brush contact resistance, and the brush resistance.

Eddy-Current Losses

If a d-c generator armature core were made of solid iron (fig. 10–11, A) and rotated rapidly in the field, excessive heating

would develop even with no-load current in the armature windings. This action would be the result of a generated voltage in the core itself. As the core rotates, it cuts the lines of magnetic field flux at the same time the copper conductors of the armature cut them. Thus, induced currents alternate through the core, first in one direction and then in the other, with accompanying generation of heat.

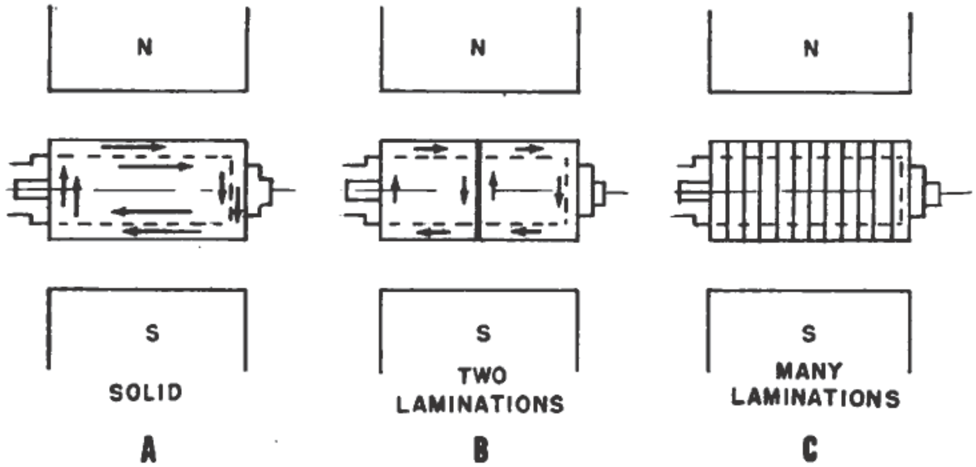


Figure 10–11.—Eddy currents in d-c generator armature cores.

These induced currents are called EDDY CURRENTS. They are kept to a low value by sectionalizing (laminating) the armature core. For example, if the core is split into two equal parts (fig. 10–11, B) and these parts are insulated from each other, the voltage induced in each section of iron is halved and the resistance of the eddy-current paths is doubled (resistance varies inversely with the cross-sectional area). If 10 volts is induced in the core (fig. 10–11, A) and the resistance of the path for eddy currents is 1 ohm, the eddy-current loss is $\frac{E^2}{R}$, or $\frac{10^2}{1}=100$ watts. The voltage in each section (fig. 10–11, B) will be $\frac{10}{2}$, or 5 volts, because the length is halved and the resistance of the eddy-current path for each section is 2 ohms. The loss in each section is $\frac{E^2}{R}$, or $\frac{5^2}{2}=12.5$ watts, and the total loss in both sections is 12.5×2, or 25 watts. This value represents one-fourth of the power loss in figure 10–11, A.

Reducing the thickness of the core to one-half its original value reduces the loss to one-fourth of the original loss. Thus, eddy-current losses vary as the square of the thickness of the core laminations.

If the armature core is sufficiently subdivided into multiple sections or laminations (fig. 10–11, C), the eddy-current loss can be reduced to a negligible value. Reducing the thickness of the laminations reduces the magnitude of the induced emf in each section and increases the resistance of the eddy-current paths. Laminations in small generator armatures are $\frac{1}{64}$ inch thick. The laminations are insulated from each other by a thin coat of lacquer or in some instances simply by the oxidation of the surfaces due to contact with the air while the laminations are being annealed. The insulation need not be high because the voltages induced are very small.

All electrical rotating machines are laminated to reduce eddy-current losses. Transformer cores are laminated for the same reason.

The eddy-current loss is also influenced by speed and flux density. Because the induced voltage, which causes the eddy currents to flow, varies with the speed and flux density, the power loss, $\frac{E^2}{R}$, varies as the square of the speed and the square of the flux density.

Hysteresis Loss

When an armature revolves in a stationary magnetic field, the magnetic particles of the armature are held in alignment with the field in varying numbers depending upon the strength of the field. If the field is that of a 2-pole generator, these magnetic particles will rotate, with respect to the particles not held in alignment, one complete turn for each revolution of the armature. The rotation of magnetic particles in the mass of iron produces friction and heat.

Heat produced in this manner is identified as magnetic HYSTERESIS LOSS. Several complete cycles of magnetization plotted on rectangular coordinates are shown in figures 7–21 and 7–22 in the chapter on magnetism. These are called HYSTERESIS LOOPS. The area of these loops is a measure of the energy expended in completing one cycle of magnetization (1 revolution of the magnetic particles).

The hysteresis loss varies with the speed of the armature and the volume of iron. Steinmetz discovered that the hysteresis loss also varies as the 1.6 power of the flux density ($B^{1.6}$). The flux density varies from approximately 50,000 lines per square inch in the armature core to 130,000 lines per square inch in the iron between the bottom of adjacent armature slots (called the TOOTH ROOT). Heat-treated silicon steel having a low hysteresis loss is used in most d-c generator armatures. After the steel has been formed to the proper shape, the laminations are heated to a dull red heat and allowed to cool. This annealing process reduces the hysteresis loss to a low value.

ARMATURE REACTION

Armature reaction in a generator is the effect on the main field of the armature acting as an electromagnet. With no armature current, the field is undistorted, as shown in figure 10–12, A. This flux is produced entirely by the ampere-turns of the main field windings. The neutral plane AB is perpendicular to the direction of the main field flux. When an armature conductor moves through this plane its path is parallel to the undistorted lines of force and the conductor does not cut through any flux. Hence no voltage is induced in the conductor. The brushes are placed on the commutator so that they short-circuit coils passing through the neutral plane. With no voltage generated in the coils, no current will flow through the local path formed momentarily between the coils and segments spanned by the brush. Therefore, no sparking at the brushes will result.

When a load is connected across the brushes, armature current flows through the armature conductors, and the armature itself becomes a source of magnetomotive force. The effect of the armature acting as an electromagnet is considered in figure 10–12, B, with the assumption that the main field coils are deenergized and full-load current is introduced to the armature circuit from an external source. The currents in the conductors on the left of the neutral plane all carry current toward the observer, and those on the right carry current away from the observer. These directions are the same as those in which the current would flow if it were under the influence of the normal emf generated in the armature with normal field excitation.

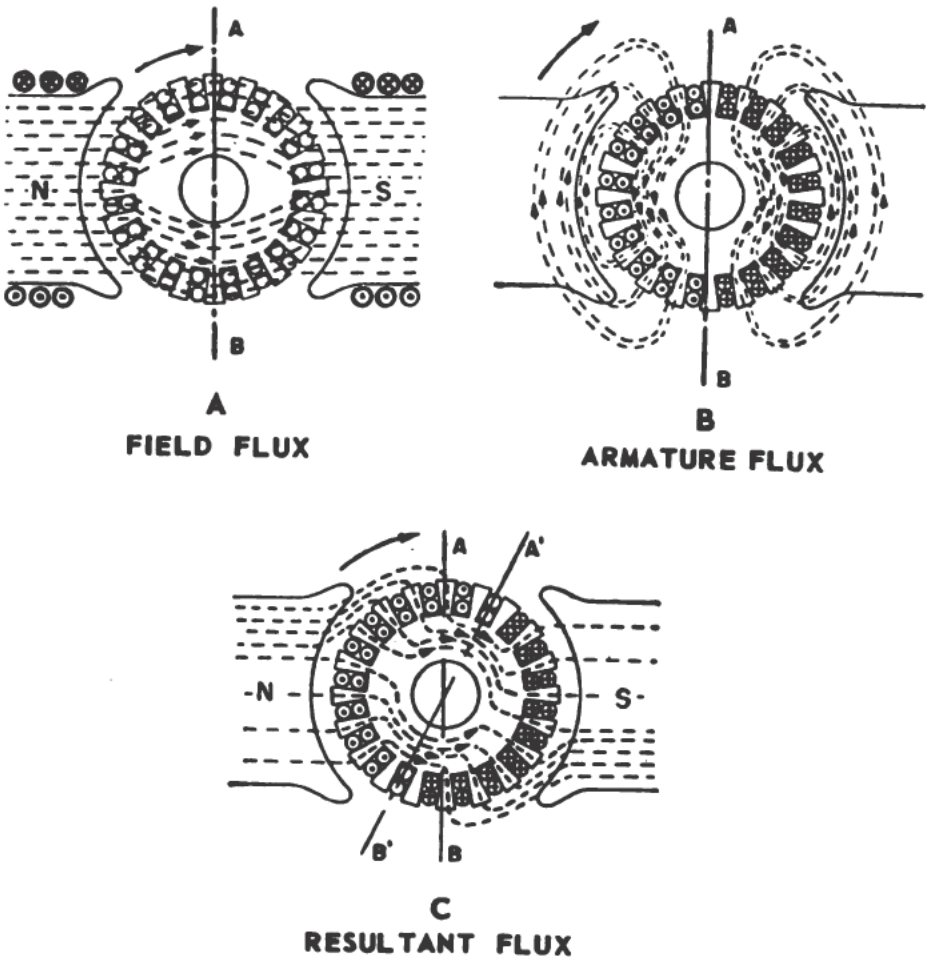
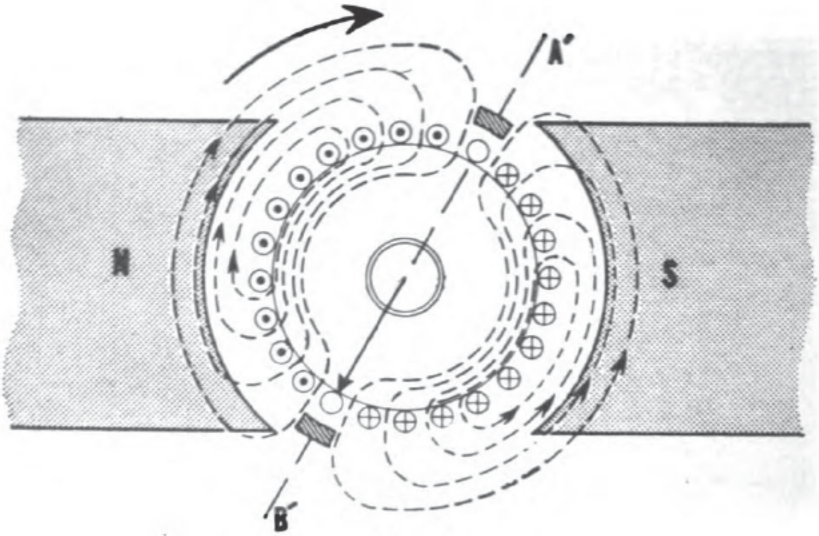


Figure 10–12.—Flux distribution in a d-c generator.

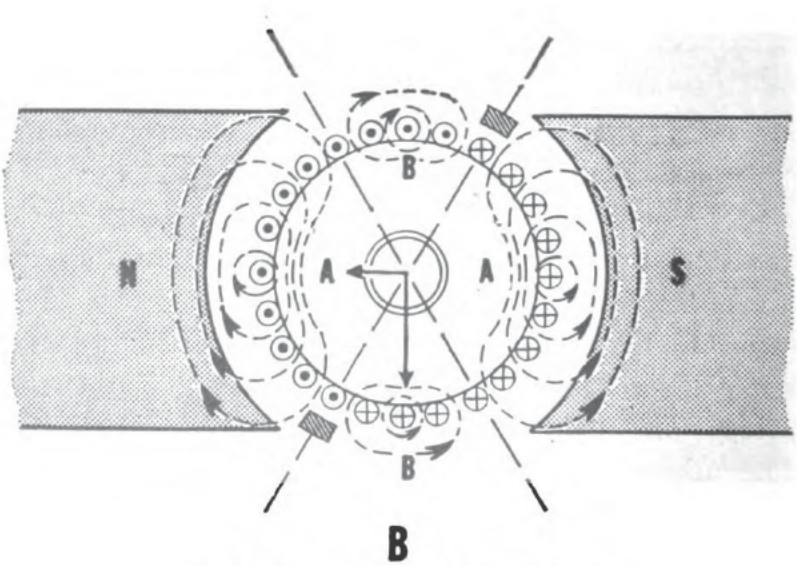
These armature current-carrying conductors establish a magnetomotive force that is perpendicular to the axis of the main field, and in the figure the force acts downward. This magnetizing action of the armature current is called CROSS MAGNETIZATION and is present only when current flows through the armature circuit. The amount of cross magnetization produced is proportional to the armature current.

When current flows in both the field and armature circuits, the two resulting magnetomotive forces act simultaneously on the main field and distort or twist it in the direction of rotation of the armature. The mechanical (no load) neutral plane, AB (fig. 10–12, C), is now advanced to the electrical (load) neutral plane, A′B′. When armature conductors move through plane A′B′ their



A

ARMATURE FLUX



B

DEMAGNETIZING AND CROSS-MAGNETIZING COMPONENTS

Figure 10–13.—Effect of brush shift on armature reaction.

paths are parallel to the distorted field and the conductors cut no flux, hence no voltage is induced in them. The brushes are therefore moved forward on the commutator to a position where they momentarily short-circuit the coils passing through the electrical neutral plane. Thus, no currents flow in the local paths formed by the coils and segments on short circuit by a brush, and no sparking at the commutator indicates the correct placement of the brushes. The amount that the neutral plane shifts is proportional to the load on the generator because the amount of cross-magnetizing magnetomotive force is directly proportional to the armature current.

When the brushes are shifted into the electrical neutral plane A′B′ the direction of the armature magnetomotive force is downward and to the left, as shown in figure 10–13, A, instead of vertically downward. The armature magnetomotive force may now be resolved into two components, as shown in figure 10–13, B.

The conductors included at the top and bottom of the armature within sectors BB produce a magnetomotive force that is directly in opposition to the main field and weakens it. This component is called the ARMATURE DEMAGNETIZING MMF. The conductors included on the right and left sides of the armature within sector AA produce a cross-magnetizing mmf at right angles to the main field axis. This cross-magnetizing force tends to distort the field in the direction of rotation. As mentioned previously, the distortion of the main field of the generator is known as ARMATURE REACTION. Armature reaction occurs in the same manner in multipolar machines.

Compensating for Armature Reaction

The effects of armature reaction are reduced in d-c machines by the use of (1) high flux density in the pole tips, (2) a compensating winding, and (3) commutation poles.

The cross-sectional area of the pole tips is reduced by building the field poles with laminations having only one tip. These laminations are alternately reversed when the pole core is stacked so that a space is left between alternate laminations at the pole tips. The reduced cross section of iron at the pole tips increases the flux density so that they become saturated and the cross-magnetizing and demagnetizing forces of the armature will not affect the flux

distribution in the pole face to as great an extent as they would at reduced flux densities.

The compensating winding consists of conductors imbedded in the pole faces parallel to the armature conductors. The winding is connected in series with the armature and is arranged so that the ampere-turns are equal in magnitude and opposite in direction to those of the armature. The magnetomotive force of the compensating winding therefore neutralizes the armature magnetomotive force, and armature reaction is eliminated. Because of the relatively high cost, compensating windings are ordinarily used only on high-speed and high-voltage generators of large capacity.

Commutating poles are discussed after the description of the process of commutation.

COMMUTATION

Commutation is the process of reversing the current in the individual armature coils and conducting the direct current to the external circuit during the brief interval of time required for each commutator segment to pass under a brush. In figure 10–14, commutation occurs simultaneously in the two coils that are undergoing momentary short circuit by the brushes—coil *B* by the negative brush, and the diametrically opposite coil by the positive brush. As mentioned previously, the brushes are placed on the commutator in a position that short-circuits the coils that are moving through the electrical neutral plane because there is no voltage generated in the coils at the time and no sparking occurs between commutator and brush.

There are two paths for current through the armature winding. If the load current is 100 amperes, each path will contain a current of 50 amperes. Thus each coil on the left side carries 50 amperes in a given direction and each coil on the right side carriers a current of 50 amperes in the opposite direction. The reversal of the current in a given coil occurs during the time that particular coil is being short-circuited by a brush. For example, as coil *A* approaches the negative brush it is carrying the full value of 50 amperes which flows through commutator segment 1 and the left half of the negative brush where it joins 50 amperes from coil *C*.

At the instant shown, the negative brush spans half of segment 1 and half of segment 2. Coil *B* is on short circuit and is moving

parallel to the field so that its generated voltage is zero, and no current flows through it. As rotation continues in a clockwise direction the negative brush spans more of segment 1 and less of segment 2. Consider, for example, the interval included between the instant shown in the figure and the instant that the negative

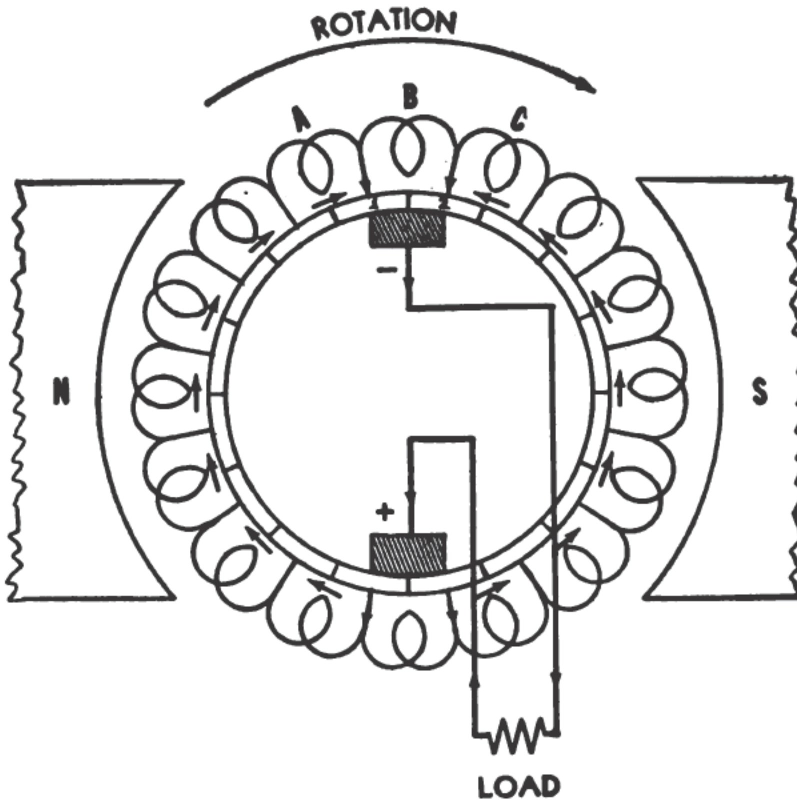


Figure 10–14.—Commutation in a d-c generator.

brush spans only segment 1. During this time the current in segment 1 increases from 50 to 100 amperes and the current in segment 2 decreases from 50 to 0 amperes. When segment 2 leaves the brush, no current flows from segment 2 to the brush and commutation is complete.

As coil *A* continues into the position of coil *B* the current in *A* decreases to zero. Thus, the current in the coils approaching the brush is reducing to zero during the brief interval of time that it takes for coil *A* to move to the position of coil *B*. During this time the flux collapses around the coil and induces an emf of self-induction which opposes the decrease of current. Thus, if the

emf of self-induction is not neutralized, the current will not decrease in coil *A* and the current in the coil lead to segment 1 will not be zero when segment 1 leaves the brush. This delay causes a spark to form between the toe of the brush and the trailing edge of the segment. As the segment breaks contact with the brush, this action burns and pits the commutator.

The reversal of current in the coils takes place very rapidly. For example, in an ordinary 4-pole generator, each coil passes through the process of commutation several thousand times per minute. It is important that commutation be accomplished without sparking to avoid excessive commutator wear.

Advancing the Brushes

The emf of self-induction in the armature coils is caused by the inductance of the coils and the changing current in them and cannot be eliminated. The effects of this emf can be neutralized, however, by introducing into the coil during the process of commutation an emf that is equal and in opposition to the induced emf. This neutralization can be accomplished by shifting the brushes in the direction of rotation or by using interpoles (commutating poles).

If the brushes are shifted in the direction of rotation until the coils undergoing commutation cut a small amount of flux from the on-coming main pole, sufficient emf is induced to neutralize the effect of the self-induced emf. This action allows the coil current to decrease to zero and increase to the desired values in the opposite direction without sparking at the brushes.

The flux necessary to generate the emf that neutralizes the emf of self-induction is called the COMMUTATING FLUX. This method of reducing sparking at the brushes is satisfactory only under steady-load conditions because the amount of commutating flux required varies with the load and, therefore, the brushes must be shifted with each change in load.

Commutating Poles

Commutating poles, or interpoles, provide the required amount of commutating flux without shifting the brushes from mechanical neutral. They are narrow auxiliary poles located midway between the main poles, as shown in figure 10–2, F. They establish a flux in the proper direction and of sufficient magnitude to pro-

duce satisfactory commutation. They do not contribute to the generated emf of the armature as a whole because the voltages generated by their fields cancel each other between brushes of opposite polarity.

The interpole magnetomotive force neutralizes that portion of the armature reaction within the zones of commutation and produces the proper flux to generate an emf in the short-circuited coil that is equal and opposite to the emf of self-induction. Thus, no sparking occurs at the brushes and "black" commutation indicates the ideal condition. Because both the armature reaction and the self-induced emf in the commutated coils vary with armature current, the interpole flux should also vary with the armature current. This condition is obtained by connecting the interpole windings in series with the armature and operating the interpole iron at flux densities well below saturation. The magnetic polarities are such that an interpole always has the same polarity as the adjacent main field pole in the direction of rotation. This relation always exists for d-c generators.

MOTOR REACTION IN A GENERATOR

Whenever a generator delivers current to a load, the load current creates an opposition force that opposes the rotation of the generator armature. An armature conductor is represented in figure 10–15. When the conductor is stationary, no voltage is

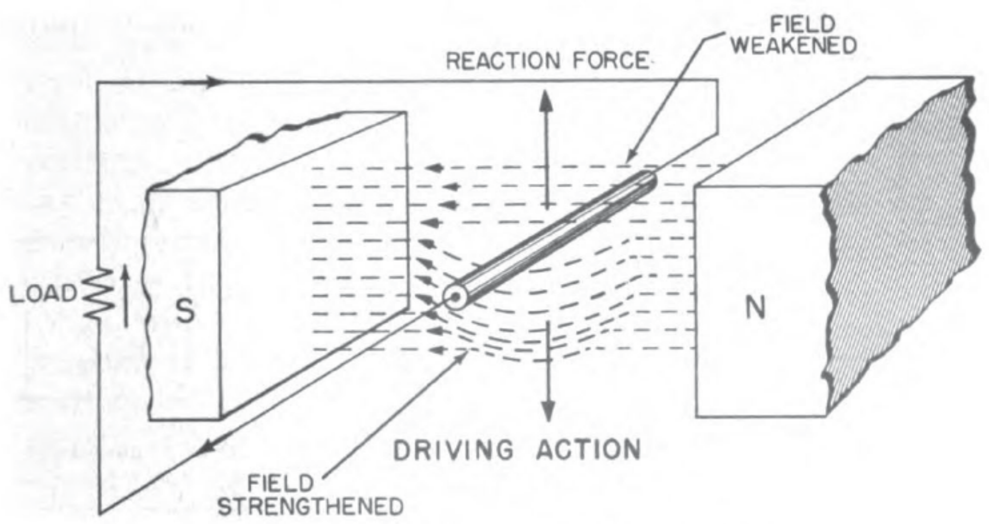


Figure 10–15.—Motor reaction in a generator.

generated and no current flows; hence, no force acts on the conductor. When the conductor is moved downward and the circuit is completed through an external load, current flows through the conductor in the direction indicated, setting up lines of force around it that have a clockwise direction.

The interaction of the conductor field and the main field of the generator weakens the field above the conductor and strengthens it below the conductor. The field consists of lines that act like stretched rubber bands. Thus, an upward reaction force is produced that acts in opposition to the downward driving force applied to the generator armature. If the current in the conductor increases, the reaction force increases, and more force must be applied to the conductor to keep it from slowing down.

With no armature current, no magnetic reaction exists and the generator input power is low. As the armature current increases, the reaction of each armature conductor against rotation increases and the driving power to maintain the generator armature speed must be increased. If the prime mover driving the generator is a gasoline engine, this effect is accomplished by opening the throttle on the carburetor. If the prime mover is a steam turbine, the main steam-admission valve is opened wider, thus permitting more steam to flow through the turbine.

D-C GENERATOR CHARACTERISTICS

Methods of Connecting the Field Windings

Usually d-c generators are classified according to the manner in which the field windings are connected to the armature circuit (fig. 10–16).

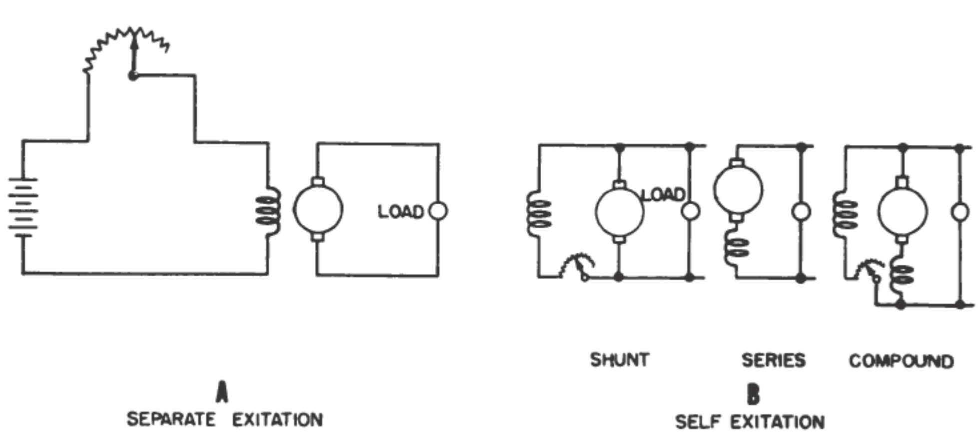


Figure 10–16.—Types of d-c generators.

A SEPARATELY EXCITED D-C GENERATOR is indicated in the simplified schematic diagram of figure 10–16, A. In this machine the field windings are energized from a separate d-c source other than its own armature.

A SHUNT GENERATOR has its field windings connected across the armature in shunt with the load, as shown in figure 10–16, B. The shunt generator is widely used in industry.

A SERIES GENERATOR has its field windings connected in series with the armature and load, as shown in figure 10–16, B. Series generators are seldom used.

COMPOUND GENERATORS contain both series and shunt field windings, as shown in figure 10–16, B. Compound generators are widely used in industry.

Field Saturation Curves

The strength of the field of a d-c generator depends on the number of ampere-turns in the field windings and the reluctance of the magnetic circuit. The number of turns is generally fixed. Hence, the ampere-turns vary directly with the field current. The generated voltage is directly proportional to the product of the field strength and the speed. Thus, if the field strength is zero or the speed is zero, the generated voltage will be zero. As the current through the field windings increases, the field flux and voltage output will increase. The field strength, however, is not directly proportional to the field current because the reluctance of the magnetic circuit varies with the degree of magnetization. With increasing flux density in the field and armature iron the permeability decreases, thereby increasing the reluctance of the magnetic circuit and it becomes more difficult to increase the voltage.

Certain operating characteristics of the d-c generator are very closely related to the no-load and the full-load field saturation curves of the machine. The no-load saturation curve is determined with no armature current and the full-load saturation is determined with full-load armature current. The speed is held constant for both curves and the field current is increased in equal steps. The terminal voltage corresponding to each value of field current is plotted and forms the field saturation curves as shown in figure 10–17. The no-load saturation curve is preferably taken with the field separately excited. However, for a shunt or a

compound generator these curves can be taken with the machine self-excited because the armature voltage drop due to the shunt field current is negligible.

With a certain field current, OA (fig. 10–17), a no-load emf of AD volts is generated, but the terminal voltage at full load is AF. The difference, DF, is caused by the armature IR drop and armature reaction. The saturation curves bend to the right at high values of field current and thereby show the tendency of the field iron to saturate. The no-load saturation curve is the magnetization curve of the machine. Because this curve is obtained with no armature current, the observed voltage is the generated emf of the machine. At zero field current the emf is not zero because of the residual flux of the machine. This fact is

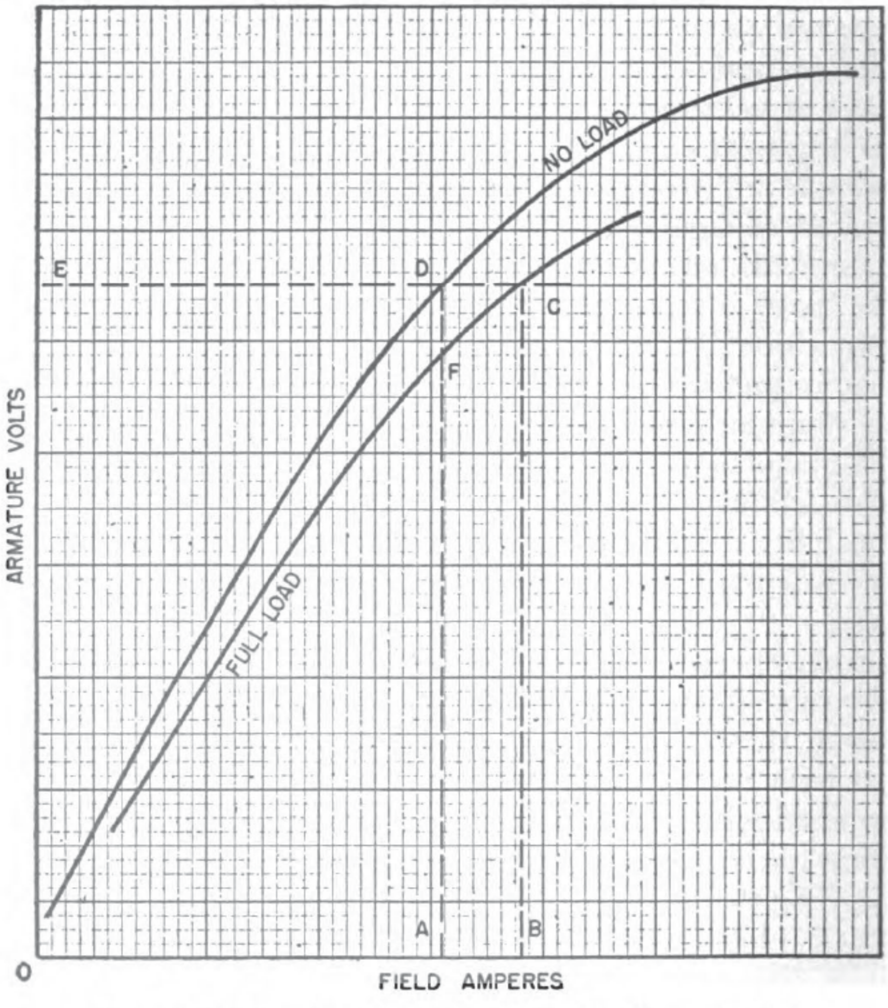


Figure 10–17.—Field saturation curves for d-c generator.

important because a self-excited generator depends on residual flux to build up its voltage. At low values of field current the voltage varies proportionally, but as the field current increases, the steel portion of the magnetic circuit becomes partly saturated and the voltage increases more slowly.

The abrupt bend in the curve is called the KNEE of the curve. Shunt generators are designed to operate at a point slightly above the knee so that a slight change in speed does not cause a great change in voltage. Compound generators operate at a lower point in order to avoid the use of a large series field winding.

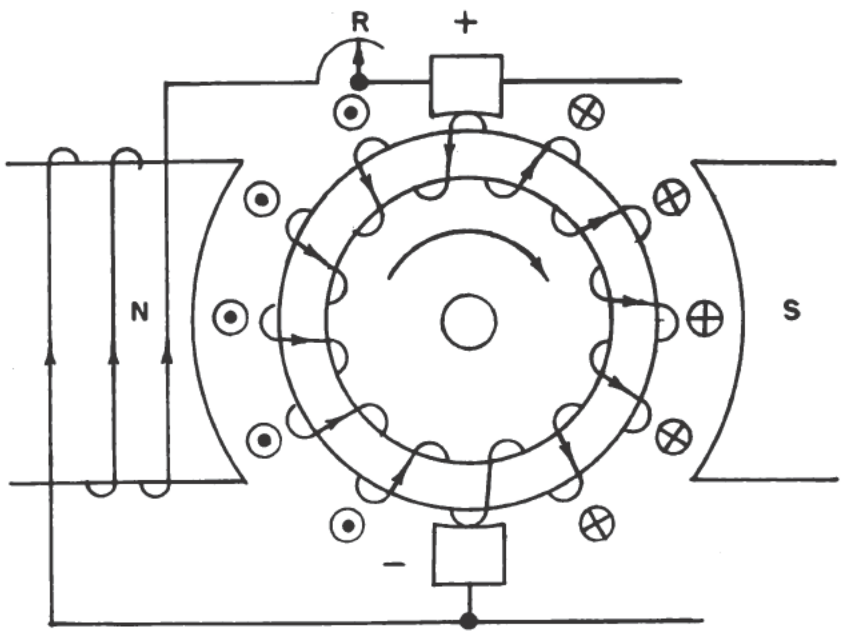
Shunt Generator

The shunt generator (fig. 10–16, B) has field coils of many turns of small size wire connected in shunt with the armature and load. The armature current is equal to the sum of the field current and the load current. The field current is small compared with the load current and is approximately constant for normal variations in load. Thus, the armature current varies directly with the load. The field flux produced by the field current is normally constant so that the terminal voltage does not vary widely with load change. Hence, the generator is essentially a constant-potential machine that delivers current to the load in accordance with the load demand.

BUILD-UP OF VOLTAGE.—After the generator is brought up to normal speed, and before any load is connected across the armature, the generator must "build up" its voltage to the rated value (fig. 10–18). A schematic wiring diagram (fig. 10–18, A) is shown without load, and with only a few turns of the field coil around one pole indicated for simplicity in tracing the direction of current flow and the polarity of the voltage generated in the armature winding.

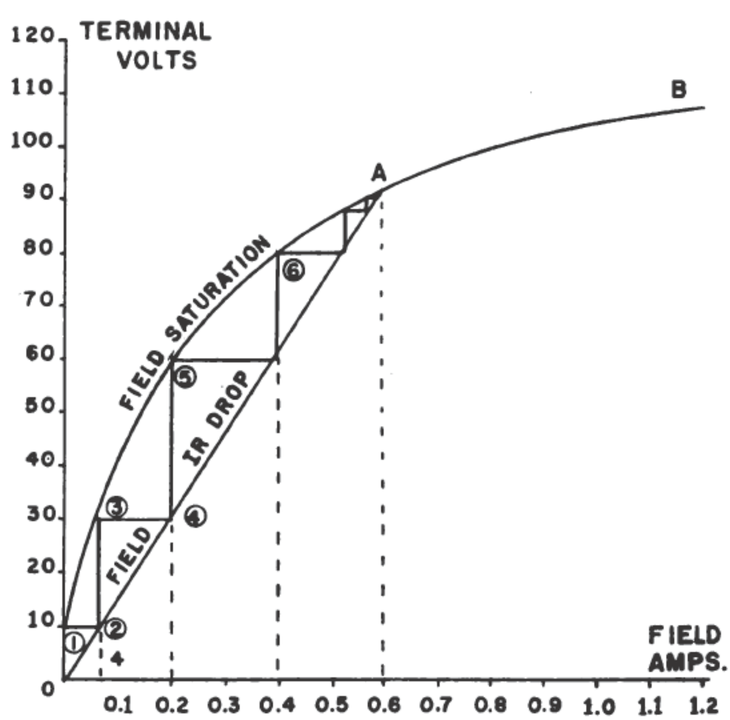
Figure 10–18, B, indicates the field saturation curve for the generator. Line *OA* represents the relation between voltage and current in the field-coil circuit for one value of field-circuit resistance. Line *OA* is called an *IR*-drop curve for the field circuit and is assumed to be a straight line on the basis of constant temperature operation. The field rheostat permits adjustment of the field current through a relatively wide range and is designed so that its resistance is at least equal to that of the field winding.

At start, the generator is brought up to rated speed. The load



SCHEMATIC WIRING DIAGRAM

A



FIELD SATURATION AND IR DROP CURVE

B

Figure 10–18.—Build-up of voltage of a shunt generator.

is not connected. The armature conductors cut the small residual field and generate about 10 volts across the brushes (point ① on the saturation curve). The left-hand rule for generator action indicates that the generated voltage is applied to the field winding in a direction to supplement the residual field. Ten volts applied across the field circuit will cause approximately 0.07 ampere to flow through the field coils (point ② on the *IR*-drop curve).

This current strengthens the field, and the armature voltage increases to 30 volts, as shown on the field saturation curve (point ③). During the process of voltage build-up the effect of the armature circuit resistance on the terminal voltage is neglected since the field current is only a small fraction of an ampere. This current flows through the armature and the accompanying *IR* drop is negligible. A generated voltage of 30 volts applied to the field circuit causes a field current of 0.2 ampere to flow, as shown by the *IR*-drop curve (point ④). This current in turn increases the field strength and generated voltage in the armature to 60 volts, as indicated on the field saturation curve (point ⑤). This voltage applied to the field causes 0.4 ampere to flow and this action increases the terminal voltage to 80 volts (point ⑥). The rise in voltage continues until the field current has increased to 0.6 ampere and the terminal voltage levels off at 90 volts (point *A*). No further increase in generated voltage occurs for the given value of field circuit resistance because the amount of field saturation flux produced is enough to generate the voltage required (90 volts) to circulate this current (0.6 ampere) through the field coils. For this condition the resistance of the field circuit is $\frac{90}{0.6}$, or 150 ohms. Field saturation limits the generated voltage to this value as determined by the setting of the field rheostat.

To increase the shunt generator terminal voltage further, it is necessary to decrease the field circuit resistance. For example, if the terminal voltage is to be increased to 110 volts, the corresponding field current from the saturation curve is 1.2 amperes (point *B*). The field circuit resistance is now represented by the slope of line *OB*, or $\frac{110}{1.2} = 91.8$ ohms. Thus, if the field resistance is decreased from 150 ohms to 91.8 ohms, the terminal voltage will increase from 90 volts to 110 volts.

INHERENT REGULATION OF SHUNT GENERATOR.—Internal changes, both electrical and magnetic, that occur in a generator automatically with load change, give the generator certain typical characteristics by which it may be identified. These internal changes are referred to as the inherent regulation of a generator. At no load, the armature current is equal to the field current. With low armature resistance and low field current there is little armature *IR* drop, and the generated voltage is equal to the terminal voltage. With load applied, the armature *IR* drop increases, but is relatively small compared with the generated voltage. Also the armature reaction voltage loss is small. Therefore, the terminal voltage decreases only slightly provided the speed is maintained at the rated value.

Load is added to a shunt generator by increasing the number of parallel paths across the generator terminals. This action reduces the total load circuit resistance with increased load. Since the terminal voltage is approximately constant, armature current increases directly with the load. Since the shunt field is in a separate circuit it receives only a slightly reduced voltage and its current does not change to any great extent.

Thus, with low armature resistance and a relatively strong field there is only a small variation in terminal voltage between no load and full load.

EXTERNAL VOLTAGE CHARACTERISTICS.—A graph of the variation in terminal voltage with load on a shunt generator is shown in curve *A* of figure 10–19. This curve shows that the terminal voltage of a shunt generator falls slightly with increase in load from the no-load condition to the full-load condition. It also shows that with heavy overload the terminal voltage falls more rapidly. The shunt field current is reduced and the magnetization of the field falls to a low value. The dotted portion of curve *A* indicates the way the terminal voltage falls beyond the breakdown point. In large generators the breakdown point occurs at several times rated load current. Generators are not designed to be operated at these large values of load current and will overheat dangerously even at twice the value of full-load current.

Curve *B* represents the external voltage characteristic of a shunt generator with constant field current for variations in load between zero and approximately 25 percent over the rated load

condition. Curve *C* represents the external voltage characteristic for the same range of load with simulated conditions of zero armature reaction and constant field current. The divergence of curve *C* from curve *D* represents the voltage variators resulting from the *IR* drop in the armature circuit. As mentioned previously, the terminal voltage of the shunt generator is prevented from varying widely with load change by providing it with a (1)

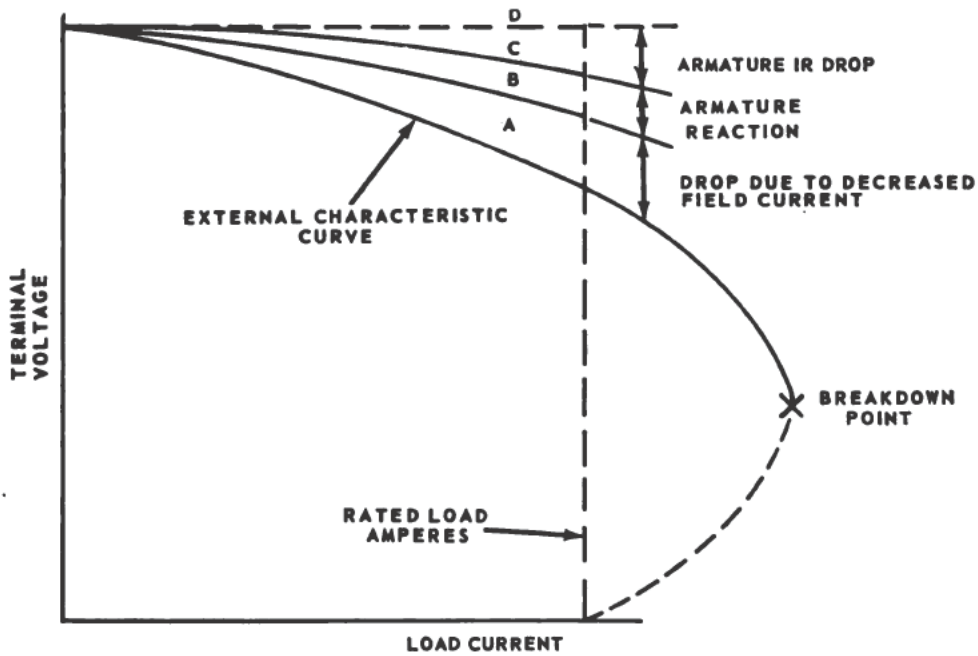


Figure 10–19.—Shunt generator external characteristics.

low armature resistance, (2) strong main field with low armature reaction, and (3) separate field circuit in shunt with the armature and load.

Compound Generator

Compound generators employ a series field winding in addition to the shunt field winding, as shown in figure 10–16, B. The series field coils are made of a relatively small number of turns of large copper conductor, either circular or rectangular in cross section, and are connected in series with the armature circuit. These coils are mounted on the same poles on which the shunt field coils are mounted and therefore contribute a magnetomotive force that influences the main field flux of the generator.

Effect of series field.—If the ampere-turns of the series field act in the same direction as those of the shunt field the combined magnetomotive force is equal to the sum of the series and shunt field components. Load is added to a compound generator in the same manner in which load is added to a shunt generator—by increasing the number of parallel paths across the generator terminals. Thus, the decrease in total load resistance with added load is accompanied by an increase in armature-circuit and series-field circuit current.

The effect of the additive series field is that of increased field flux with increased load. The extent of the increased field flux depends on the degree of saturation of the field iron as determined by the shunt field current. Thus, the terminal voltage of the generator may increase with load or it may decrease depending upon the influence of the series field coils. This influence is referred to as the degree of compounding.

For example, a flat-compound generator is one in which the no-load and full-load voltages have the same value. An under-compound generator is one in which the full-load voltage is less than the no-load value. An over-compound generator is one in which the full-load voltage is higher than the no-load value. The way the terminal voltage changes with increasing load depends upon the degree of compounding.

A variable shunt is connected across the series field coils to permit adjustment of the degree of compounding. This shunt is called a diverter. Decreasing the diverter resistance increases the amount of armature circuit current that is bypassed around the series field coils, thereby reducing the degree of compounding.

A field rheostat in the shunt field coil circuit permits adjustment of the no-load voltage of the compound generator. The diverter across the series field coils permits adjustment of the full-load voltage.

External voltage characteristics.—The variation of terminal voltage with load is indicated in the external characteristic curves of figure 10–20. Curve *A* is the graph of terminal voltage versus armature current for a flat-compound generator. The no-load and full-load voltages are the same. Neither the diverter nor the field rheostat are altered in this test. The speed is maintained constant at the rated value. The hump in the curve is caused by the increased influence of the series ampere-turns on

the field iron at half-load when the degree of saturation is reduced and the armature reaction and armature IR drop are approximately half their normal values.

Curve B is the external characteristic of an over-compound generator. As load is added to this machine its terminal voltage increases so that the field load voltage is higher than the no-load value. Such a characteristic might be desirable where the genera-

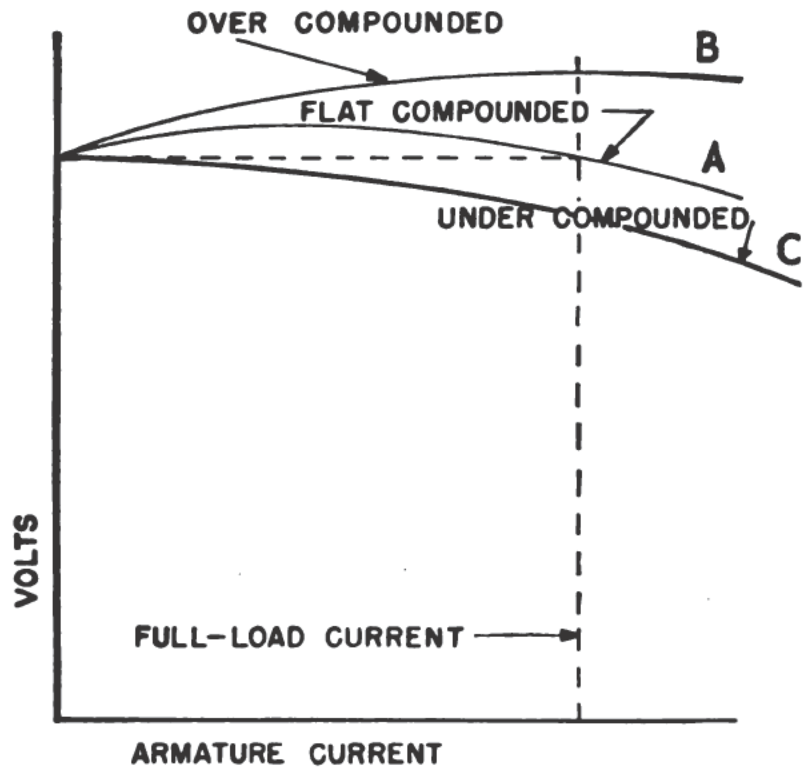


Figure 10–20.—Compound generator external characteristics.

tor is located some distance from the load and the rise in voltage compensates for the voltage loss in the feeder. This action holds the load voltage approximately constant from no-load to full-load by increasing the generator terminal voltage an amount that is just equal to the voltage drop in the feeder at full load.

Curve C represents the external characteristic for an under-compound generator. The Navy uses a so-called stabilized shunt generator that has a small series field winding of a few turns in the coils to partially compensate for the voltage loss due to armature IR drop and armature reaction. Thus, the external characteristic is almost the same as that of a shunt generator except that

the terminal voltage does not fall off quite as rapidly with increased load.

The stabilized shunt generator is the only type of compound generator commonly used in the Navy. This generator is used to supply general lighting and power aboard ship.

VOLTAGE REGULATION

The external characteristic of a generator is sometimes called the VOLTAGE-REGULATION CURVE of the machine. The regulation of a generator refers to the VOLTAGE CHANGE that takes place when the load is changed. It is usually expressed as the change in voltage from no-load to full-load voltage in percent of full-load voltage. Expressed as a formula,

$$\frac{E_{nL}-E_{fL}}{E_{fL}}\times 100=\text{percent regulation,}$$

where E_{nL} is the no-load terminal voltage and E_{fL} is the full-load terminal voltage of the generator. For example, the percent regulation of a generator having a no-load voltage of 237 volts and a full-load voltage of 230 volts is

$$\frac{237-230}{230}\times 100\text{, or 3 percent.}$$

Voltage Control

Flat-compound generators were formerly used to supply the ship's d-c electric power because they provided a more constant voltage under varying load conditions. Shunt generators are simpler in design and have greater reliability when operating in parallel. The stabilized shunt generator represents a compromise between the two types. As mentioned previously, it has a very light series winding and a slightly drooping voltage characteristic and is generally used to supply the ship's d-c electric power requirements.

Voltage control is either (1) manual or (2) automatic. In most cases the process involves changing the resistance of the field rheostat. When the load changes are infrequent and small, manual control is sufficient to hold the d-c system voltage to the desired value. When the load changes are frequent and large, it may be desirable to employ some form of automatic control.

Inherent voltage regulation should not be confused with voltage control. As described previously, voltage regulation is an internal action occurring within the generator whenever the load changes. Voltage control is a super-imposed action usually by external adjustment. Certain electromechanical devices are used extensively in automatic voltage regulators. These include solenoids, relays, carbon piles, rocking disks, and tilted plates.

MANUAL OPERATION.—A hand-operated field rheostat (fig. 10–21) connected in series with the shunt field circuit provides the simplest method of controlling the terminal voltage of a d-c generator.

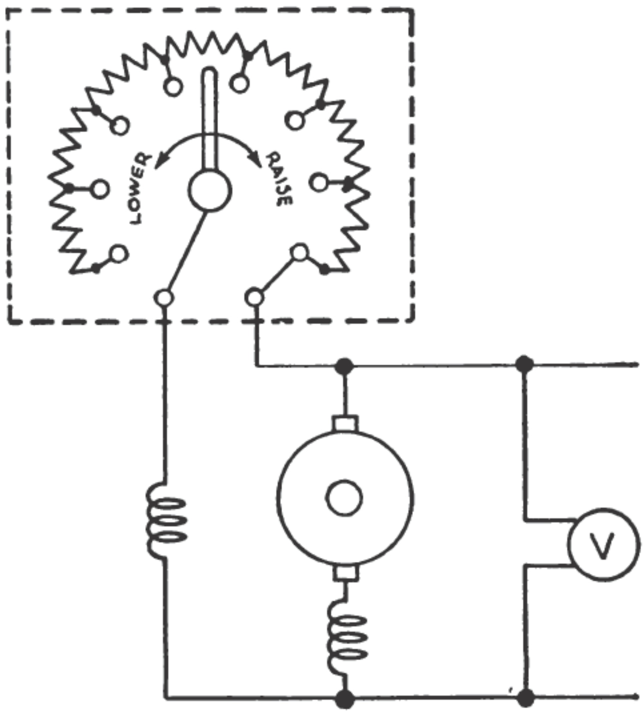


Figure 10–21.—Hand-operated field rheostat.

One form of field rheostat contains tapped resistors with leads to a multiterminal switch. The arm of the switch may be rotated through an arc to make contact with the various resistor taps, thereby varying the amount of resistance in the field circuit. Rotating the arm in the direction of the LOWER arrow increases the resistance and lowers the terminal voltage. Rotating the arm in the direction of the RAISE arrow, decreases the resistance and increases the terminal voltage.

Field rheostats for generators of moderate size employ resistors of alloy wire having a high specific resistance and a low temperature coefficient. These alloys include copper, nickel, manganese, and chromium and are marketed under trade names such as nichrome, advance, manganin, and so forth. Large generator field rheostats use cast-iron grids and a motor-operated switching mechanism.

Field rheostats for some equipments are made of carbon in the form of rods and disks. The disks are stacked and mechanical pressure is applied across the stack. Increasing the pressure decreases the contact resistance between the disks and thus lowers the resistance of the stack. Decreasing the pressure increases the resistance.

AUTOMATIC OPERATION.—Several types of automatic voltage regulators are used to control the terminal voltage of d-c generators. One of the earliest types was the vibrating regulator. More recent types include the tilted plate regulator and the rocking disk regulator.

The VIBRATING REGULATOR operates on the principle that an intermittent short circuit applied across the generator field rheostat will cause the terminal voltage of the generator to pulsate within narrow voltage limits and thus it will maintain an average steady value of voltage that is independent of load change. A simplified circuit is shown in figure 10–22.

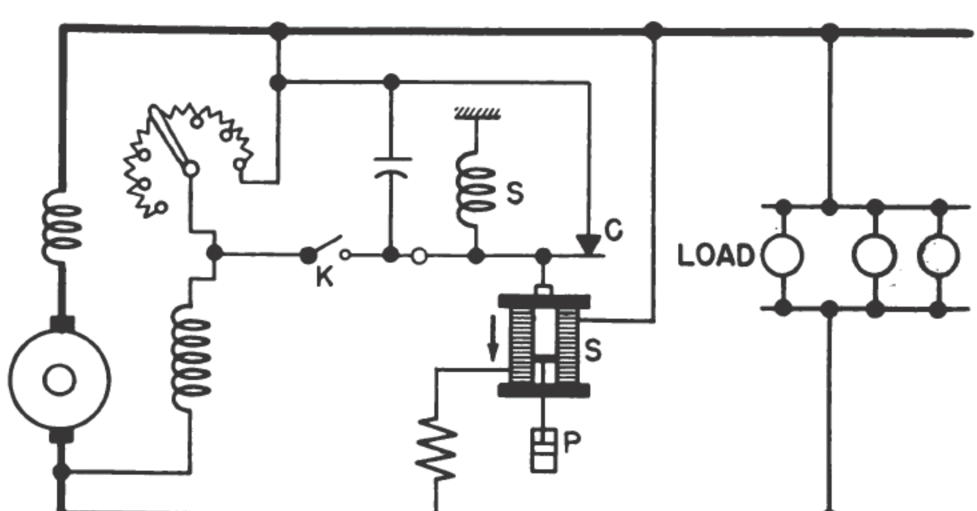


Figure 10–22.—Vibrating type of voltage regulator.

With the generator running at normal speed and switch K open, the field rheostat is adjusted so that the terminal voltage is about 60 percent of normal. Solenoid S is weak and contacts C are held closed by the spring. When K is closed, a short circuit is placed across the field rheostat. This action causes the field current to increase and the terminal voltage to rise.

When the terminal voltage rises above a certain critical value, for example 111 volts, the solenoid downward pull exceeds the spring tension and contact C opens, thus reinserting the field rheostat in the field circuit and reducing the field current and terminal voltage.

When the terminal voltage falls below a certain critical voltage, for example 109 volts, the solenoid armature contact C is closed again by the spring. The field rheostat is now shorted and the terminal voltage starts to rise. The cycle repeats and the action is rapid and continuous. The average voltage of 110 volts is maintained with or without load change.

The dashpot, *P*, provides smoother operation by acting as a damper to prevent hunting. The capacitor across contacts C eliminates sparking. Added load causes the field rheostat to be shorted for a longer period of time and thus the solenoid armature vibrates more slowly. If the load is reduced and the terminal voltage tends to rise, the armature vibrates more rapidly. Thus the regulator holds the terminal voltage to a steady value for any change in load from no load to full load on the generator.

The TILTED PLATE REGULATOR (fig. 10–23) consists of two or more stacks of graphite plates mounted on metallic supports at the center of each stack. Each plate is balanced on its fulcrum (the metallic support) at the center. On one end of the stack the plates are separated by mica spacers. On the other end they are separated by silver contacts. Before the silver contacts are closed, the path for field circuit current is from graphite plate to graphite plate through the connecting fulcrums at the center of the stack. Between the graphite plates and their supporting fulcrums there exists a certain amount of contact resistance. A weight acting on a lever tilts the plate in a manner that brings the contacts together. A solenoid, which responds to terminal voltage change, tilts the plates in the opposite direction. Bringing the silver contacts together shorts out the resistance of the stack. Opening the silver contacts increases the field circuit resistance.

The regulator responds to load change and is not designed to overshoot in the manner of the vibrating regulator previously described. For example, at rated voltage the solenoid is in balance with the lever weight and the arm does not move. An increase in load decreases the terminal voltage only slightly because the lever weight immediately overcomes the solenoid and

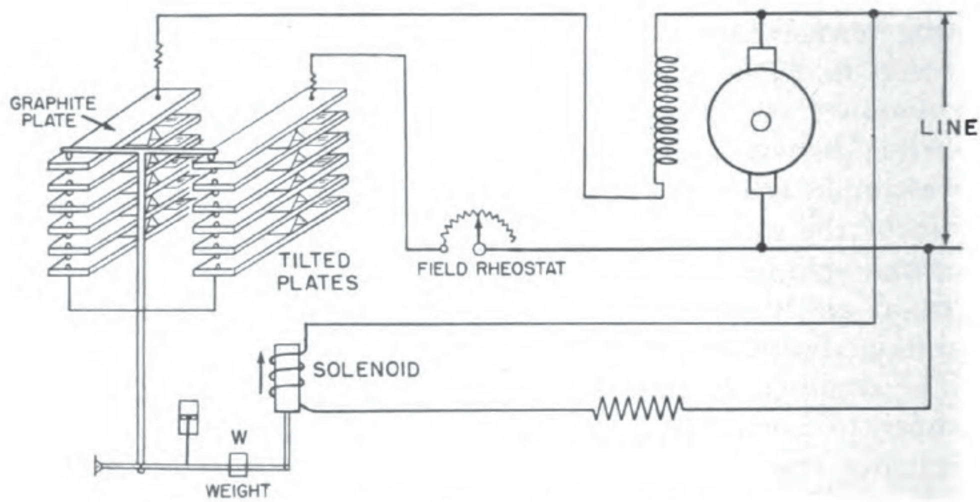


Figure 10–23.—Tilted plate voltage regulator.

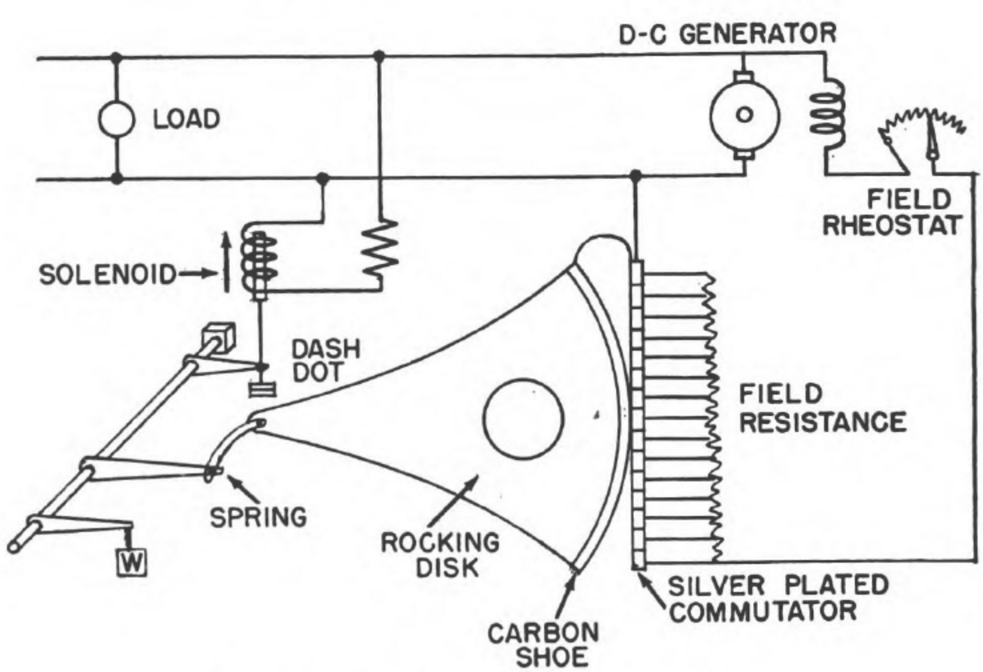


Figure 10–24.—Rocking disk voltage regulator.

tilts the plates in a direction to lower the resistance, thereby strengthening the field and the generated voltage. This action checks the fall of terminal voltage with load.

The ROCKING DISK REGULATOR (fig. 10–24) is another device that is quick acting and sensitive to slight changes in terminal voltage. It may be designed to be in a constant-motion condition of overshooting the voltage a slight amount through each cycle of operation or it may be operated in a static condition in which load change alone causes it to operate.

A spring presses the rocking disk against the flat surface of a silver-plated commutator, the segments of which connect to taps on the field circuit resistors. The disk has a carbon shoe which makes contact with the segments one at a time from top to bottom as the disk is rocked downward. A small range of motion of the spring is converted into a long range of motion of the contact point of the carbon shoe with the various commutator segments. For a constant load on the generator the lever weight, which tends to rock the disk downward, and the solenoid, which tends to rock it upward, are in a state of balance. Thus, the shoe is in contact with one commutator segment and a portion of the field resistors is short-circuited.

When the load on the generator increases, the terminal voltage of the generator starts to fall. The solenoid is weakened and the weight rocks the disk downward, thus short-circuiting more of the field resistors and increasing the generated voltage. The decrease in terminal voltage is almost instantly checked by the increase in field strength and generated voltage, and the weight is now in balance with the solenoid at a new position where the disk contacts a commutator segment nearer the lower end of its travel.

A decrease in load causes the terminal voltage to start to rise and this strengthens the solenoid. Thus the disk rocks in an upward direction and more resistance is inserted in the field circuit, with accompanying decrease in field strength and generated voltage.

The decrease in generated voltage checks the rise in terminal voltage. For example, at one load the generated voltage is 120 volts, the internal voltage loss is 10 volts, and the terminal voltage is $120-10$, or 110 volts. With decrease in load the internal voltage loss might decrease to 5 volts, and without automatic regulation the terminal voltage would increase to $120-5$, or 115 volts.

However, with automatic regulation the terminal voltage momentarily rises to 111 volts and the regulator causes the generated voltage to fall from 120 volts to 111+5, or 116 volts, and to become stable at 115 volts when the terminal voltage is again 115−5, or 110 volts.

REGULATOR FOR VARIABLE-SPEED GENERATOR

If the speed of a shunt generator varies, the output voltage will vary. In cars, trucks, small boats, and aircraft, the generator charges the battery and provides auxiliary electric power for lights, radio, and other equipment. The generator is usually driven from the main source of power, which may be a variable-speed internal-combustion engine. In naval aircraft, carbon-pile voltage regulators are usually employed to maintain a constant generator terminal voltage and also to equalize generator loads in installations involving two or more generators operating in parallel. In cars, trucks, small boats, and some types of airplanes, if the generator is of the shunt type, a 3-unit regulator is often employed. One unit consists of a vibrating voltage regulator that places an intermittent short-circuit across a resistor in series with the field. This action is similar to the vibrating-type voltage regulator described earlier in this chapter. The second unit is a current regulator that limits the output current at increased speeds. If the voltage of the generator falls below that of the battery, the battery will discharge through the generator armature; thereby tending to drive the generator as a motor. This action is called "motoring" the generator, and unless it is prevented, will discharge the battery in a short time. The third unit in the regulator is a reverse-current cutout that prevents the battery from motoring the generator at low speeds by disconnecting the battery from the generator.

The action of vibrating contact $C1$ in the voltage-regulator unit (fig. 10–25) places an intermittent short circuit across $R1$ and $L2$. When the generator is not operating, spring $S1$ holds $C1$ closed. $C2$ is also closed by $S2$, and the shunt field is connected directly across the armature.

When the generator is started, its terminal voltage will rise as the generator comes up to speed, and the armature will supply the field with current through closed contacts $C2$ and $C1$.

As the terminal voltage rises, the current flow through $L1$ increases and the iron core becomes more strongly magnetized. At a certain speed and voltage, when the magnetic attraction on the movable arm becomes strong enough to overcome the tension of spring $S1$, contact points $C1$ are separated. The field current now flows through $R1$ and $L2$. Because resistance is added to

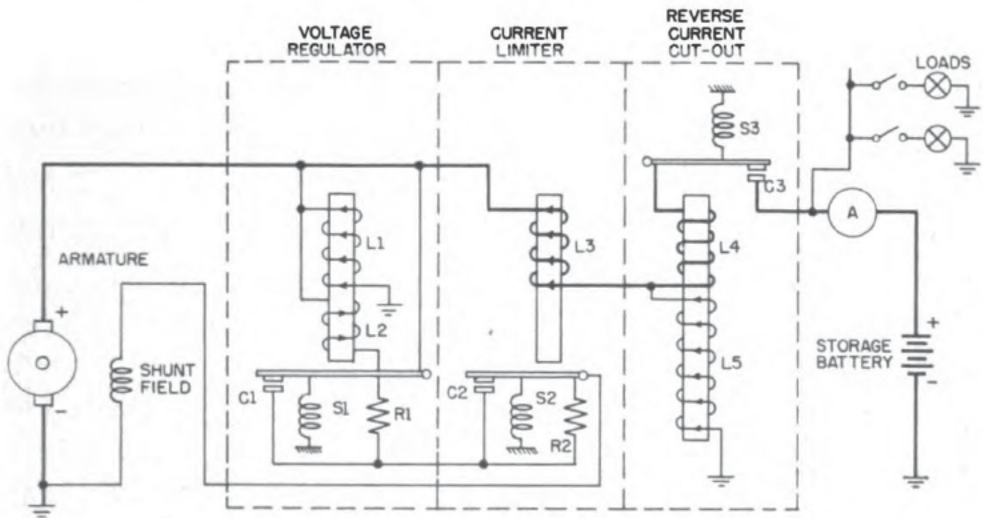


Figure 10–25.—Regulator for variable-speed generator.

the field circuit, the field is momentarily weakened and the rise in terminal voltage is checked. Additionally, because the $L2$ winding is opposed to the $L1$ winding, the magnetic pull of $L1$ against $S1$ is partially neutralized, and spring $S1$ closes contact $C1$. $R1$ and $L2$ are again shorted out of the circuit, the field current again increases, the output voltage increases, and $C1$ is opened because of the action of $L1$. The cycle of events occurs very rapidly, many times per second. The terminal voltage of the generator thus varies slightly but rapidly above and below an average value determined by the tension of spring $S1$, which may be adjusted.

The purpose of the vibrator-type current limiter is to limit the output current of the generator automatically to its maximum rated value in order to protect the generator. As shown in figure 10–25, $L3$ is in series with the main line and load. Thus, the amount of current flowing in the line determines when $C2$ will be opened and $R2$ placed in series with the generator field. By contrast, the voltage regulator is actuated by line voltage, where-

as the current limiter is actuated by line current. Spring $S2$ holds contact $C2$ closed until the current through the main line and $L3$ exceeds a certain value, as determined by the tension of spring $S2$, and causes $C2$ to be opened. The increase in current is due to an increase in load. This action inserts $R2$ into the field circuit of the generator and decreases the field current and the generated voltage. When the generated voltage is decreased, the generator current is reduced. The core of $L3$ is partly demagnetized, and the spring closes the contact points. This causes the generator voltage and current to rise until the current reaches a value sufficient to start the cycle again. A certain minimum value of load current is necessary to cause the current limiter to vibrate.

The purpose of the reverse-current cutout relay (fig. 10–25) is to disconnect the battery automatically from the generator whenever the generator voltage is less than the battery voltage. If this device were not used in the generator circuit, an especially harmful action would occur if the engine were shut off. The battery would discharge through the generator. This would tend to make the generator operate as a motor, but because the generator is coupled to the engine it could not rotate such a heavy load. Under this condition, the generator winding may be severely damaged by excessive current.

There are two windings, $L4$ and $L5$, on the soft-iron core. The current winding, $L4$, consisting of a few turns of heavy wire, is in series with the line and carries the entire line current. The voltage winding, $L5$, consisting of a large number of turns of fine wire, is shunted across the generator terminals.

When the generator is not operating, the contacts, $C3$, are held open by spring $S3$. As the generator voltage builds up, $L5$ magnetizes the iron core. When the current (as a result of the generated voltage) produces sufficient magnetism in the iron core, contact $C3$ is closed, as shown. The battery then receives a charging current. The coil spring, $S3$, is so adjusted that the voltage winding will not close the contact points until the voltage of the generator is in excess of the normal voltage of the battery. The charging current passing through $L4$ aids the current in $L5$ in holding the contacts tightly closed. Unlike $C1$ and $C2$, contacts $C3$ do not vibrate. When the generator slows down, or for any other cause the generator voltage decreases to a certain value

below that of the battery, the current reverses through $L4$ and the ampere-turns of $L4$ oppose those of $L5$. Thus, a momentary discharge current from the battery reduces the magnetism of the core and $C3$ is opened, thereby preventing the battery from discharging into the generator and motoring it. $C3$ will not close again until the generator terminal voltage exceeds that of the battery by a predetermined value.

PARALLEL OPERATION

The generation and supply of electric power aboard ship is furnished by several generators in separate parts of the ship. These generators are operated in parallel. This arrangement is better than power supplied from a single large unit because parallel operation permits the number of generators in service to be varied in accordance with the load demand and they may be operated at or near their rated capacities. This practice increases the economy of operation because of increased efficiency. Also if a unit becomes disabled, continuity of electric service is maintained by other units of the plant that are in operating condition, and repairs underway are completed more readily.

In naval vessels that are provided with d-c electric power systems, the lighting circuits are designed to operate on 115 volts while most motors and other relatively large power-consuming devices are designed to operate on 230 volts. To avoid the use of separate generators for lighting and for power, a 3-wire generator is used that supplies both types of circuits simultaneously. The 3-wire generator is similar to a 2-wire generator except that the armature winding is tapped at points 180 electrical degrees (1-pole span) apart and these are brought out to slip rings on the back of the armature. These rings are connected by means of brushes to a reactance coil the center tap of which connects with the neutral, or third, wire of the 3-wire system.

The reactance coil (or balance coil) provides a method of establishing the potential of the neutral wire, which must be 115 volts positive with respect to one of the outer wires and 115 volts negative with respect to the other. Alternating current is produced in the armature of the d-c generator; and this is taken from the machine through the sliprings to which the balance coil is attached. The action of the alternating current in the balance coil is similar to that occurring in the primary winding of the single-

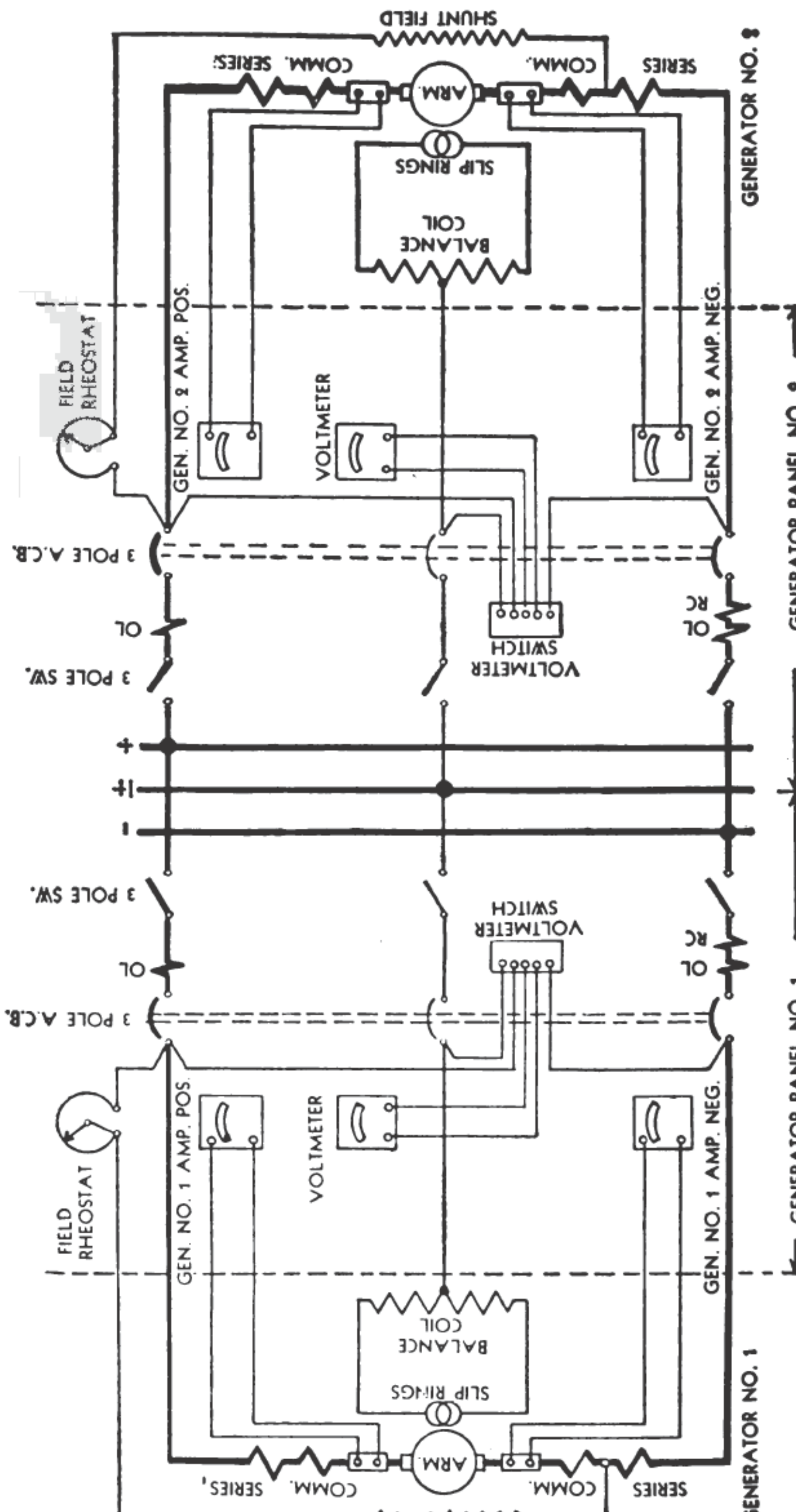


Figure 10–26.—Parallel operation of d-c generators.

phase transformer described in chapter 14. The potential of the center point on the balance coil is at the electrical center of the total voltage generated by the machine. Hence, this point can serve as the neutral, or center, between the positive and negative brushes. The neutral current passes through a portion of the balance coil, which has low resistance to direct current.

The connections for operating stabilized shunt generators in parallel are shown in figure 10–26. Each generator develops 230 volts across the two outside (positive and negative) leads, and 115 volts across either outside lead and neutral. Usually one ammeter and one voltmeter are provided on each generator panel.

Assume that generator No. 1 is supplying the load and it is desired to put generator No. 2 in service. The procedure for paralleling the d-c generators is as follows:

1. The brush rigging and armature of the incoming generator should be inspected for loose gear, and the field rheostat should be adjusted to the lowest voltage position.
2. The prime mover of the incoming generator (No. 2) is brought up to rated speed.
3. The voltage of the incoming machine is adjusted to about 2 volts higher than the bus voltage.
4. Closing the circuit breaker and switch places the generator in parallel with generator No. 1. The ammeter will indicate that generator No. 2 is now carrying a small portion of the load.
5. The field of generator No. 2 is strengthened until the load which it supplies is the proper value. At the same time the field of generator No. 1 should be weakened to maintain the normal voltage.

The procedure for removing a generator from its parallel connection with another is as follows:

1. The field of the generator being secured is weakened at the same time the field of the remaining generator is strengthened until the load on the outgoing machine is about 5 percent of the rated load current.
2. The circuit breaker is tripped and the switch is opened.

QUIZ

1. What type of energy conversion takes place in the d-c generator?
2. What two important functions are performed by the frame of the d-c generator?

3. What is the function of the field windings of a d-c generator?
4. Why is the field current in the d-c generator of figure 10–2 small at start?
5. Why does a high-voltage d-c generator require more commutator segments than a low-voltage generator?
6. If a given coil of wire is rotated in a magnetic field, upon what two factors will the induced emf depend?
7. In practical generators, how is the ripple in the output waveform reduced?
8. Why is there no circulating current in the generator armatures shown schematically in figure 10–6?
9. What is the pitch of an armature coil?
10. What are three reasons why Gramme-ring armatures have been replaced by drum-type armatures?
11. Why do fractional-pitch coils have reduced emf compared with the emf of full-pitch coils?
12. What will be the generated voltage of a 4-pole d-c generator with a simplex-lap armature winding having 10^5 lines of magnetic flux per pole, 440 armature conductors, and a speed of 3,600 rpm?
13. In a single simplex wave winding, what is the relation between the number of coils connected in series between adjacent commutator bars and the number of pairs of poles in the field?
14. In a single simplex wave wound armature there are always how many paths for current?
15. What is the advantage of using four brushes instead of two in the single simplex wave-wound generator shown in figure 10–10?
16. What is the voltage generated in a 4-pole d-c generator armature having a simplex wave winding, 10^6 lines of magnetic flux per pole, 4,000 armature conductors, and a speed of 600 rpm?
17. What are the three losses in every d-c generator armature?
18. What is the limiting factor in load on a d-c generator?
19. Why is a d-c generator armature core laminated?
20. Reducing the armature lamination thickness from ⅛ inch to $\frac{1}{16}$ inch will have what relative effect on the eddy-current loss?
21. What is the name applied to the heat loss that occurs when magnetic particles are rotated in a mass of iron?
22. How does armature reaction affect the magnetic field of a shunt generator?
23. How are the effects of armature reaction reduced in a d-c generator?
24. How does a compensating winding eliminate armature reaction?

25. What is the process called by which the current in the individual armature coils of a d-c generator is reversed and fed to the external circuit during the brief interval required for each commutator segment to pass under a brush?

26. What is the effect on the commutator of a failure to neutralize the emf of self-induction in the armature coils during the time they are being commutated?

27. What is the disadvantage of using manual-brush shift to compensate for the effects of a widely varying load?

28. How is the commutating field flux provided in a d-c generator without having to shift the brushes?

29. What relative turning effort is required to drive a shunt generator when the armature current is doubled?

30. What provides the initial voltage that enables a d-c generator to commence building up its voltage?

31. Why is the field strength of a d-c generator not always directly proportional to the field current?

32. What is the function of the field rheostat in the circuit of figure 10–18?

33. In the shunt generator shown in figure 10–18, what is the resistance of the field circuit when the generator terminal voltage is 60 volts?

34. Why does the terminal voltage of a shunt generator driven at constant speed vary only slightly with increased load?

35. What are the three factors that cause the drop in the external characteristic curve of a shunt generator?

36. In a compound generator, what is the effect of the additive series field?

37. Compare the magnitude of the no-load with that of the full-load voltage for (a) under-compound, (b) flat-compound, and (c) over-compound generators.

38. What is the percent regulation of a d-c generator when the no-load voltage is 250 volts and the full-load voltage is 240 volts?

39. The stabilized shunt generator represents a compromise between what two types of generators?

40. In the circuit represented by figure 10–21, how is the terminal voltage affected by a reduction in the field rheostat resistance?

41. In the circuit of figure 10–22, what is the effect on the output voltage when contact *C* is closed?

42. What sequence of events follows a reduction in terminal voltage because of increased load when the tilted-plate voltage regulator is used?

43. In the circuit of figure 10–25, when $C1$ is open, what is the effect of $L2$ on $L1$?
44. What is the purpose of the vibrator-type current limiter, as shown in figure 10–25?
45. What is the purpose of the reverse-current cutout relay shown in figure 10–25?

CHAPTER 11

DIRECT-CURRENT MOTORS

INTRODUCTION

Many kinds of work are performed by electric motors aboard ship. These include turning fan blades, running hydraulic pumps, turning rudders, training and elevating guns, and sometimes turning the propellers. Every one of these jobs consumes mechanical power. The electric motor converts electric power into mechanical power. Electric power is the simplest, safest, and most economical type of power, and for these reasons it is widely used aboard ship. For example, compare the use of a steam engine to operate the anchor winch with that of an electric motor to do the same job. Running a steam pipe to the anchor winch cuts through many bulkheads and decks. Watertight integrity is impaired. However, an electric cable can make the same run, and the kick pipes and stuffing tubes through decks and bulkheads will preserve watertight integrity throughout the entire run. Suppose enemy fire should burst a steam line between the boiler and the steering engines. Power is drained off the boilers, and men may be injured by escaping steam. If an electric cable is damaged, an open circuit might result with no casualties sustained by personnel. The worst that might happen to the cable would be a short circuit, in which event fuses or circuit breakers would quickly open the circuit and render it harmless. There are many reasons for using electric motors. They are safe, convenient, easily controlled, and easily supplied with power.

The construction of a d-c motor is essentially the same as that of the d-c generator shown in figures 10–1 and 10–2. The d-c generator converts mechanical energy into electrical energy, and the d-c motor converts the electrical energy back into mechanical energy. A d-c generator may be made to function as a motor by applying a suitable source of direct voltage across the normal

output electrical terminals. Electrical power is thus applied, and output mechanical power is obtained via the armature shaft.)

There are various types of d-c motors (depending on the way the field coils are connected), each having characteristics that are advantageous under given load conditions.

SHUNT MOTORS have the field coils connected in parallel with the armature circuit. This type of motor with constant potential applied, develops variable torque at an essentially constant speed, under changing load conditions. Such loads are found in machineshop drives. They include lathes, milling machines, drills, planers, shapers, and so forth.

SERIES MOTORS have the field coils connected in series with the armature circuit. This type of motor with constant potential applied, develops variable torque with a widely varying speed, under changing load conditions. That is, the speed is low under heavy loads, but becomes excessively high under light loads. Such loads are the drive on electric cranes, hoists, winches, and certain types of vehicles (for example, electric trucks). Series motors are also used extensively to start internal combustion engines.

COMPOUND MOTORS have one set of field coils in parallel with the armature circuit and another set of field coils in series with the armature circuit. This type of motor is a compromise between shunt and series motors. It develops an increased starting torque over that of the shunt motor and has less variation in speed than the series motor. Shunt, series, and compound motors are all d-c motors designed to operate from constant-potential variable-current d-c sources.

STABILIZED SHUNT MOTORS have a light series winding in addition to the shunt field. The action is similar to ordinary shunt motors except that stabilized shunt motors have less field iron and are lighter in weight. Without the stabilizing series field, these motors would increase in speed when the load increases, because of the weakened field due to armature reaction. With the stabilizing winding, they have the characteristics of a shunt motor with a strong field and low armature resistance.

Schematic diagrams of the four basic types of d-c motors are shown in figure 11–1.

Direct-current motors may also be classified in other ways. For example, they may be classified according to the degree of enclosure as OPEN (fig. 11–2, A), DRIP-PROOF (fig. 11–2, B), ENCLOSED (fig. 11–2, C), and so forth.

The open-type motor shown in figure 11–2, A, has end bells which offer little or no restriction to ventilation.

The drip-proof motor shown in figure 11–2, B, is protected from falling moisture and dirt from any direction up to a 45° angle with respect to the vertical. A motor of this type has all ventilation openings protected with wire screens or perforated covers, as shown in the figure. The openings do not exceed an area of one-half square inch.

An enclosed motor like the one in figure 11–2, C, is totally enclosed except for openings provided for the admission and discharge of air. These openings are connected to inlet and outlet ducts or pipes.

Other motor types include a spray-tight motor so constructed that a stream of water from a hose may be played upon it from any direction without leakage into the motor.

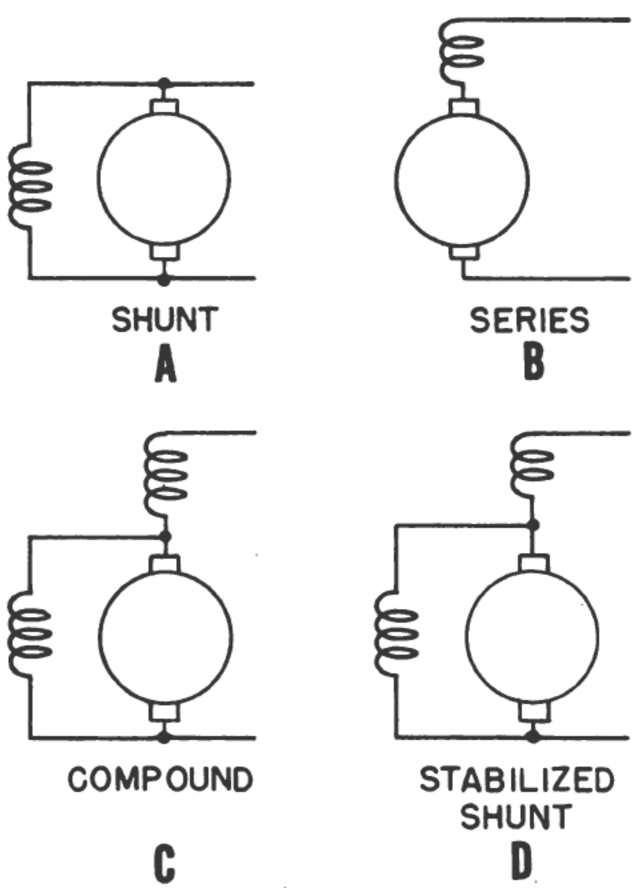


Figure 11–1.—Schematic diagrams of four types of d-c motors.

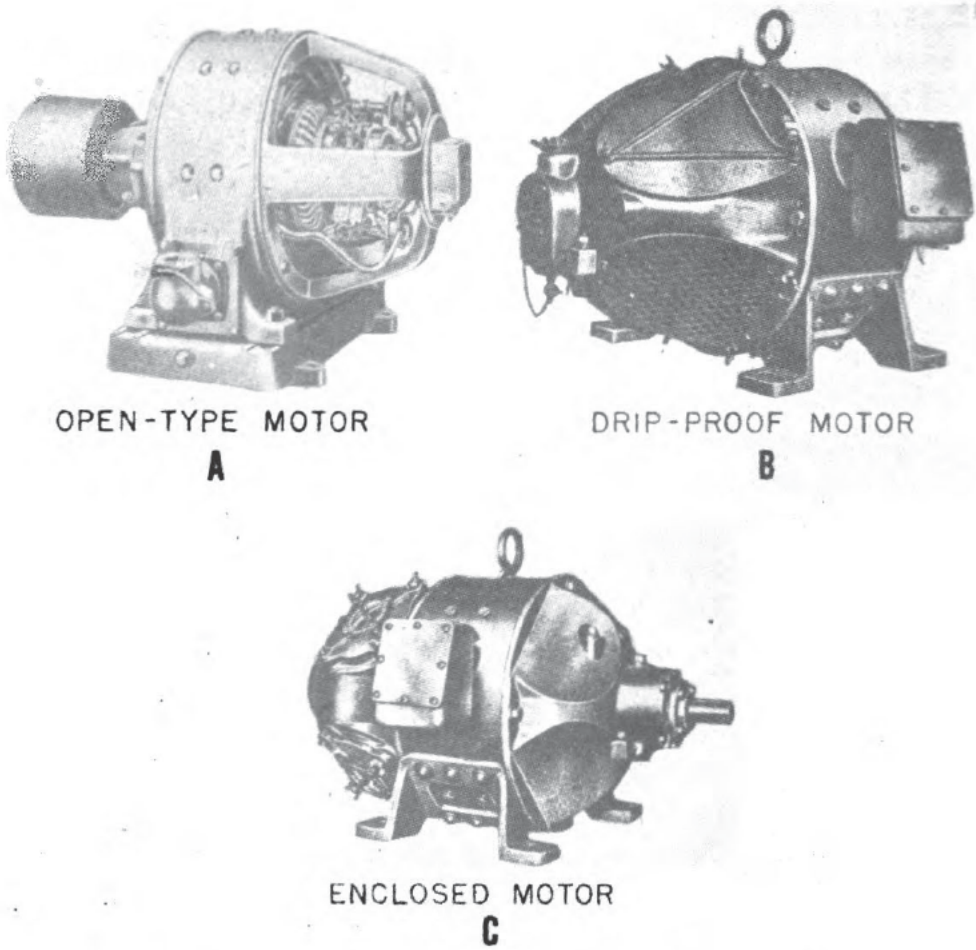


Figure 11–2.—Types of d-c motors classified according to enclosure.

A submersible motor is one that will operate under water.

With reference to cooling, the following types of motors are found aboard ship:

1. NATURAL VENTILATED MOTORS, cooled by the natural circulation of the air caused by the rotation of the armature.
2. SELF-VENTILATED MOTORS, cooled by a fan attached to the armature of the motor.
3. SEPARATELY VENTILATED MOTORS, cooled by an independent fan or blower apart from the motor.

With reference to speed, motors are classified as:

1. CONSTANT-SPEED MOTORS in which the speed varies only slightly between no load and full load. A shunt motor is a constant-speed motor.

2. MULTISPEED MOTORS which can be operated at any one of several definite speeds, each speed being nearly constant with load change. Such motors cannot be operated at intermediate speeds. A motor with two windings on the armature is an example of a multispeed motor.
3. ADJUSTABLE-SPEED MOTORS whose speed can be varied gradually over a wide range, but when once adjusted remains at nearly constant speed with load change.
4. VARYING SPEED MOTORS in which the speed varies with the load. Ordinarily the speed decreases as the load increases. The series motor is an example of this type of motor.
5. ADJUSTABLE VARYING SPEED MOTORS in which the speed may be adjusted over a wide range for any given load; but if the speed is adjusted at a given load, the speed will vary with any change in load.

Motors are also classified according to the type of duty they are to perform. For example, a CONTINUOUS-DUTY MOTOR is capable of operating continuously at its rated output without exceeding specified temperature limits. An INTERMITTENT-DUTY MOTOR is capable of being operated at its rated output for a limited period without exceeding its specified temperature limit.

Most d-c motors on Navy vessels are designed to operate on constant-potential 2-wire circuits of either 115 volts or 230 volts. These circuits may be combined in a 3-wire d-c supply with 230 volts between each outside wire, and 115 volts between each outside wire and middle or neutral wire.

PRINCIPLES OF D-C MOTORS

Force Acting on a Conductor

The operation of a d-c motor depends on the principle that a current-carrying conductor placed in, and at right angles to, a magnetic field tends to move at right angles to the direction of the field, as shown in figure 11–3. This action was previously described under the operating principle of the D'Arsonval meter movement in chapter 9.

The magnetic field between a north and a south pole of a magnet is shown in figure 11–3, A. The lines of force, comprising the field extend from the north pole to the south pole. A cross section of a current-carrying conductor is shown in figure 11–3, B.

The plus sign in the wire indicates that the electron flow is away from the observer. The direction of the flux loops around the wire is counterclockwise, as shown. This follows from the left-hand flux rule which states that if the conductor is grasped in the left hand with the thumb extended in the direction of the

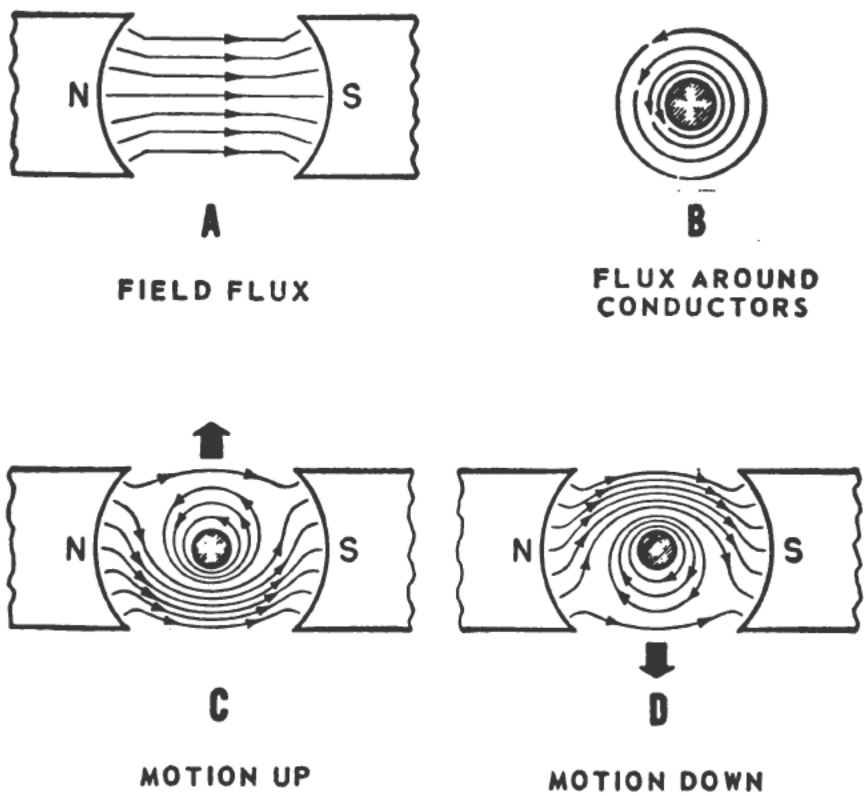


Figure 11–3.—Force acting on a current-carrying conductor in a magnetic field.

current flow, the fingers will curve around the conductor in the direction of the magnetic flux.

If the conductor (carrying the electron flow away from the observer) is placed between the poles of the magnet, as in figure 11–3, C, both fields will be distorted. Above the wire the field is weakened, and the conductor tends to move upward. The force exerted upward depends on the strength of the field between the poles and on the strength of the current flowing through the wire.

If the current through the conductor is reversed, as in figure 11–3, D, the direction of the flux around the wire is reversed.

The field below the conductor is now weakened, and the conductor tends to move downward.

A convenient method of determining the direction of motion of a current-carrying conductor in a magnetic field is by the use of the right-hand motor rule, as explained in chapter 9 and shown in figure 9–2, A.

Practical d-c motors depend for their operation on the interaction between the field flux and a large number of current-carrying conductors. As in d-c generators, the conductors are wound in slots in the armature; and the armature is mounted in bearings and is free to rotate in the magnetic field. An armature with two slots and two conductors (that is, a single conductor wound in the two slots) is shown in figure 11–4.

Figure 11–4, A, shows the uniform distribution of the main field flux when the field magnets are energized, and no current flows through the armature. The flux is concentrated in the air gap between the field and armature because of the relatively high permeability of the soft-iron armature core and the low reluctance of the magnetic circuit.

Figure 11–4, B, shows the magnetic fields surrounding the two active conductors when current flows through the armature coil in the direction indicated, and the field coils are not energized. The direction of the magnetic fields surrounding the two conductors is determined by means of the left-hand flux rule.

Figure 11–4, C, shows the resultant magnetic field produced by the interaction of the main field magnetomotive force and the armature magnetomotive force. Note that the flux is strengthened below the conductor at the north pole and above the conductor at the south pole because the lines are in the same directions at these points. Conversely, the flux is weakened above the conductor at the north pole and below the conductor at the south pole because the lines are opposite in direction and tend to cancel each other. The lines of force act like stretched rubber bands that tend to contract, with the result that the armature rotates in a clockwise direction. If either the direction of current through the armature coil or the polarity of the field is reversed (but not both), the direction of the force on the armature conductors reverses. If both the field polarity and the armature current are reversed, rotation continues in the same direction.

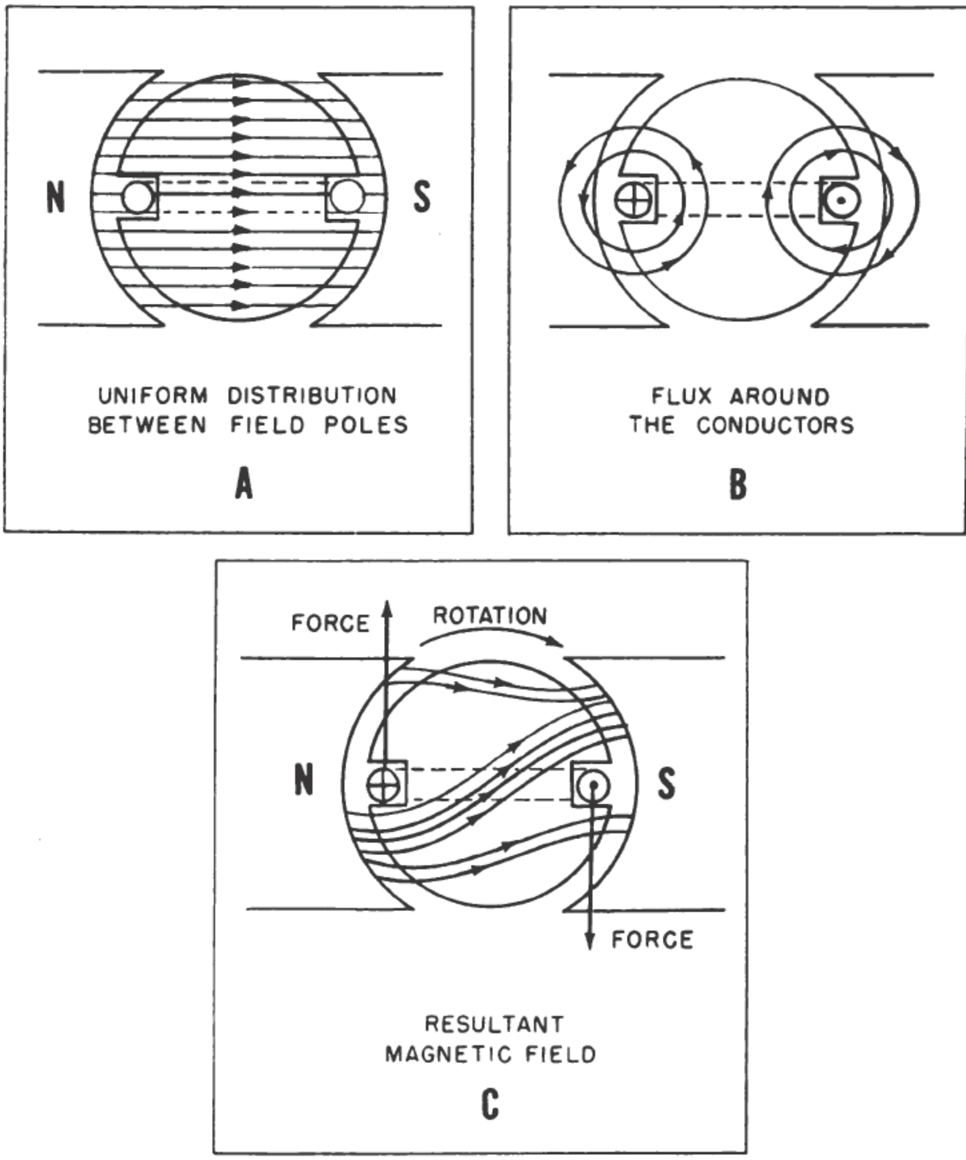


Figure 11–4.—Two-conductor armature.

A practical d-c motor (like the d-c generator, fig. 10–6, B) has many coils in the armature winding. The armature has many slots into which are inserted many turns of wire. This increases the number of armature conductors and thus produces greater torque and output. Power output is also increased by increasing the number of poles, as shown in the d-c generator windings in figures 10–8 and 10–10.

As in the case of the d-c generator, the d-c motor is equipped with a commutator and brushes.

The force, F, acting on a current-carrying conductor in a magnetic field is directly proportional to the field strength, the active length of the conductor (that part of the conductor contained in the armature slot and lying under a pole face), and the current flowing through it. The force, the conductor, and the field are assumed to be mutually perpendicular. This relationship is expressed algebraically as

$$F = \frac{8.85 \times BLI}{10^8},$$

where F is the force in pounds, B the flux density in lines per square inch, L the active length of the conductor in inches, I the current in amperes flowing through the conductor, and 8.85 is a constant that must be used when the above units are employed.

In a given motor, the length of the conductor is a fixed quantity; therefore, the only variables are the current and the flux. If the field flux is constant, the force acting on the conductor varies directly as the armature current. In other words, F is proportional to I. The following example will illustrate how the force acting on a conductor is calculated.

If a conductor having an active length of 10 inches and a current of 30 amperes is placed in a uniform field of 37,700 lines per square inch, what force will be exerted on the conductor? Thus

$$F = \frac{8.85BLI}{10^8} = \frac{8.85 \times 37{,}700 \times 10 \times 30}{10^8} = 1 \text{ pound.}$$

If the conductor is wound around an armature (fig. 11–4, C) and the armature current flows in the directions shown, there will be an upward force of one pound on the left of the armature and a downward force of one pound on the right of the armature. The net force acting to turn the armature is the sum of these two forces, or two pounds.

Torque

The TORQUE (or twist) on the armature is the product of the force acting at the surface of the armature times the perpendicular distance to the line of action of the force from the center of rotation of the armature. Assume the radius of the armature being considered is 1.5 feet, the torque exerted by each conductor

is $1 \times 1.5 = 1.5$ pound-feet; and the total torque exerted by both conductors is $1.5 + 1.5 = 3$ pound-feet.

The armature conductors for a motor are assembled in coils and connected to the commutator exactly as in a generator. In the 2-pole motor (fig. 11–5, A), the current divides equally in the two paths through the armature. Current flows in one direction in the conductors under the north pole and in the opposite direction in the conductors under the south pole. To develop a continuous motor torque the current in a coil must reverse when the coil passes the dead center position (top and bottom). The function of the commutator is to reverse the current at the proper time to maintain the current flow in the same direction in all conductors under a given pole. The total torque is the sum of the individual torques contributed by all the armature conductors. If there are 200 active conductors each developing a force of 1 pound the total torque will be $200 \times 1 \times$ the average moment arm.

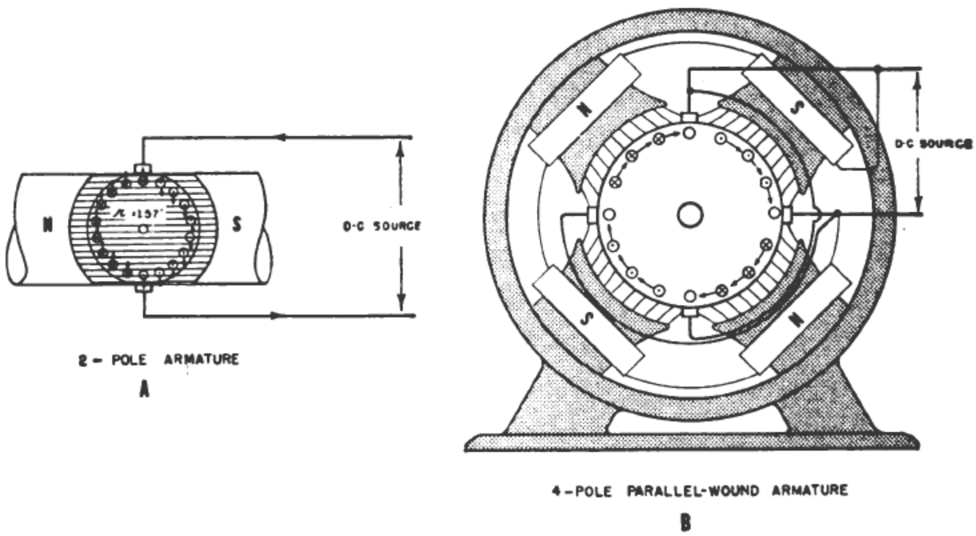


Figure 11–5.—Torque developed in a motor armature.

The AVERAGE MOMENT ARM is the average of all the perpendicular distances to the line of action of the forces from the center of rotation. If the field is assumed to consist of uniformly spaced horizontal lines of flux, the average moment arm will be 0.637 of the maximum moment arm, or radius, of the armature.

If the radius of the armature is 1.57 feet, the average moment arm is $0.637 \times 1.57 = 1$ foot. The total torque exerted by 200

armature conductors developing a force of 1 pound each is $200 \times 1 \times 1 = 200$ pound-feet.

In a 4-pole parallel-wound armature, the current divides into four paths as shown in figure 11–5, B. The current flows in opposite directions under alternate poles, as in the 2-pole armature, and develops a unidirectional force on all of the armature conductors. The total torque produced by the armature current is equal to the sum of the individual torques developed by each conductor. This torque, T, is proportional to the product of armature current and field strength.

$$T = K_t \Phi I_a, \tag{11–1}$$

where K_t is a constant that includes the number of armature conductors, number of paths, and other factors which are constant for a particular machine; Φ the flux per pole and I_a the total armature current. When the speed of a motor is constant, the generated torque due to the armature current is just equal to the retarding torque caused by the combined effect of the friction losses in the motor and the mechanical load. The torque developed by the motor armature is a steady one and does not pulsate as in the case of reciprocating steam engines or internal combustion engines. This may be readily seen by comparing the acceleration of an electric locomotive with that of a steam locomotive.

Horsepower of a Motor

If the number of revolutions that the armature of a motor makes per minute (or per second), the effective armature radius at which the force acts, and the total force acting at and tangent to this effective radius are known, the horsepower may be determined. Briefly, the horsepower may be determined in the following manner.

Work is accomplished when force acts through distance. For example, when a force of one pound acts through a distance of one foot, one foot-pound of work is accomplished. If 33,000 foot-pounds of work is done in one minute, one horsepower of work is accomplished.

Assume that an armature makes 100 revolutions per minute and that the effective radius is 1.59 feet. The circumference (the

distance through which the force moves in one revolution of the armature) is

$$\text{circumference} = 2\pi r$$
$$= 2 \times 3.14 \times 1.59 = 10 \text{ feet}.$$

Assuming that a total effective force of 200 pounds acts on the armature tangent to the 10-foot circumference, the work done in one revolution is

$$\text{work} = \text{force} \times \text{distance} = 200 \times 10 = 2{,}000 \text{ foot-pounds}.$$

The work done in 100 revolutions (one minute) is

$$2{,}000 \times 100 = 200{,}000 \text{ foot-pounds}.$$

The horsepower is therefore,

$$\text{hp} = \frac{\text{foot-pounds per minute}}{33{,}000} = \frac{200{,}000}{33{,}000} = 6.06 \text{ horsepower}.$$

The horsepower developed by a motor armature may be derived from the general expression,

$$\text{hp} = \frac{FV}{33{,}000}, \tag{11-2}$$

where F is the total force in pounds tangent to the effective circumference of the armature, and V the velocity of a point on this circumference in feet per minute. Velocity is determined as

$$V = 2\pi rN, \tag{11-3}$$

where r is the effective radius of the armature in feet, and N is the armature speed in revolutions per minute. The effective circumference is equal to $2\pi r$.

Substituting equation (11–3) in equation (11–2),

$$\text{hp} = \frac{2\pi rNF}{33{,}000}. \tag{11-4}$$

The torque in pound-feet produced by the motor armature is

$$T = rF. \tag{11-5}$$

Substituting equation (11–5) in equation (11–4) and dividing numerator and denominator by 2π,

$$\text{hp}=\frac{NT}{5{,}252}. \tag{11-6}$$

Substituting the values $N=100$ rpm and $T=1.59\times 200=318$ pound-feet of the preceding example in equation (11–6), the result is

$$\text{hp}=\frac{100\times 318}{5{,}252}=6.06.$$

Thus, the horsepower developed by a motor depends on its speed and torque. Large, slow-speed motors develop a large torque; whereas other much smaller motors of the same horsepower rating operate at reduced torque and increased speed.

Counter Emf

The MOTOR ACTION of a generator was treated in chapter 10; the GENERATOR ACTION of a motor is now considered. By applying the right-hand motor rule to an armature conductor carrying current in the direction indicated in figure 11–6, it will be found that the conductor is forced upward. As the conductor is moved up through the field, it cuts lines of flux and has a voltage induced in it. Applying the left-hand generator rule, it is found that this generated voltage is in opposition to the impressed emf. This counter voltage is induced in the windings of any rotating motor

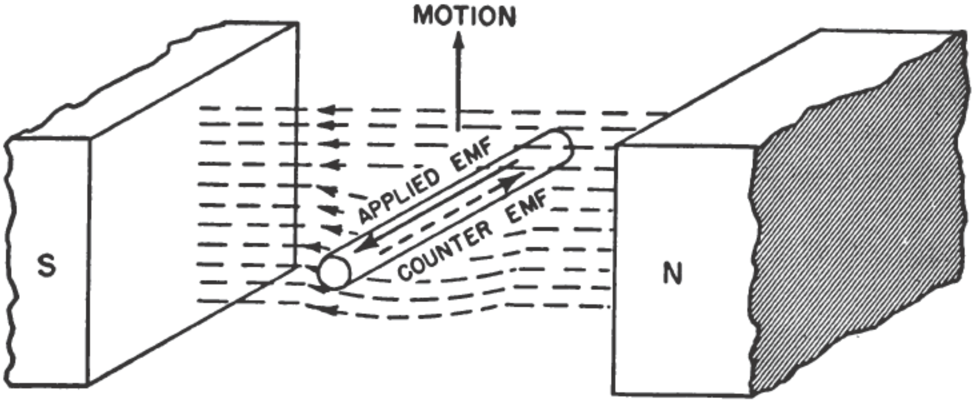


Figure 11–6.—Generator action in a motor.

armature and always opposes the impressed voltage. It is called COUNTER ELECTROMOTIVE FORCE and is directly proportional to the speed of the armature and the field strength. That is, the counter emf is increased or decreased if the speed is increased or decreased respectively; the same is true if the field strength is increased or decreased.

The EFFECTIVE VOLTAGE (IR drop) in the armature is equal to the impressed voltage minus the counter emf. The armature IR drop varies directly with the current flowing in the armature and the resistance of the armature.

To produce an armature current, I_a, in an armature of resistance, R_a, requires an effective voltage of I_aR_a.

The current flowing through the armature can be found by the equation

$$I_a=\frac{E_a-E_c}{R_a},$$

where I_a is the current flowing through the armature, E_a the impressed (or applied) voltage across the armature, E_c the counter emf, and R_a the armature resistance. This equation can be transposed and written as

$$E_c=E_a-I_aR_a. \tag{11–7}$$

In the case of the generator, the generated emf is equal to the terminal voltage plus the armature resistance drop; and in the case of the motor the generated or counter emf is equal to the terminal voltage minus the voltage drop in the armature resistance.

Expressing E_a in terms of E_c and I_aR_a,

$$E_a=E_c+I_aR_a.$$

This formula is valid for any d-c motor regardless of speed or load.

The counter emf, E_c, that is induced in a motor armature may be determined by an equation similar to that used to determine the generator voltage in a generator in chapter 10. This equation is

$$E_c=\frac{\Phi ZNP}{10^8p}, \tag{11–8}$$

where Φ is the number of lines of flux per pole, Z the number of face conductors, N the armature speed in revolutions per second,

P the number of field poles, and p the number of parallel paths through the armature circuit.

Armature Reaction

The armature current in a generator flows in the same direction as the generated emf, but the armature current in a motor flows in the opposite direction to that of the counter emf. Assume that the field of the motor (fig. 11–7, A) is of the same polarity as the field of the generator (fig. 10–12, A). For the same direction of armature rotation, the armature flux of the motor (fig. 11–7, B) is in the opposite direction to that of the generator (fig. 10–12, B). In a motor the main field flux is

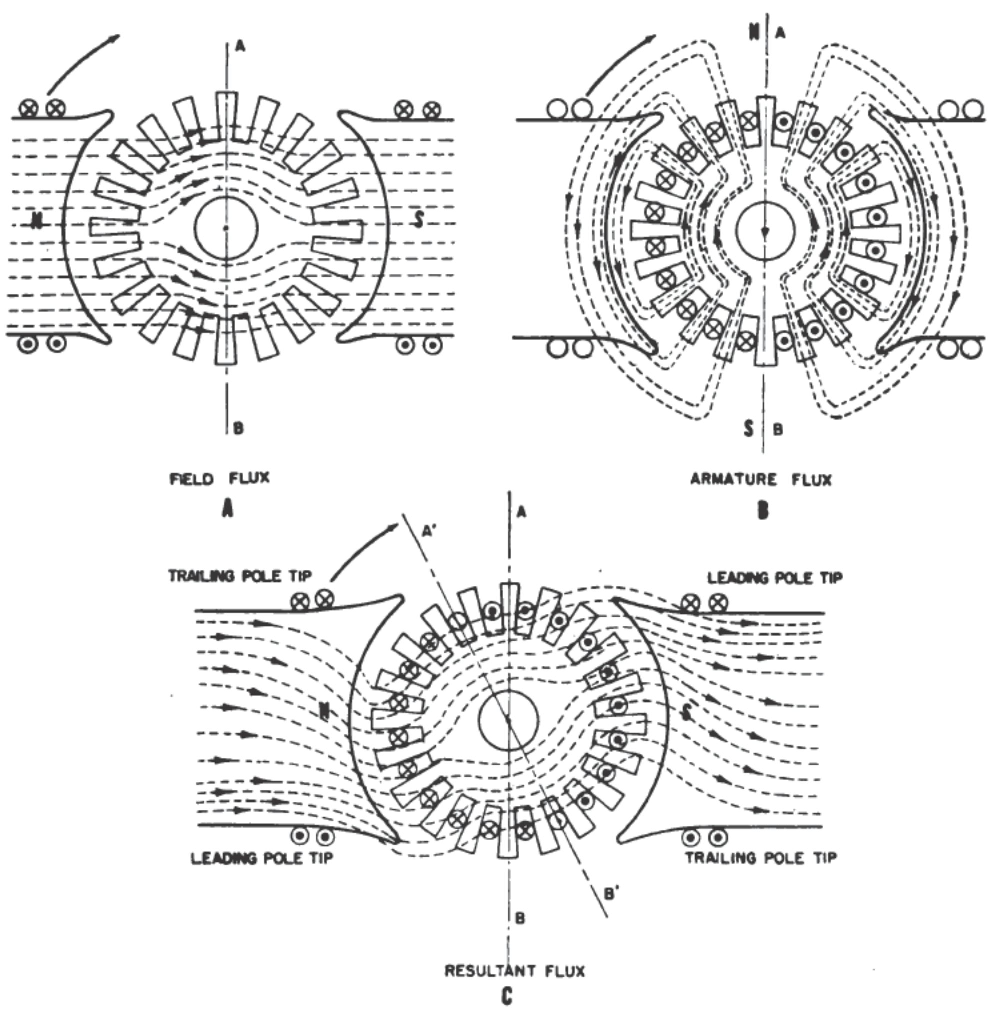


Figure 11–7.—Armature reaction in a motor.

always distorted in the opposite direction to rotation (fig. 11–7, C), whereas in a generator the main field flux is distorted (fig. 10–12, C) in the same direction as that of rotation. Note that the resultant field in the motor (fig. 11–7, C) is strengthened at the leading pole tips and weakened at the trailing pole tips. This action causes the electrical neutral plane to be shifted back to *A′B′*. Thus, to obtain good commutation in a motor without interpoles, it is necessary to shift the brushes from the mechanical neutral, *AB,* in a direction opposite to that of the armature rotation.

The armature reaction is overcome in a motor by the same methods as for the generator—that is, by the use of laminated pole tips with slotted pole pieces, and compensating windings. In each case the effect produced is the same as that produced in the generator, but it is in the opposite direction.

Commutation

The brush axis, *A′B′* (fig. 11–7, C), could be made to coincide exactly with the neutral plane of the combined field. This would eliminate sparking at the brushes if it were not for the self-induced emf in the commutated coils. Because of the necessity for neutralizing this emf to eliminate sparking, the brushes must be set slightly behind the neutral plane in a motor. Thus, in both the generator and the motor, the brushes must be moved from the mechanical neutral slightly beyond the electrical neutral plane to neutralize the effect of self-inductance.

The current flowing in the armature coils of a motor (fig. 11–8) must be periodically reversed when the coils are being short-circuited by the brushes in order to maintain a unidirectional torque as the coils move under alternate poles. When coil *A* is short circuited by a brush, its current immediately starts to decrease to zero; from zero, the current then builds up to a maximum in the reversed direction by the time the coil moves from position ① to position ②, where the brush no longer short circuits it. As a result of the changing current, a self-induced voltage is set up in the shorted coil, which tends to keep the current from changing. To obtain sparkless commutation it is necessary to overcome this self-induced voltage.

In any motor the current flows as a result of the applied voltage, and the counter emf opposes the flow of current. As the

current in the commutated coil decreases to zero, the emf of self-induction tends to keep the current flowing in the same direction, as indicated by the arrow in position ①. Therefore, it aids the applied voltage during this portion of the cycle. As the coil moves into position ② the current increases in the opposite direction, and the self-induced voltage again opposes this current

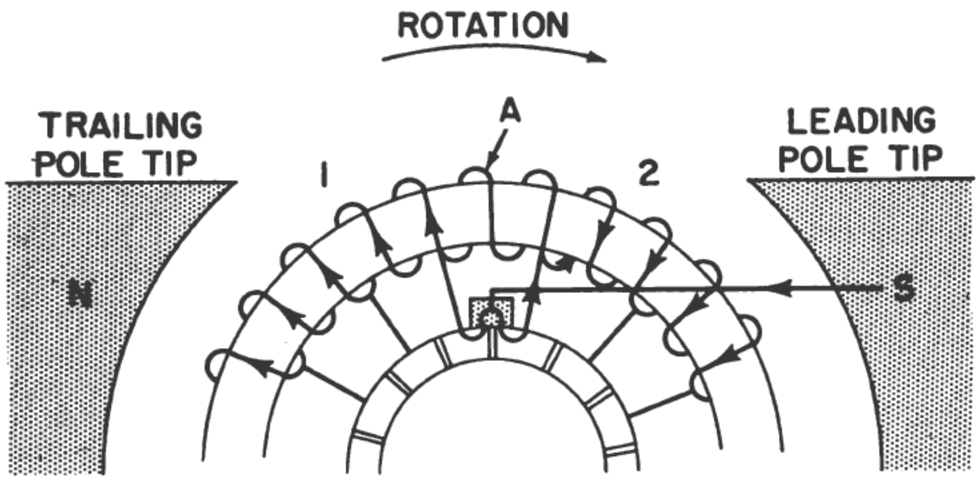


Figure 11–8.—Commutation in a motor.

increase. To overcome the self-induced voltage, another voltage is introduced into the coil, which opposes the self-induced voltage. The counter emf is in the proper direction to accomplish this, but the coil being commutated is in the neutral plane and consequently has no counter emf generated in it.

When the brushes are shifted against the direction of rotation, the coils that are being shorted by the brushes still cut lines of flux from the previous north pole and have a small amount of counter emf generated in them. Because this counter emf opposes the applied voltage it also opposes the self-induced voltage, thus resulting in rapid reversal of coil current and in sparkless commutation.

When the load on the motor increases, armature reaction increases and the electrical neutral plane is shifted further in the direction opposite to that of the armature rotation. To maintain sparkless commutation, the plane of the brushes will have to be shifted slightly beyond the electrical neutral plane. When the load is reduced, the brushes are shifted in the opposite direction. Thus, for sparkless commutation, it is necessary to manually shift the brushes when the load varies.

Commutating Poles

Commutating poles are as important in motors as in generators. Nearly all motors of more than 1 horsepower depend on COMMUTATING POLES, sometimes called INTERPOLES, rather than on the shifting of the brushes to obtain sparkless commutation. Essentially the only difference between the interpole generator and the interpole motor is that in the generator the interpole has the same polarity as the main pole AHEAD of it in the direction of rotation, but in the motor the interpole has the same polarity as the main pole BACK of it.

Without interpoles the emf of self-induction is overcome by commutating the coil while it is cutting the flux of the main pole it is just leaving. Therefore, if the interpole is to overcome the emf of self-induction, it must have the same polarity as the

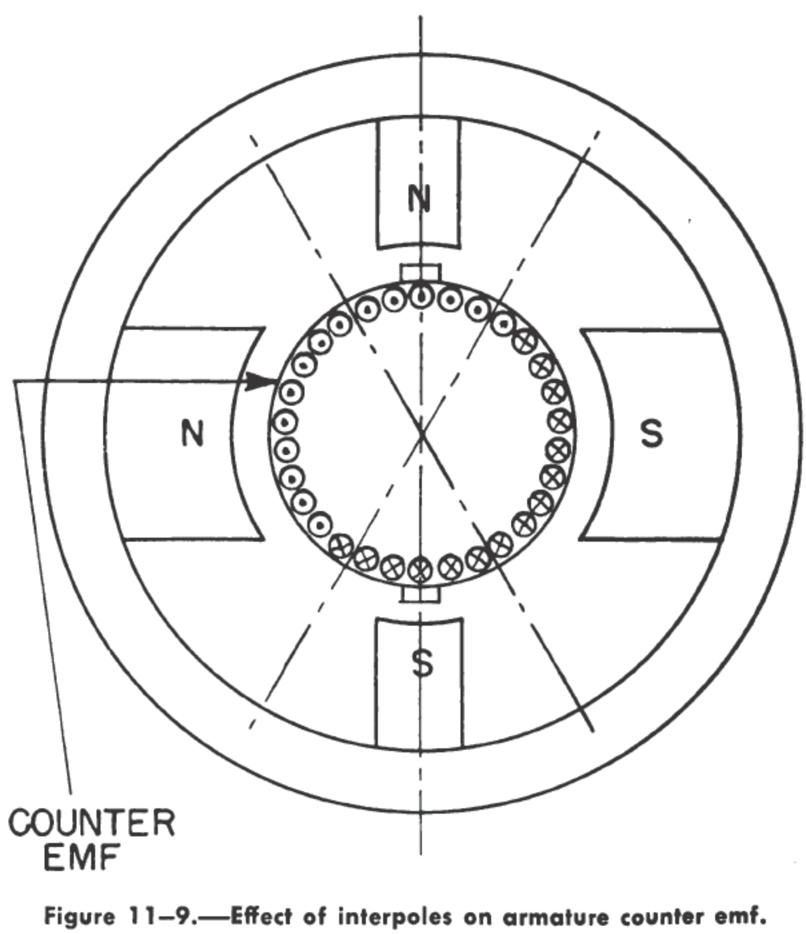


Figure 11–9.—Effect of interpoles on armature counter emf.

main pole it is leaving. This is a north pole in the example of figure 11–8.

The interpole series field coil in a motor is connected to carry the armature current, as in the generator. As the load varies, the interpole flux varies, and commutation is automatic with load change. It is not necessary to shift the brushes when there is an increase or decrease in load. The brushes are located on the no-load neutral and remain in that position for all conditions of load.

The motor may be reversed by reversing the direction of the current in the armature. When the armature current is reversed, the current through the interpole is also reversed, and therefore the interpole still has the proper polarity to provide automatic commutation.

The direction of the counter emf in the interpole motor with clockwise rotation (fig. 11–9) illustrates that coils under the interpoles do not contribute to the armature counter voltage, but that only those coils under the main poles contribute to it. The total counter voltage generated in the conductors on each half of the armature is equal to the algebraic sum of the voltages generated in each conductor between the upper and lower brush. Because those under the upper north interpole generate an equal and opposite voltage to those under the lower south interpole, the remaining conductors under the main field poles generate the armature counter emf that controls the armature load current and the speed-torque characteristics of the motor. It is important that the brushes be set in the correct neutral plane, otherwise the commutation will be impaired and the motor characteristics will be altered.

Speed Regulation

SPEED REGULATION is the ability of a motor to maintain its speed when a load is applied. It is an inherent characteristic of a motor and remains the same as long as the applied voltage does not vary—that is, unless a physical, or mechanical, change is made in the machine. The speed regulation of a motor is a comparison of its no-load speed to its full-load speed and is expressed as a percentage of full-load speed. Thus,

$$\text{percent speed regulation} = \frac{\text{no-load speed} - \text{full-load speed}}{\text{full-load speed}} \times 100.$$

For example, if the no-load speed of a shunt motor is 1,600 rpm and the full-load speed is 1,500 rpm the speed regulation is

$$\frac{1{,}600-1{,}500}{1{,}500}\times 100=6.6 \text{ percent.}$$

The LOWER the speed-regulation percentage figure of a motor, the more constant the speed will be under varying load conditions and the BETTER will be the speed regulation. The HIGHER the speed-regulation percentage figure, the POORER is the speed regulation.

SPEED CONTROL refers to the external means of varying the speed of a motor under any load.

SHUNT MOTORS

Speed Regulation of Shunt Motors

The field circuit of a shunt motor is connected across the line and is thus in parallel with the armature the same as it is in the shunt generator (fig. 10–16, B). If the supply voltage is constant, the current through the field coils, and consequently the field flux, will be constant.

When there is no load on the shunt motor, the only torque necessary is that required to overcome bearing friction and windage. The rotation of the armature coils through the field flux establishes a counter emf that limits the armature current to the relatively small value required to establish the necessary torque to run the motor on no load.

When an external load is applied to the shunt motor it tends to slow down slightly. The slight decrease in speed causes a corresponding decrease in counter emf. If the armature resistance is low, the resulting increase in armature current and torque will be relatively large. Therefore, the torque is increased until it matches the resisting torque of the load. The speed of the motor will then remain constant at the new value as long as the load is constant.

Conversely, if the load on the shunt motor is reduced, the motor tends to speed up slightly. The increased speed causes a corresponding increase in counter emf and a relatively large decrease in armature current and torque.

Thus, it may be seen that the amount of current flowing through the armature of a shunt motor depends on the load on the motor. The larger the load, the larger the current; and conversely, the smaller the load, the smaller the current. The change in speed causes a change in counter emf and armature current in each case.

Figure 11–10 indicates graphically what happens when the load is applied or removed from a shunt motor. In this figure it is assumed that the field strength remains constant.

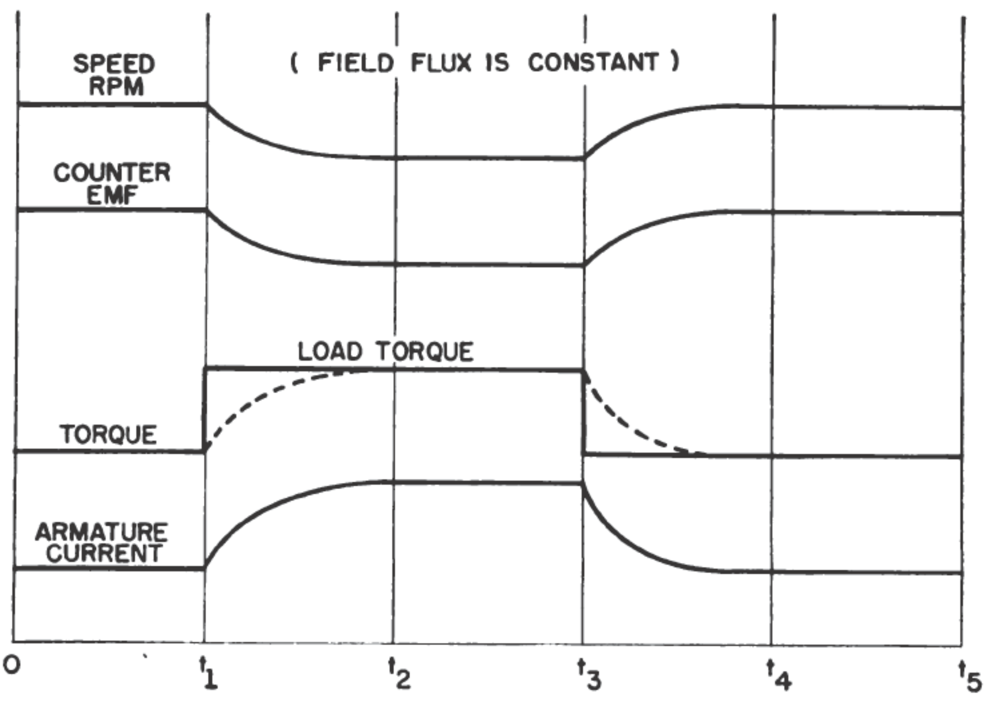


Figure 11–10.—Shunt motor load-speed-torque-emf relationships with respect to time.

During the interval between *0* and t_1 the motor operates under conditions of equilibrium—that is, the speed, counter emf, torque, and armature current have steady values. At t_1 a load is applied to the motor. Instantly, the load-torque curve rises, but the inertia of the armature prevents the generated torque from rising instantly with it. Because the load torque exceeds the generated torque in the motor armature, the motor speed is reduced until a balance is again obtained. When the generated torque is again equal to the load torque deceleration ceases and the motor operates at a constant reduced speed.

At time t_2 this state of equilibrium, corresponding to the new load, is reached. At time t_3 the load is suddenly removed. The load torque drops instantly, but the armature inertia prevents the generated torque from falling instantly with it. Likewise, the armature inertia prevents the armature speed from increasing suddenly. Because the generated torque exceeds the load torque, the motor accelerates. As the speed increases, the counter emf increases proportionately. This opposes the applied emf, and the armature current is reduced until the generated torque and load torque are again in a state of equilibrium. The speed then levels off to a constant value.

The operational features of a shunt motor are indicated in table 11. The variation of line current, armature current, counter emf, speed, torque, and efficiency, with varying load conditions are indicated. The values are based on a 1-horsepower 100-volt shunt motor having a field current of 1 ampere and an armature resistance of 1 ohm.

TABLE 11.—*Operational features of a shunt motor*

I Line (*amperes*)	I Armature (*amperes*)	Counter emf (*volts*)	Speed (*rpm*)	Torque (*lb-ft*)	Output $0.142NT$ (*watts*)	Input (*watts*)	Efficiency (*percent*)	Load
[1] 2	[1] 1	[1] 99	[1] 990	[1] 0.7	98.4	200	49.2	Light.
4	3	97	970	2.1	289	400	72.2	Do.
6	5	95	950	3.5	472	600	78.5	Do.
8	7	93	930	4.9	646	800	80.8	Do.
10	9	91	910	6.3	815	1,000	81.5	Normal.
21	20	80	800	14	1,590	2,100	76	Over.

[1] Assumed.

When the armature current is 1 ampere the armature IR drop is 1 volt, and the counter emf is the difference between the applied voltage and the IR drop, or $100-1=99$ volts. The speed at this load is assumed to be 990 rpm. At this load the motor torque is assumed to be 0.7 pound-foot. The output power, P_o, in watts is

$$P_o=\frac{NT}{5{,}252}\times 746=0.142NT=0.142\times 990\times 0.7=98.4 \text{ watts.}$$

The constant 746 is used to convert horsepower output into an equivalent number of watts output (1 horsepower=746 watts).

The total input current to the motor is equal to the sum of the field current and the armature current, or $1+1=2$ amperes. The input power is the product of the applied voltage and the total input current, or $100\times 2=200$ watts. The efficiency is

$$\frac{\text{output}}{\text{input}}\times 100=\frac{98.4}{200}=49.2 \text{ percent at this load.}$$

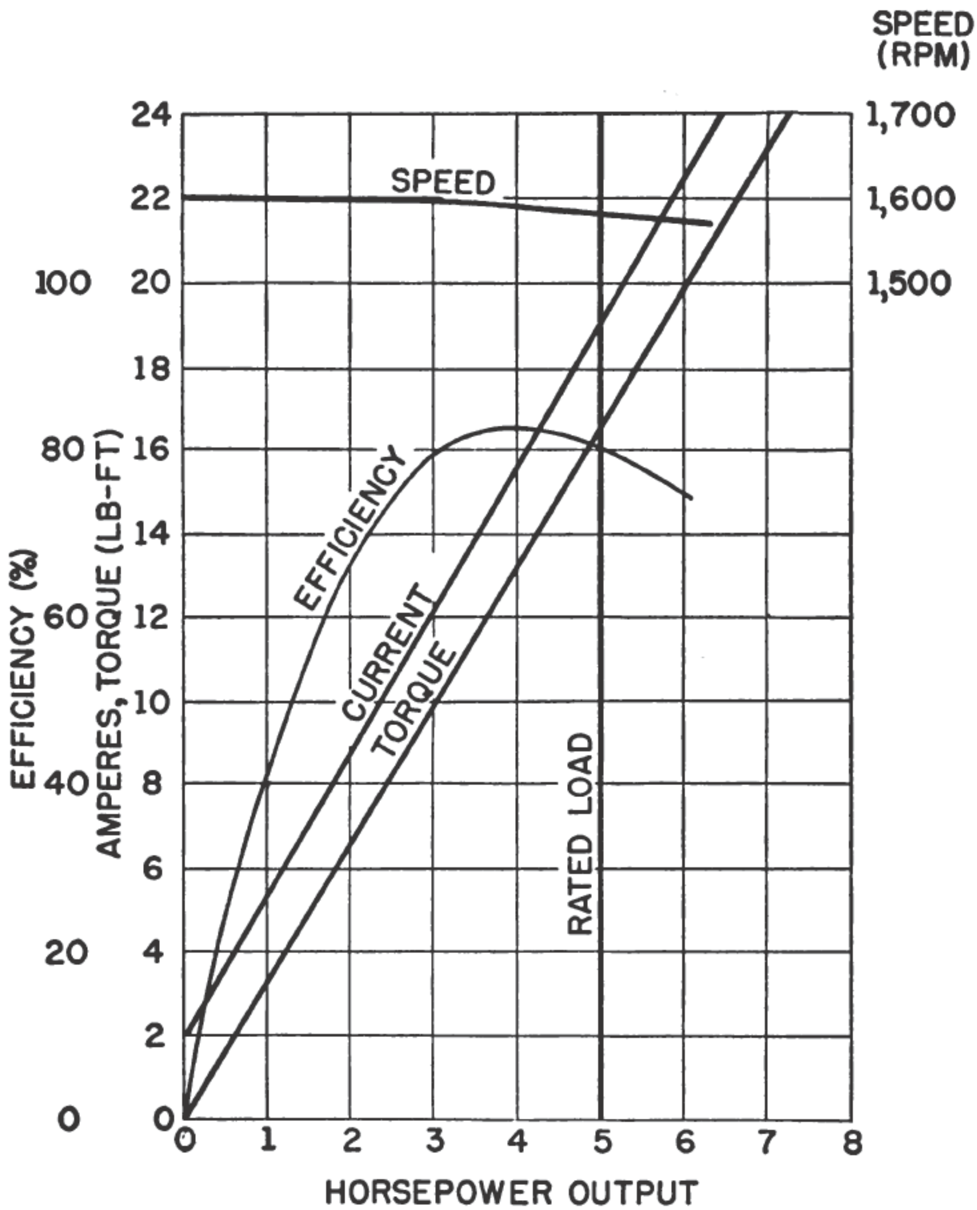


Figure 11–11.—Characteristic curves of a shunt motor.

The torque calculations for other values of load are based on the proportionality between armature current and torque. For example, if the torque is 0.7 pound-foot when the armature current is 1 ampere, the torque for an armature current of 2 amperes will be $2 \times 0.7 = 1.4$ pound-feet.

The speed calculations are based on the proportionality between speed and counter voltage. For example, when the armature current is 2 amperes the armature *IR* drop is $2 \times 1 = 2$ volts, and the counter emf is $100 - 2 = 98$ volts. If the speed is 990 rpm when the counter emf is 99 volts, the speed at 98 volts counter emf will be 980 rpm.

The characteristic curves of a 5-horsepower 230-volt shunt motor are shown in figure 11–11. These curves include speed, torque, efficiency, and input current. The speed curve is almost a horizontal line, and the regulation is good. The torque varies directly with output load and armature current because the field is approximately constant over the load range.

The speed regulation of a shunt motor is generally better than the voltage regulation of the same machine when operating as a shunt generator. This action is due to the fact that armature reaction in a generator weakens the field and lowers the terminal voltage, whereas armature reaction in a motor weakens the field and tends to increase the motor speed. Armature resistance is low, and a decrease in field strength and counter emf (for example, of 5 percent) might result in an increase in armature current and accelerating force on the armature conductors of 50 percent with resulting increase in armature speed. In general, however, armature reaction in a motor weakens the field only slightly and does not cause the speed to increase with load, but to decrease less than it would if the armature reaction were not present.

Uses of Shunt Motors

From the preceding considerations it is evident that a shunt motor is essentially a constant-speed machine. Although the speed may be varied by varying the current through the field winding (for example, by means of a field rheostat), the speed remains nearly constant for a given field current.

The constant-speed characteristic makes the use of shunt motors desirable for driving machine tools or any other device that requires a constant-speed driving source.

Where there is a wide variation in load, or where the motor must start under a heavy load, series motors have desirable features not found in shunt motors.

SERIES MOTORS

Speed Regulation of Series Motors

The field coils of a series motor are connected in series with the armature like those of the series generator in figure 10–16, B. With relatively low flux density in the field iron, the series field strength is proportional to armature current I_a. The torque developed by the motor armature is proportional to $I_a\Phi$; and because Φ is also proportional to I_a, the torque is proportional to I_a^2.

If the supply voltage is constant, the armature current and the field flux will be constant only if the load is constant. If there were no load (this is never done) on the motor, the armature would speed up to such an extent that the windings might be thrown from the slots and the commutator destroyed by the excessive centrifugal forces. The reason why the speed becomes excessive on no load is explained in the following manner.

The speed of a series motor may be expressed mathematically as

$$\text{rpm}=\frac{K[E-I_a(R_a+R_s)]}{\Phi}; \qquad (11\text{–}9)$$

Where K is a constant that depends on such factors as the number of poles, the number of current paths, and the total number of armature conductors; E is the voltage applied across the entire motor; I_a is the armature current; R_a is the armature resistance; R_s is the resistance of the series field; and Φ is the field strength. The factor $E-I_a(R_a+R_s)$ represents mathematically the counter emf, E_c. The applied voltage must then be equal to the counter voltage plus the voltage drop in the armature, or

$$E=E_c+I_a(R_a+R_s). \qquad (11\text{–}10)$$

The armature always tends to rotate at such a speed that the sum of the counter voltage, E_c, and the I_aR_a drop in the armature will equal the applied voltage, E. If the load is removed from the motor, the armature will speed up and a higher counter emf will be induced in the armature. This reduces the current through

the armature and the field. The weakened field causes the armature to turn faster. The counter emf is thus increased, and the speed is further increased until the machine is destroyed. For this reason, series motors are never belt-connected to their loads. The belt might come off and the motor would then overspeed and destroy itself. Series motors are always connected to their loads directly, or through gears.

As the load increases, the armature slows down and the counter emf is reduced. The current through the armature is increased and likewise the field strength is increased. This reduces the speed to a very low value. The armature current, however, is not excessive because the torque developed depends on BOTH the field flux and the armature current.

The operational features of a series motor are indicated in table 12. The variation of line current, counter emf, speed and torque under varying load conditions are indicated. The values are based on a 0.5-horsepower 100-volt series motor having a field resistance of 1 ohm and an armature resistance of 1 ohm.

TABLE 12.—*Operational features of a series motor*

I Armature (*amperes*)	Counter emf (*volts*)	Speed (*rpm*)	Torque (*lb-ft*)	Output 0.142 *NT* (*watts*)	Input (*watts*)	Efficiency (*percent*)
[1] 5	[1] 90	[1] 900	[1] 3	383.4	500	76
10	80	400	12	681.6	1,000	68
15	70	233	27	893.3	1,500	59
20	60	150	48	1,022.4	2,000	51
25	50	100	75	1,065.0	2,500	42

[1] Assumed.

When the armature current is 5 amperes, the armature and field IR drop is 10 volts, and the counter emf is the difference between the applied voltage and the IR drop, or $100-10=90$ volts. At this load the speed is assumed to be 900 rpm and the motor torque is assumed to be 3 pound-feet. The output power, P_o, in watts is

$$P_o=0.142NT=0.142\times900\times3=383.4 \text{ watts.}$$

The input power is the product of the applied voltage and the current through the field and the armature, or $100\times5=500$

watts. The efficiency (the output divided by the input) is $\frac{383.4}{500}$, or 76.8 percent.

The speed calculations are based on the following considerations. As has been explained (see equation 11–8), the counter emf, E_c, is determined as

$$E_c = \frac{\Phi ZNP}{10^8 p}.$$

It is also determined as

$$E_c = E - I_a(R_a + R_f).$$

These equations may be equated as

$$\frac{\Phi ZNP}{10^8 p} = E - I_a(R_a + R_f).$$

Assume that when 5 amperes flow through the field coil (and the armature) $\Phi = 10^6$ lines of force. Assume also that $Z = 600$ conductors. There are 2 poles ($P = 2$) and 2 paths ($p = 2$) through the armature. If N is divided by 60, the number of revolutions will be determined for an interval of 1 minute. Thus,

$$\frac{10^6 \times 600 \times N \times 2}{10^8 \times 60 \times 2} = 100 - 5(1+1),$$

and,

$$N = 900 \text{ rpm}.$$

If the current is increased to 10 amperes (doubled), the flux is also assumed to double. Thus,

$$\frac{2 \times 10^6 \times 600 \times N}{10^8 \times 60} = 100 - 10(1+1).$$

The number of revolutions per minute becomes

$$N = 400 \text{ rpm}.$$

The torque varies as the square of the current. Thus, when the current is doubled, the torque is increased four times.

The characteristic curves of a series motor are shown in figure 11–12. As in the case of the shunt motor, these curves include

speed, torque, efficiency, and input current. As stated previously, the torque varies as the square of the armature current (below saturation) and the speed decreases rapidly as the load is increased.

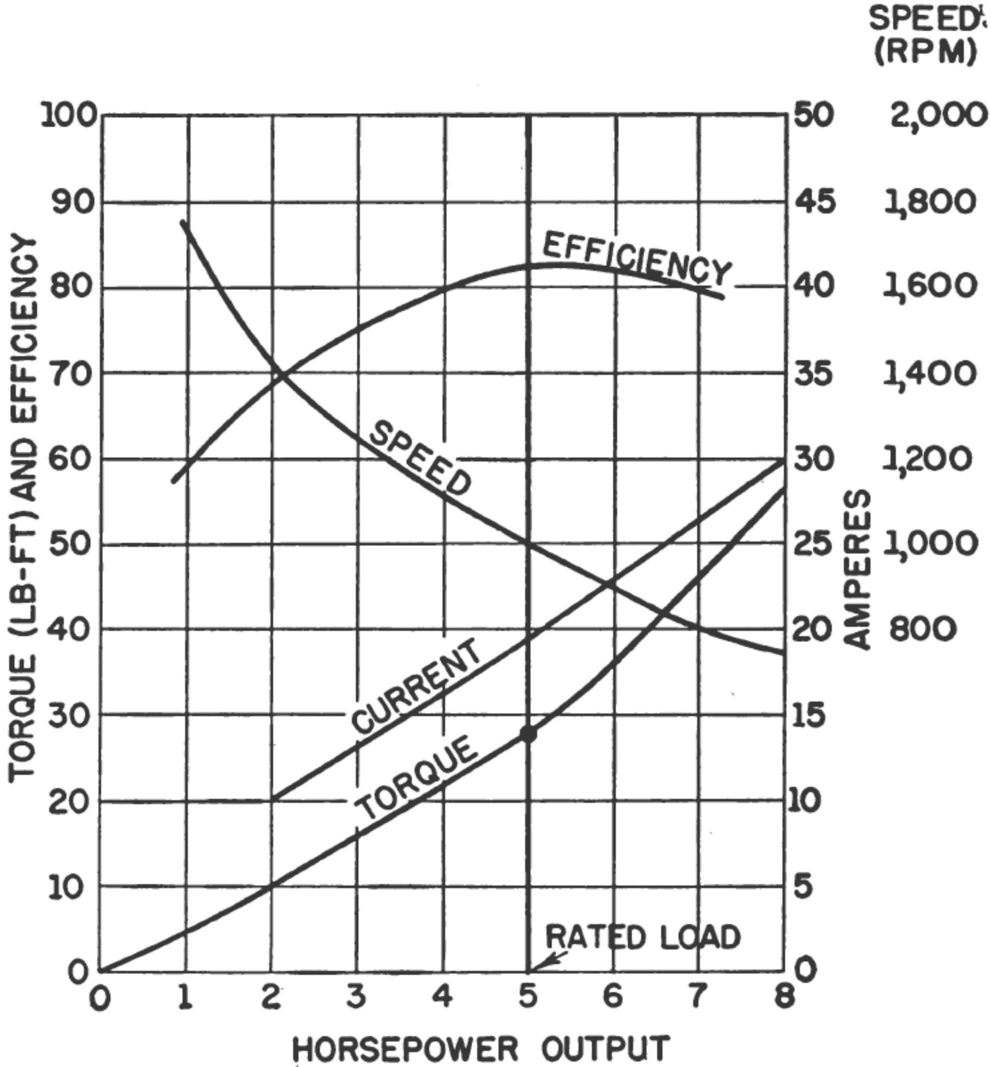


Figure 11–12.—Characteristic curves of a series motor.

Uses of Series Motors

As indicated in figure 11–12, the torque increases with load. When the load on a series motor is increased, the speed and the counter emf decrease and the armature current and field strength increase. There is therefore an increase in torque with decrease in speed with the result that the increased load on the motor is limited by the decrease in speed.

When a heavy load is suddenly thrown on a shunt motor, it attempts to take on the load at only slightly reduced speed and counter emf. The flux remains essentially constant and therefore the increased torque is proportional to the increase in armature current. With heavy overload the armature current becomes excessive and the temperature increases to a very high value. The shunt motor cannot slow down appreciably on heavy load, as can the series motor; hence the shunt motor is more susceptible to overload.

For example, assume that a d-c motor is used to move an electric truck up an incline. If it is a shunt motor it will attempt to maintain almost the same speed up the incline as on the level and consequently excessive current may be drawn through the armature, since the field flux remains almost constant. If it is a series motor, the speed must decrease more than the flux increases in order to have a reduction in counter emf and an increase in armature current $(E_c - \frac{\Phi ZNP}{10^8 p})$. The decrease in speed protects the series motor from excessive overload, and the armature current is limited by the counter voltage and the combined resistance of the armature and series field.

The series motor is therefore used where there is a wide variation in both torque and speed such as in traction equipment, blower equipment, hoists, cranes, and so forth.

COMPOUND MOTORS

Compound motors, like compound generators, have both a shunt and a series field. In most cases the series winding is connected so that its field aids that of the shunt winding, as shown in figure 11–13, A. Motors of this type are called CUMULATIVE COMPOUND motors. If the series winding is connected so that its field opposes that of the shunt winding, as shown in figure 11–13, B, the motor is called a DIFFERENTIAL COMPOUND motor. Under full load, the ampere-turns of the shunt coil are greater than the ampere-turns of the series coil.

In the CUMULATIVE COMPOUND motor the speed decreases (when a load is added) more rapidly than it does in a SHUNT motor, but less rapidly than in a series motor. Series field strength increases

as in a series motor. Transposing equation (11–10) for armature current,

$$I_a = \frac{E - E_c}{R_a + R_s}.$$

To increase I_a, E_c must decrease. As in the series motor, the decrease in speed is necessary to allow the counter emf to decrease at the same time the field increases. Because the torque varies directly as the product of the armature current and the field flux

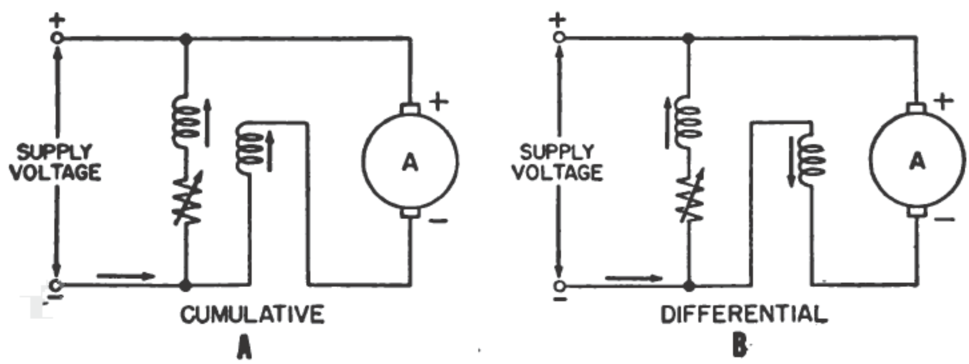


Figure 11–13.—Types of compound motors.

(equation 11–1), it is evident that the cumulative compound motor has greater starting torque than does the shunt motor for equal values of armature current and shunt field strength. The performance of this type of motor approaches that of a shunt motor as the ratio of the ampere-turns of the shunt winding to the ampere-turns of the series winding becomes greater. The performance approaches that of a series motor as this ratio becomes smaller.

If the load is removed from this type of motor it tends to speed up, and the counter emf increases. The current in the series field is reduced, and the greater portion of the field flux is produced by the shunt field coils. The compound motor then has characteristics similar to a shunt motor; and unlike the series motor, there is a definite no-load speed.

Because the total flux increases when there is an increase in load, there is a greater proportional increase in torque than in armature current. Therefore, for a given torque increase, this type of motor requires less increase in armature current than the shunt motor, but more than the series motor.

In some operations it is desirable to use the cumulative series winding to obtain good starting torque; and when the motor comes up to speed, the series winding is shorted out. The motor then has the improved speed regulation of a shunt motor.

In a differential compound motor, because of the opposition of the series field to the shunt field, the flux decreases as the load and armature current increase. From equation (11–9), it is seen that a decrease in flux causes an increase in speed. However, because the speed is proportional to $\frac{E_c}{\Phi}$, if both factors vary in the same proportion, the speed will remain constant. This action may occur in the differential compound motor. If more turns are added to the series coil, it is possible to cause the motor to run faster as the load is increased.

More armature current is required of the differential compound motor than of the shunt motor for the same increase in torque. This results from the fact that an increase in armature and series-field current reduces the field flux. Since the torque acting on the armature is proportional to the product of armature current and field strength, a decrease in field strength must be accompanied by a disproportionate increase in armature current in order that the product will increase.

Under heavy load the speed of the differential compound motor is unstable; and if the overload current is very heavy, the direction of rotation may be reversed. Thereafter, the motor will run as a series motor with the danger of overspeed on no load that is the inherent characteristic of all series motors.

The characteristics of the differential compound motor are somewhat similar to those of the shunt motor, only exaggerated in scope. Thus, the starting torque is very low, and the speed regulation is very good if the load is not excessive. However, because of some of the undesirable features, this type of motor does not have wide use.

Figure 11–14, A, shows the relative speed-load characteristics of shunt motors, cumulative compound motors, and differential compound motors. Although the speed of each of the motors is reduced when the current through the armature is increased (load is increased), the cumulative motor has the greatest decrease in speed. Likewise, in figure 11–14, B, the cumulative motor has the greatest increase in torque with increase in load.

Cumulative compound motors are used under conditions where large starting torque is necessary, where a relatively large change in speed can be tolerated, and where the load may be removed from the motor with safety. These motors are therefore used for hoists, punches, shears, and so forth.

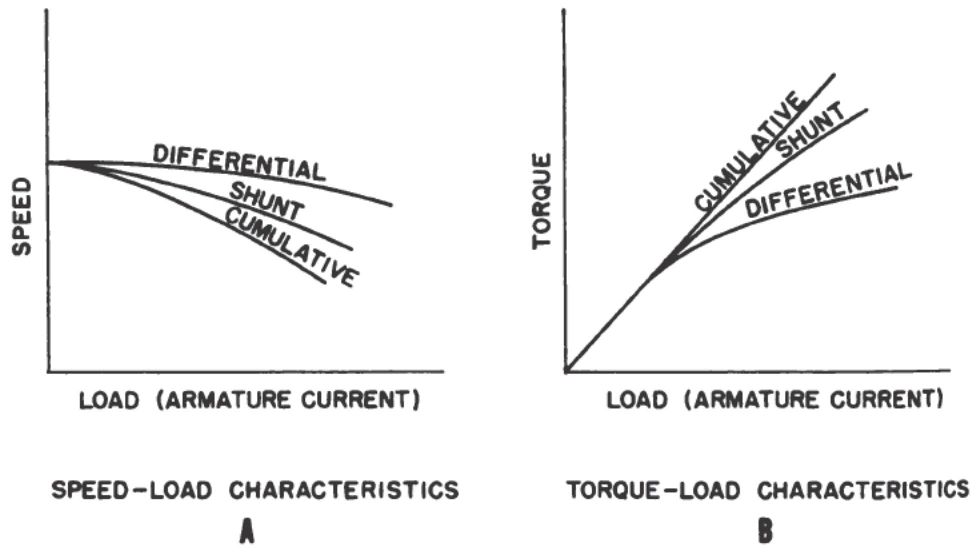


Figure 11–14.—Speed-load and torque-load characteristics of shunt and compound motors.

MANUAL STARTERS

Because the armature resistance of many motors is low (0.05 to 0.5 ohm) and because the counter emf does not exist until the armature begins to turn, it is necessary to employ an external resistance in series with the armature to keep the initial armature current to a safe value. As the armature begins to turn, the counter emf increases. Because the counter emf opposes the applied voltage, the armature current is reduced. The resistance in series with the armature is then reduced either manually or automatically as the motor comes up to normal speed and full voltage is applied across the armature.

The relation between armature current, I_a, applied voltage, E_a, counter emf, E_c, armature resistance, R_a, and starting resistance, R_s, for a shunt motor is

$$I_a=\frac{E_a-E_c}{R_a+R_s}.$$

Assume that for the shunt motor shown in figure 11–15, $E_a=100$ volts, $R_a=0.1$ ohm, $R_s=0.9$ ohm, and the normal full-load armature current is 10 amperes. Without the starting resistance, the armature current would be

$$I_a=\frac{E_a}{R_a}=\frac{100}{0.1}=1{,}000 \text{ amperes.}$$

This is 100 times the normal current for the motor and would obviously burn up the armature insulation and cause excessive acceleration forces.

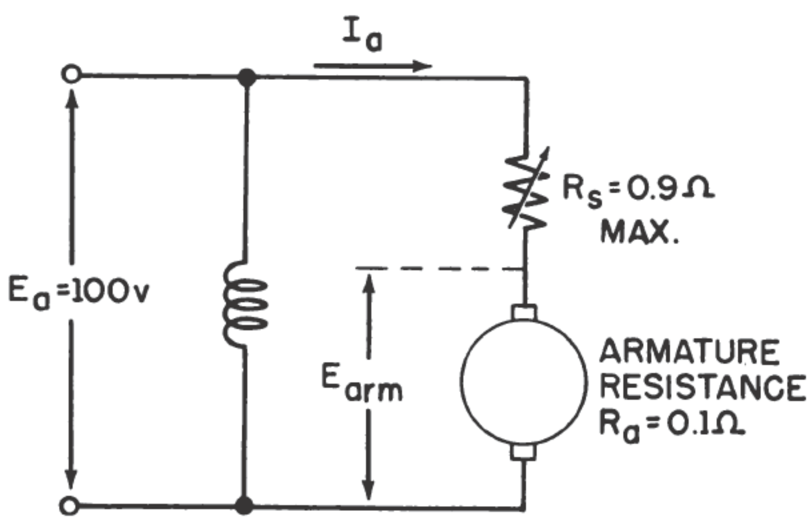


Figure 11–15.—Shunt motor with manual starting resistance.

In order to limit the initial surge of current to a maximum of 100 amperes (arbitrarily, 10 times the full-load current), the starting resistance is

$$R_s=\frac{E_a-I_aR_a}{I_a}=\frac{100-100\times 0.1}{100}=0.9 \text{ ohm.}$$

The relation of E_a; I_a; the starting resistance, R_s; motor speed, and counter emf during the accelerating period is shown in the curves of figure 11–16 and in table 13.

Initially, the full starting resistance of 0.9 ohm is inserted in series with the armature resistance of 0.1. The speed and counter emf are zero, and the armature current is limited to 100 amperes

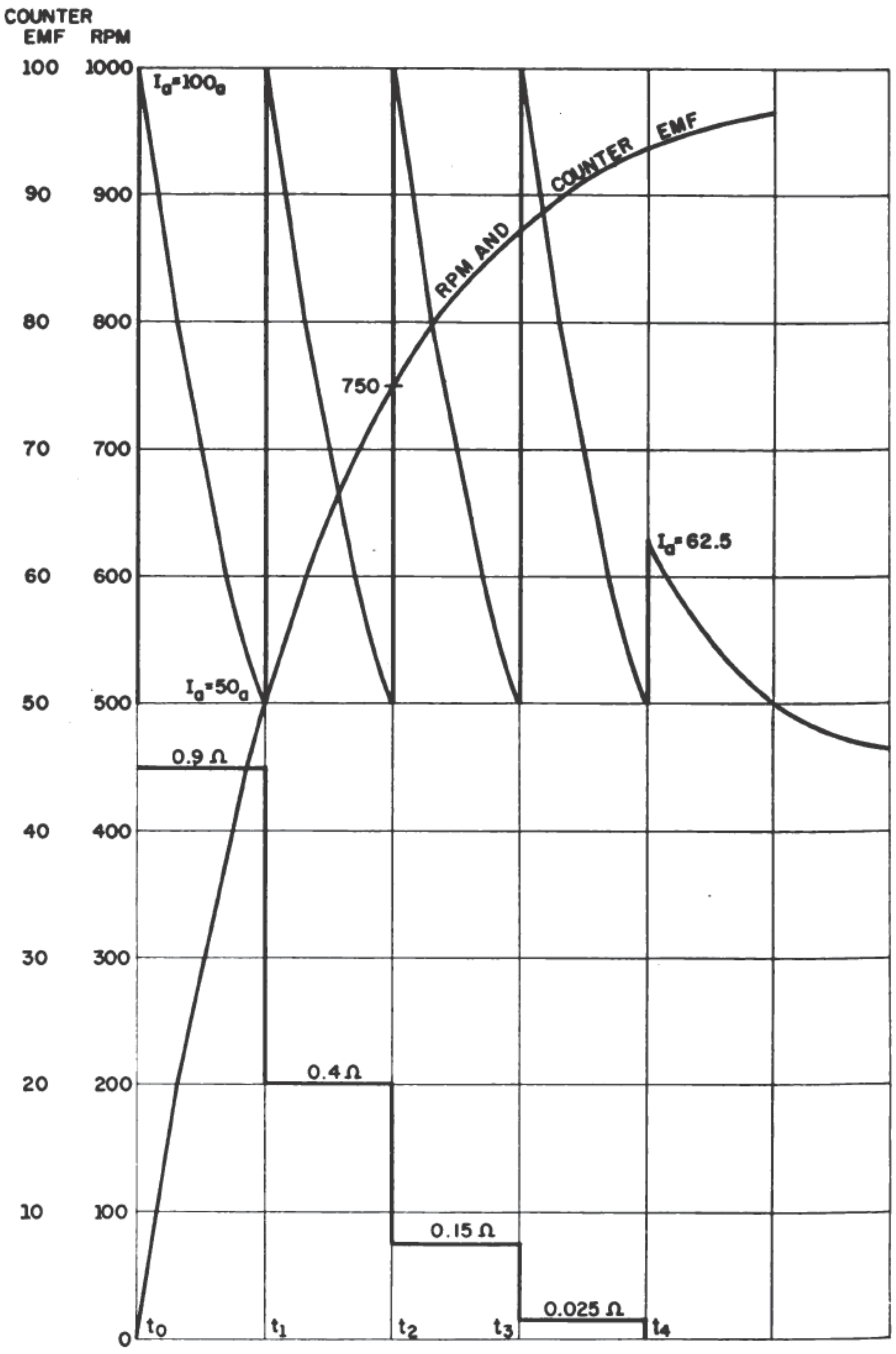


Figure 11–16.—Shunt motor starting curves.

TABLE 13.—*Starting characteristics of a shunt motor*

Armature current (*amperes*)	Starting resistance (*ohms*)	Armature volts ($E_a - I_a R_s$)	Speed (*rpm*)	Counter emf volts [$E_a - I_a(R_a + R_s)$]
100	0.9	10	0	0
50	0.9	55	500	50
100	0.4	60	500	50
50	0.4	80	750	75
100	0.15	85	750	75
50	0.15	92.5	875	87.5
100	0.025	97.5	875	87.5
50	0.025	98.75	937.5	93.75
62.5	0	100	937.5	93.75
30	0	100	970	97

by the armature resistance of 0.1 ohm, acting in series with the starting resistance of 0.9 ohm. As the speed and counter emf increase, the counter emf subtracts a greater magnitude of voltage from the applied emf, and the armature current is reduced from 100 amperes to some lower value. When the speed is 500 rpm, the counter emf is 50 volts and the current is

$$I_a = \frac{100-50}{0.1+0.9} = 50 \text{ amperes.}$$

Suppose that at the instant t_1, when the motor speed is 500 rpm, the starting switch is moved to the 0.4-ohm tap. The counter emf of 50 volts subtracts from the 100 volts applied, leaving 50 volts to cause the armature current to again increase to $\frac{100-50}{0.4+0.1}$ $=100$ amperes at time t_1. The armature speed and counter emf cannot increase suddenly, but rise gradually during the interval from t_1 to t_2. Therefore, the armature current is decreased gradually, and at t_2, when the counter emf is 75 volts and the speed is 750 rpm, the current is

$$I_a = \frac{100-75}{0.4+0.1} = 50 \text{ amperes.}$$

Moving the starting switch to the 0.15-ohm tap, the 0.025-ohm tap, and the 0-ohm tap at instants t_2, t_3, and t_4 causes the motor to

accelerate to a speed of 970 rpm with variations in the armature current as indicated in the figure and in the table.

AUTOMATIC STARTERS

If a motor is at a remote location, or if a large motor is to be started, it is generally desirable to use an automatic starter to bring the motor up to operating speed. Although small motors are started by means of manually operated starters, difficulties are encountered when starting large motors. In the manually-operated starter, the skill with which the operator regulates the magnitude of the starting current is dependent upon his experience. Too much time may be taken by an overcautious operator or the maximum permissible current for the motor may be exceeded by a careless one. In order that time shall not be wasted and the motor receive maximum safe current during the acceleration period, an automatic device should be used that will interpret the conditions of the load and act accordingly.

The following automatic starters will be discussed: (1) The time-element type, (2) the counter emf type, (3) the shunt current-limit type, and (4) the series current-limit type.

Time-Element Starter

A time-element starter is one in which the resistance in series with the armature is reduced a certain amount during each succeeding unit of time regardless of the load on the motor. One type of time-element starter is shown in figure 11–17. Initially, the full value of the starting resistance is inserted in series with the armature, and its current is therefore limited to a safe value. When the switch is closed, the full-line voltage is impressed across the accelerating solenoid and across the shunt field. The solenoid immediately begins to draw the iron core upwards, and the starting resistance is gradually cut out of the armature circuit. The speed with which the resistance is cut out depends on the size of the orifice (the hole through which the oil flows) in the dashpot. The smaller the orifice, the greater the time delay. The time delay should be such that as the speed and the counter emf builds up, the starting current is maintained at the maximum safe value during the accelerating period. During the time required to

reach full speed, the resistance is completely removed from the circuit.

The advantages of the time-element type of automatic starter are that its cost is low, the wiring is simple, and starting is generally sure. The disadvantages are that the amount of load can-

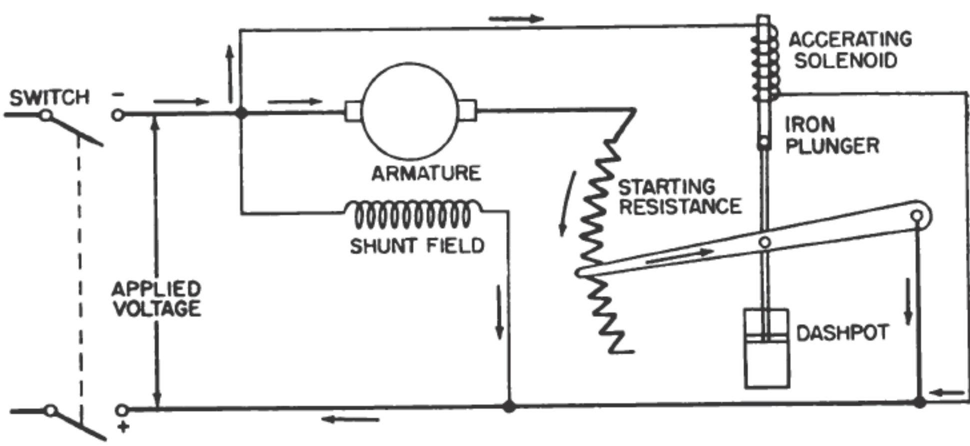


Figure 11–17.—Time-element starter.

not be sensed—that is, the resistance is removed at the same rate whether the load is heavy or light, the motor is not protected on overload, and the dashpot is a source of trouble.

Counter Emf Starter

The counter emf starter depends for its operation on the counter emf developed across the armature. As may be seen from figure 11–18, A, the solenoid that activates the contacts is connected directly across the armature. It therefore responds to armature voltage, which is only slightly higher than the counter emf.

At start (fig. 11–18, B), time t_0, the armature current flowing through the starting resistance causes a reduced voltage to be applied to the armature and the accelerating contactor will not immediately close its contacts due to low voltage across its operating coil. As the motor accelerates, the rising counter emf causes the armature current and voltage drop across the starting resistance to reduce. At the same time, the voltage across the armature increases. When this voltage rises to the proper value, time t_1, the accelerating contactor closes and shorts out the starting resistance, thus applying full voltage across the armature. The accom-

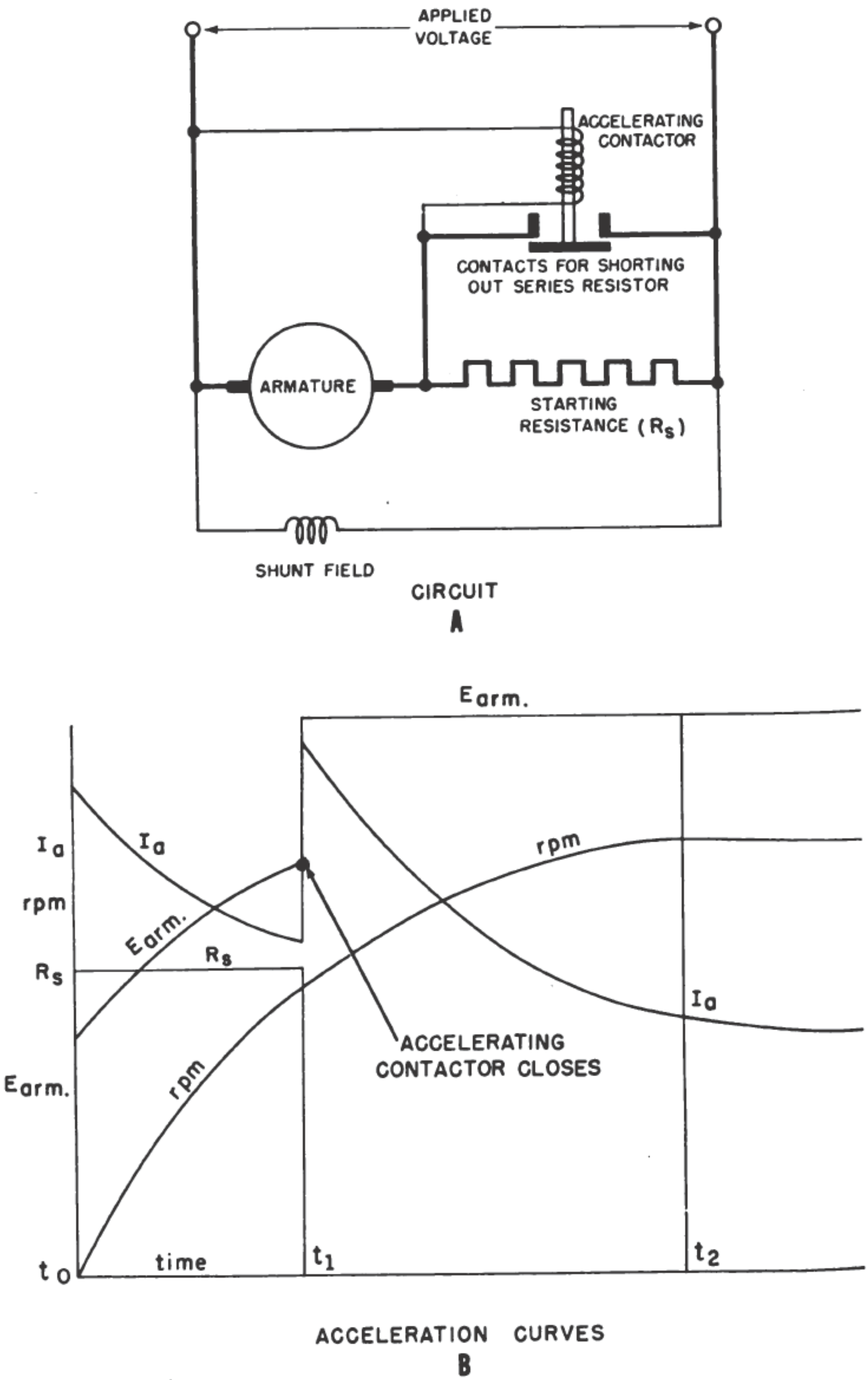


Figure 11-18.—Counter emf starter.

panying increase in starting current causes continued acceleration until the motor comes up to rated speed, and the armature current is again reduced to the normal value. For simplicity, only one resistor and starting contactor are shown, although large motors usually employ several contactors and several steps of starting resistance. If the load is heavy, the acceleration is slower and the rise in voltage on the accelerating contactor operating coil is delayed so that the starting resistance is not cut out until the rise in speed and counter emf permits the contactor to close.

The advantages of the counter emf starter are that the load is interpreted and the resistance cut out accordingly, the cost is low, the wiring is simple, and the dashpot is not used.

The disadvantages of the counter emf starter are that if the line voltage varies, the acceleration may become erratic. For example, if the line voltage rises, the starting resistance may be removed too soon; and if it falls, the starting contactor may not be activated at all. The starting contactor coils are designed to close the contactor on one value of voltage and to operate continuously on an increased voltage. Thus, they are sensitive to voltage change and will not operate properly if line voltage fluctuations occur.

Shunt Current-Limit Starter

The disadvantages of the counter emf starter are overcome in the shunt current-limit starter. This motor starter employes accelerating contactors with shunt-type operating coils wound for full-line voltage. Each contactor includes an interlocking series relay, the operation of which is limited by the motor starting current. When the series relay closes its contacts, the shunt operating coil of the accelerating contactor is energized and a section of starting resistance is cut out of the armature circuit. This type of motor starter thus derives its name from the fact that the accelerating contactors are shunt operated and the closure of the series relays is limited by the magnitude of the armature current.

Figure 11–19 is a simplified schematic diagram of a shunt current-limit starter operating in connection with a pushbutton starter with no-voltage release holding-contacts. When the "ON" button is pressed down momentarily, the no-voltage release coil is energized. The holding relay contacts and the line contacts

are closed. The series relay is mechanically released, and its contacts would close were it not for the fact that the armature current flows through the series coil and holds its core up, thus keeping its contacts open. Thus, it may be seen that current flows from the negative terminal of the voltage source through the armature, through the starting resistor, through the series relay coil, and back to the positive terminal of the voltage source.

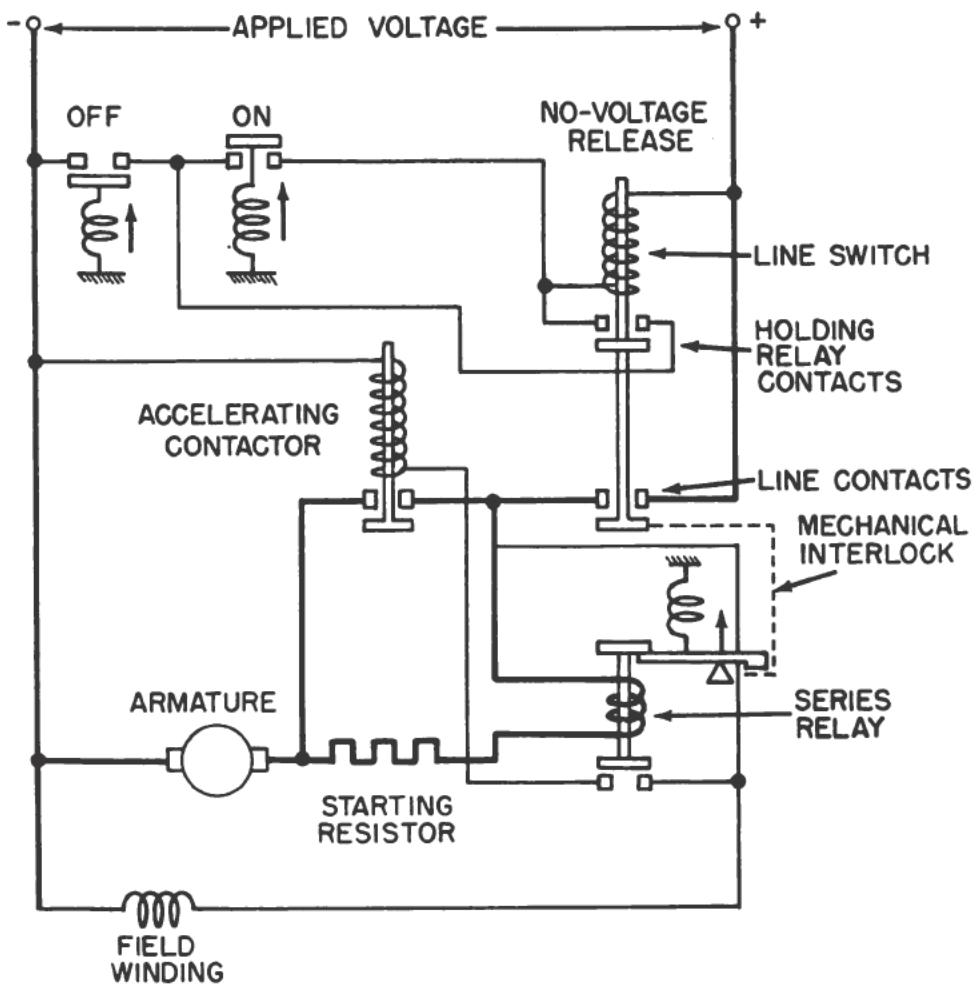


Figure 11–19.—Shunt current-limit starter.

As the motor speeds up, the counter emf increases, and the armature current is reduced accordingly. The field of the series relay is thereby weakened and the relay contacts are closed. This action energizes the coil of the accelerating contactor and its contacts close, thus shorting out the starting resistor and the series relay.

The full-line voltage is then safely applied across the armature. The series relay and accelerating contactor contacts remain closed until the line contacts are opened.

When the "OFF" button is pressed, current through the no-voltage release coil is interrupted and the holding relay contacts, the line contacts, and the series relay contacts are opened. Line voltage is thus removed from the motor.

Although for simplicity only one starting resistor is shown in a practical system, two or three are commonly used. Additional accelerating contactors and interlocking series relays are then used to remove the starting resistance in steps as the motor comes up to speed.

The advantage of the shunt current-limit starter is that starting resistance is cut out of the armature circuit only after the speed has built up and the motor current has been reduced tc a safe value irrespective of line voltage fluctuations. If the load is too great for the motor to accelerate normally, the counter emf cannot build up sufficiently to reduce the current through the series relay to cause its contacts to fall closed. Therefore, the starting resistance remains in the circuit and the armature is protected against excessive current.

The disadvantage is that the accelerating units are complicated and expensive.

Series Current-Limit Starter

The series current-limit type of motor starter was designed to accelerate motors and to provide the same protection afforded by the shunt current-limit type of starter, but to do the job with less complicated equipment. The interlocking series relays are omitted and the accelerating contactors are designed to lock open on the initial in-rush of starting current. When the current falls to a predetermined value the contactor closes and shorts out the starting resistance.

In the series current-limit starter, the relay coil which operates the accelerating contactor is series wound—that is, it is connected in series with the armature circuit. The simplified circuit, showing only one starting resistor, is shown in figure 11–20, A.

One type of magnetically operated switch that makes this type of starter possible is shown in figure 11–20, B. The initial current through the coil causes the accelerating contactor to lock open.

When the current falls to a certain predetermined value, the switch will close.

The coil is wound with few turns of heavy wire. When current flows through the coil, the movable armature is acted upon by two forces, one across the main air gap, the other across the auxiliary air gap. The force across the main air gap tends to

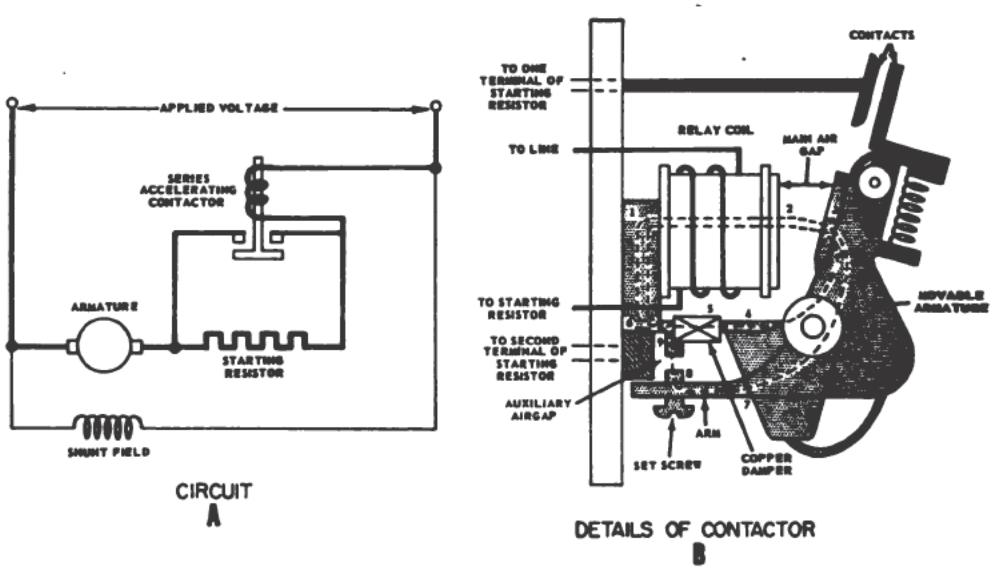


Figure 11–20.—Series current-limit starter.

close the contactor; that across the auxiliary air gap tends to open it. The flux paths are first 1, 2, 3, 4, 5, 6, 1; and second 1, 2, 3, 7, 8, 9, 6, 1. The first path includes only the main air gap. The second includes both gaps. When a small amount of current flows through the coil, the flux path is principally through 1, 2, 3, 4, 5, and 6 and the force exerted across the main air gap tends to close the switch. When a large amount of current flows through the coil, the flux path is through 1, 2, 3, 7, 8, and 9. This path includes both the main and auxiliary air gaps. Because the auxiliary gap is smaller, the force exerted across this gap to hold the contactor open exceeds the force across the main gap to close it and the contactor locks open.

To prevent the flux from rushing through the shorter path, 4, 5, 6, and closing the contactor before it has time to lock open, a short-circuited coil (damper) is placed around this flux path, as shown in figure 11–20, B. As the flux links this coil, the induced current in the copper damper sets up a counter magnetomotive

force that opposes the original flux and causes it to take the path through 7, 8, and 9 during the time that the current is rising. When the series operating coil current levels off at a high value, the flux path 4, 5, and 6 becomes saturated and sufficient flux is established through 7, 8, and 9 to maintain the contactor in the locked-out condition.

This type of contactor is especially satisfactory when the motor is always under load. The principal disadvantage is that the switch may drop open on light load. However, a shunt holding coil may be used to prevent this.

MOTOR EFFICIENCY

The efficiency of any type of machine is the ratio of the output power to the input power. This ratio is commonly expressed as a percent. Because all machines have some losses, the efficiency will never be 100 percent. Expressed as a percentage, efficiency becomes

$$\text{efficiency} = \frac{\text{output}}{\text{input}} \times 100,$$

$$\text{efficiency} = \frac{\text{output}}{\text{output} + \text{losses}} \times 100,$$

or

$$\text{efficiency} = \frac{\text{input} - \text{losses}}{\text{input}} \times 100.$$

The first equation is general. The second is advantageous when applied to electric generators and transformers. The third is most useful when applied to electric motors. The output must be expressed in the same units as the input. For example, if the output is expressed in horsepower and the input is expressed in watts, they can be changed to common units by means of the relation,

$$746 \text{ watts} = 1 \text{ horsepower}.$$

There are various types of mechanical and electrical losses in d-c motors, some of which are common to other types of motors.

One loss is in BEARING FRICTION. This type of friction is reduced by proper oiling. Roller bearings have less friction loss than sleeve bearings. However, at excessive speed or under excessive load the loss due to bearing friction may be appreciable.

A loss is also introduced by BRUSH FRICTION. This may be reduced by the use of a well-polished commutator and properly fitted brushes. WINDAGE LOSS is that due to the resistance of the air to a rapidly revolving armature. IRON LOSSES include eddy-current loss and hysteresis in the armature iron. COPPER LOSSES include the I^2R loss in the armature and field windings.

As an example, assume that the motor shown in figure 11–21 is supplied with 10 amperes at 100 volts.

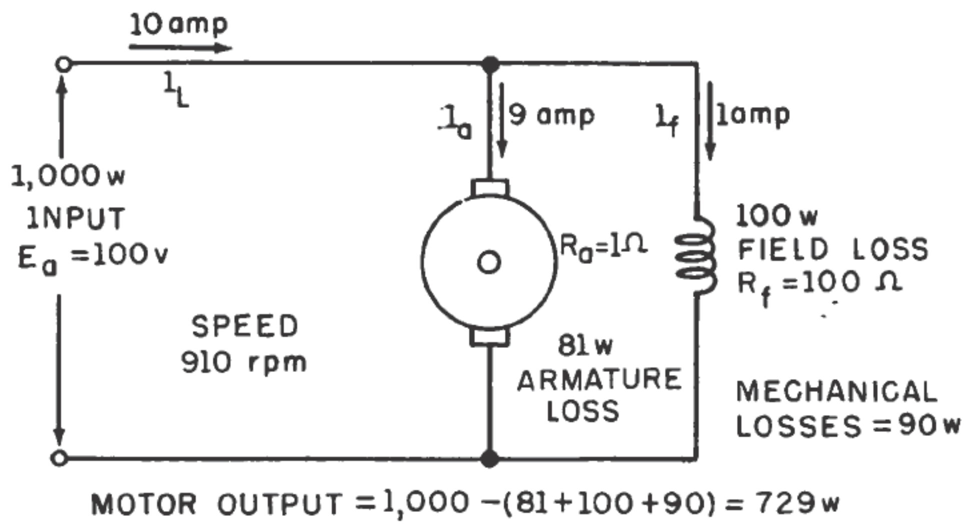


Figure 11–21.—Distribution of losses in a motor.

The COPPER LOSS in the field is

$$I_f^2 \times R_f = 1^2 \times 100 = 100 \text{ watts.}$$

The armature current is $10-1$, or 9 amperes and the COPPER LOSS in the armature is

$$I_a^2 \times R_a = 9^2 \times 1 = 81 \text{ watts.}$$

Assume that the total MECHANICAL LOSS (friction, windage, and so forth) is 90 watts. The TOTAL LOSS is therefore 271 watts. The input is

$$E_a \times I_L = 100 \times 10 = 1{,}000 \text{ watts.}$$

The output is

$$\text{input} - \text{losses} = 1{,}000 - 271 = 729 \text{ watts.}$$

The efficiency is

$$\frac{\text{output}}{\text{input}}\times 100=\frac{729}{1{,}000}\times 100=72.9 \text{ percent.}$$

If it is determined (by means of a prony brake test) that the power developed by the motor is 0.976 horsepower, the efficiency is also determined as

$$\text{efficiency}=\frac{\text{output}}{\text{input}}\times 100$$

$$=\frac{0.976\times 746}{1{,}000}\times 100=72.9 \text{ percent.}$$

SPEED CONTROL

The speed, N, of a d-c motor is directly proportional to the armature counter emf, E_c, and inversely proportional to the field strength, Φ. Transposing equation (11–8) for N, $N=\frac{E_c 10^8 p}{\Phi Z P}$. Such factors as the numbers of poles, P, the number of current paths, p, and the total number of armature conductors, Z, can be lumped into a single constant, K. Simplified, the expression becomes

$$\text{rpm}=\frac{KE_c}{\Phi}.$$

In a shunt motor

$$E_c=E_a-I_aR_a,$$

and

$$\text{rpm}=\frac{K(E_a-I_aR_a)}{\Phi}, \qquad (11\text{–}11)$$

where E_a is the voltage applied across the armature. This is similar to, but not identical with, equation (11–9) for a series motor.

From equation 11–11, it is seen that the speed may be INCREASED by DECREASING the field strength, and vice versa, or the speed may be increased by increasing E_a, and vice versa. Practically, these variations are most simply accomplished by inserting a field rheostat in series with the field circuit, or a variable resistor in series with the armature circuit.

Speed Control by Armature Series Resistance

Figure 11–22 shows how speed control may be accomplished by means of a variable resistor in series with the armature. The circuit is the same as that used in the motor starter circuit in figure 11–15. However, the wattage rating is different. The starting resistance should only be inserted for a short interval, whereas the speed control resistance can remain in the circuit indefinitely.

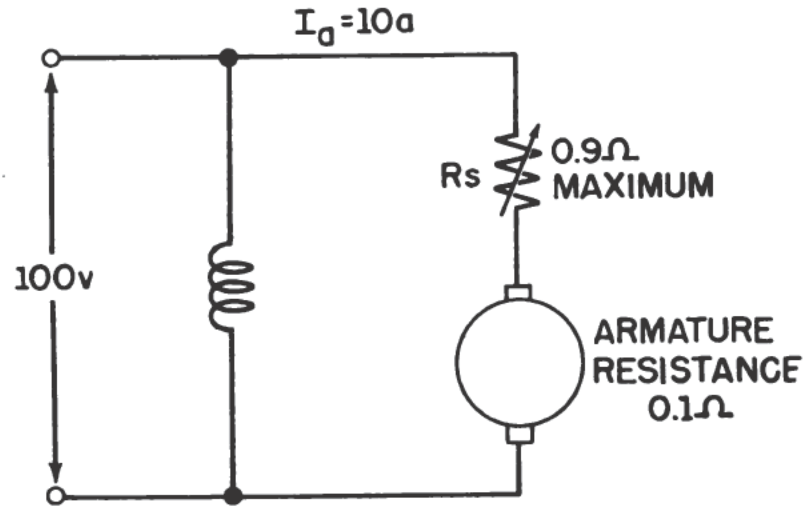


Figure 11–22.—Speed control by means of armature series resistor.

In the example of the motor shown in figure 11–22, assume that $\frac{K}{\Phi}$, as indicated in the previous equation, is replaced by the number 10. The equation for speed in rpm then becomes

$$\text{rpm}=10[E_a-I_a(R_s-R_a)].$$

When R_s is set at zero, the resistance, R_a, offered by the armature is 0.1 ohm, and I_a is assumed to be 10 amperes. Therefore,

$$\text{rpm}=10[100-10(0+0.1)]=990.$$

When R_s is increased to 0.1 ohm, I_a is lowered. The motor speed is reduced, the counter emf is reduced, and I_a is assumed to come back to the original value of 10 amperes. At the stable condition, the new speed is

$$\text{rpm}=10[100-10(0.1+0.1)]=980.$$

The rheostatic losses involved in this method of speed control are appreciable at low speeds and the resultant reduction in efficiency makes this type of control undesirable if the motor is to be operated at greatly reduced speed for prolonged intervals.

Speed Control by Adjusting the Field Strength

A more economical method of speed control is by rheostatic adjustment of the field current. If the field strength is weakened, the speed of the motor is increased; and if the strength of the field is increased, the speed of the motor is decreased.

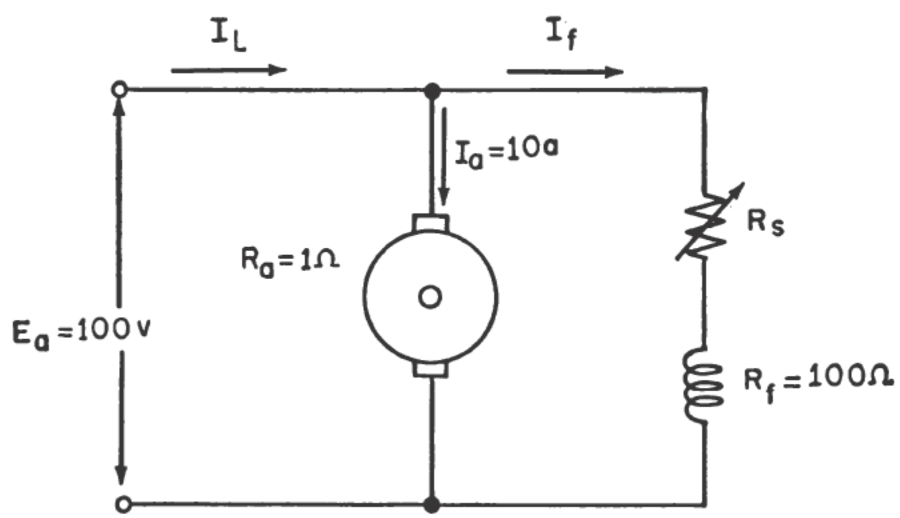


Figure 11–23.—Speed control by varying the strength of the field.

Figure 11–23 indicates a method of varying the strength of the field by means of a field rheostat.

When the field rheostat is cut out, the field current is 1 ampere. The counter emf is 90 volts, and the armature current is 10 amperes. The MEASURED speed is 900 rpm. When the resistance of the field rheostat is increased to 11.1 ohms, the field current is reduced to 0.9 amperes and the field flux is reduced. This causes a reduction in counter emf and a sharp increase in armature current, which causes an increase in force on the armature and an increase in armature speed. As the speed builds up, the counter emf builds up to 90 volts again, and the armature current is again reduced to 10 amperes.

The speed corresponding to a field current of 0.9 ampere may

be computed if it is recalled that the speed varies inversely with the flux and that the flux (below saturation) varies directly with the current. Thus, if the speed is 900 rpm when the field current is 1 ampere it will be $900 \times \frac{1}{0.9} = 1{,}000$ rpm when the field current is 0.9 ampere.

Speed Control by Ward-Leonard System

As has been explained in the previous paragraphs, the speed of a d-c motor can be controlled by at least two methods—either the armature voltage can be varied, as in figure 11–22, or the field voltage can be varied as in figure 11–23. The first method gives a range of speeds below the rated full-load speed. The second, gives a range of speed above the rated full-load speed. The first method takes away the constant speed characteristics from the shunt motor when the load varies. The second method permits control with a physically smaller resistor and greatly reduced power and gives essentially constant speed characteristics at any speed setting of the field rheostat provided the field is not weakened excessively. With a very weak field the motor may stall. The inherent difficulties of both methods are removed by the Ward-Leonard method shown in figure 11–24.

The d-c motor armature whose speed is to be controlled, is fed

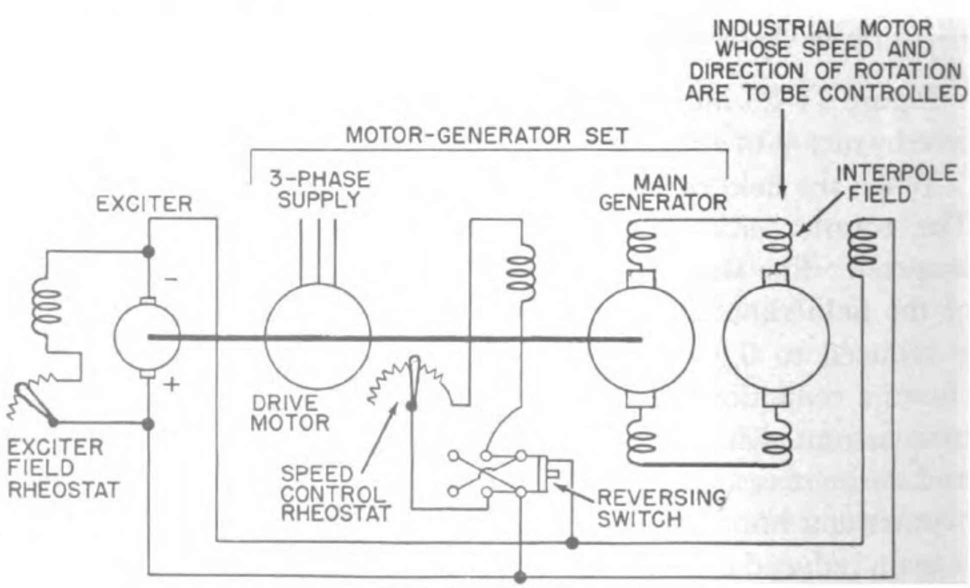


Figure 11–24.—Ward-Leonard speed control system.

directly from a d-c generator armature which is driven by a constant-speed prime mover. The d-c field supply to the generator is variable in both magnitude and polarity by means of a rheostat and reversing switch, as shown. Therefore, the motor armature is supplied by a generator having smoothly varying voltage output from zero to full-load value. The motor field is supplied with a constant voltage from the same source as that supplying the generator fields. The generator drive power could be from a single-phase or three-phase a-c motor, from an engine, or other constant-speed source. In the same way, the d-c field supply can be supplied from a rectifier, from an exciter on the end of the generator shaft, or from any other suitable d-c source. In the figure, the field exciter is a direct-connected unit on the shaft of the motor-generator set. The drive motor for the M-G (MOTOR-GENERATOR) set is shown as a 3-phase motor. Both the main generator and the speed-control motor have commutating poles, and in some instances, compensating windings.

The advantages of this type of speed control are that it does away with the armature rheostatic losses and instability of speed with variable loads.

The disadvantage of this method of speed control is the initial expense of the added equipment—that is, the M-G set with its control equipment.

QUIZ

1. How is the field winding of a shunt motor connected (series or parallel) to the motor armature?
2. How is the field winding of a series motor connected (series or parallel) to the motor armature?
3. How does a compound motor differ from (a) A shunt motor with respect to torque? (b) A series motor with respect to speed?
4. Name three methods of cooling electric motors aboard ship.
5. In figure 11–4, C, where, with respect to the two conductors, would the resultant flux be concentrated if (a) The poles were interchanged and the direction of conductor current remained unchanged? (b) The poles were interchanged and the direction of conductor current were reversed?
6. The force, *F*, acting on a current-carrying conductor in a magnetic field is directly proportional to what three factors?

7. If a conductor with an active length of 5 inches and a current of 50 amperes flowing through it is placed in a uniform magnetic field of 20,000 lines per square inch, what will be the force in pounds exerted on the conductor?
8. What is the total torque in pound-feet developed by an armature that has 40 conductors if the force on each conductor is 0.5 pound and the average moment arm of the armature is 0.8 feet?
9. Give the formula for torque in terms of armature current, I_a, and field strength, Φ.
10. What is the function of the commutator in a d-c motor?
11. When the speed of a motor is constant, what is the relation between the generated torque due to the armature current and the retarding torque because of friction and the load?
12. (a) How many foot-pounds of work are equivalent to 1 horsepower-hour?
    (b) How many foot-pounds per second are equivalent to 1 horsepower?
13. If a 1,650-rpm motor has an effective moment arm of 0.319 foot and a total effective force of 100 pounds acts on the armature, what is the developed horsepower?
14. The counter emf in the armature of a shunt motor is proportional to the product of armature ______________ and field ______________.
15. Armature reaction in a motor distorts the field in what relative direction (forward or backward)?
16. In what relative direction (forward or backward) are the brushes shifted in a noninterpole-shunt motor when (a) the load increases and (b) when the load decreases?
17. On what plane are the brushes located in a shunt motor that employs interpoles?
18. How is the interpole flux made to vary directly with the changes in load on a shunt motor?
19. Does the interpole field of a d-c motor contribute to the counter emf generated in the motor armature?
20. What is the effect of moving the brushes away from the no-load neutral in a shunt motor that has interpoles?
21. How does a load increase affect the speed of a shunt motor?
22. What is the percent speed regulation of a shunt motor that has a no-load speed of 1,800 rpm and a full-load speed of 1,700 rpm?
23. What is the relation between the armature current and the load on a shunt motor?

24. What is the relative torque developed by a shunt motor when the armature current doubles?

25. What is the relative change in field strength of a shunt motor between no-load and full-load conditions?

26. Why is the speed regulation of a shunt motor better than the voltage regulation of the same machine when operated as a shunt generator?

27. What are the speed-torque characteristics of a shunt motor?

28. (a) What type of d-c motor is best suited for machine-tool loads?
    (b) Why?

29. What relation exists between torque and armature current, I_a, in a series motor if the field is worked below saturation?

30. What are the speed-torque characteristics of a series motor?

31. Give two examples of the type of loads that may be driven by a series motor.

32. (a) Find the speed in rpm of a 2-pole series motor if the armature contains 200 face conductors, the field strength is 10^6 lines of force, the applied voltage is 100 volts, the armature current is 5 amps, and the combined resistance of the armature and field is 0.2 ohm.
    (b) Find the speed of this motor if the armature current doubles.

33. For equal values of armature current and shunt field strength, why does the cumulative compound motor have greater starting torque than does the shunt motor?

34. In a differential compound motor, why does the flux decrease as the armature current increases?

35. Why is starting resistance needed on many motors?

36. What governs the speed with which the starting resistance is cut out in a time-element starter of the type shown in figure 11–17?

37. When the load is heavy on a motor that employs a counter emf starter, why may the operation of the accelerating contactors be delayed?

38. What is the main advantage of the shunt current-limit starter?

39. In the series current-limit type of accelerating contactor illustrated in figure 11–20, what prevents the initial rise of flux from closing the contactor before it has time to lock open?

40. What is the efficiency (in percent) of a motor that develops an output of 2 horsepower when the input is 1,865 watts?

41. What is the effect on the speed of a shunt motor when (a) The shunt field is weakened? (b) The armature voltage is reduced?

42. What is the disadvantage of using a variable resistor in series with the armature of a shunt motor that operates on variable load?

43. What are the two stated advantages of the Ward-Leonard drive?

CHAPTER 12

INTRODUCTION TO ALTERNATING-CURRENT ELECTRICITY

BASIC A-C GENERATOR

A direct current has been described as the movement of free electrons from atom to atom along the conductor. The electrons flow out of the negative terminal of the source, through the load, and back to the positive terminal. The circuit is completed through the source and the direction of curent flow is always one way in all parts of the circuit.

An alternating current consists of electrons that move first in one direction and then in the other. The direction of flow changes periodically. Because most of the theory of electric power and communications deals with currents that surge back and forth in a particular manner known as a sine-wave variation, the sine wave is described in considerable detail in this chapter.

Alternating current has certain advantages over direct current. When alternating current is used, it is possible to build larger central station generators with their higher operating efficiency and lower cost and weight per kilowatt of capacity. It is also possible to convert voltage and current from one value into another with less loss by the use of transformers. As a result, power may be transmitted more economically over long distances by using transmission voltages up to about 220,000 volts and reducing these voltages to a usable level by means of transformers in consumer areas. A-c motors for constant-speed variable-torque industrial applications are cheaper and simpler to build. It is possible to generate electric power at higher initial voltages ranging from 2,300 volts to 25,000 volts.

In 1883 there were no a-c applications in the U. S. Navy. By 1913, however, the collier *Jupiter* was completely equipped with a-c generators for turbo-electric propulsion. In 1923 the Navy had five battleships equipped with turbo-electric drive, each ship

using a-c generators totaling about 160,000 kw in generating capacity.

Since that time the perfection of gear-reduction drives has made possible the widespread use of turbine gear drives with consequent reduction in the capacity of a-c generating equipment. Today the electricity used aboard ship requires an a-c generating capacity in accordance with the type of ship, the battleship being the largest with approximately 10,000 kw.

Just as a current flowing in a conductor produces a magnetic field around the conductor, the reverse of this process is true. A voltage can be generated in a circuit by moving a conductor so that it cuts across lines of magnetic force or, conversely, by moving the lines of force so that they cut across the conductor. The a-c generator converts mechanical energy into electrical energy by utilizing this principle of electro-magnetic induction. A simple 2-pole a-c generator is shown in figure 12–1.

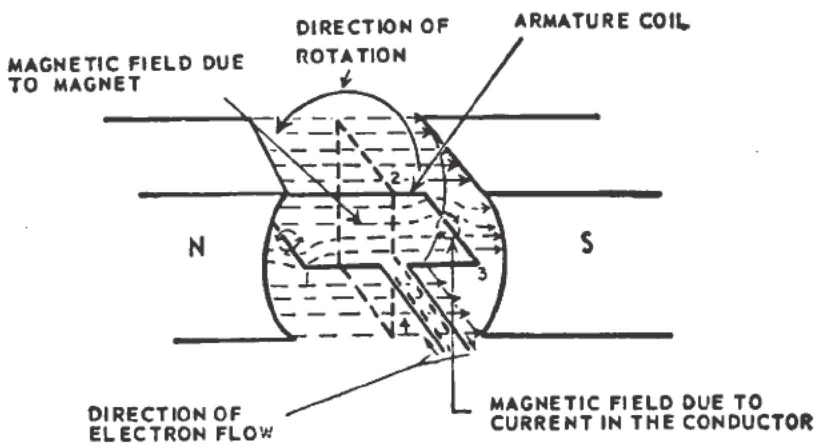


Figure 12–1.—Simple 2-pole a-c generator.

A single loop of wire (called the armature) is arranged on a shaft so that it can be rotated through the field existing between the north and south poles of a magnet. When the armature is rotated, the two sides of the loop that are parallel to the pole faces cut across the magnetic lines of force and generate a voltage in them. The instantaneous direction of this induced emf depends on the polarity of the field and the direction of rotation of the loop. If an external circuit is connected to the two ends of the loop, the voltages generated in the two sides of the loop are additive in causing electron flow around the circuit.

The amount of voltage generated depends upon the rate at which the lines of force are cut by the turn or turns of a rotating coil. When the loop sides are in positions 1 and 3 (fig. 12–1), they are MOVING perpendicularly to the lines of force and the induced emf is maximum.

Cycle

When the loop sides are MOVING parallel to the lines of force (positions 2 and 4), the induced emf is zero. As rotation continues, the two sides of the loop interchange their positions and the generated voltage in each of them is in the opposite direction. One complete revolution of the loop results in one cycle of induced a-c voltage.

Frequency

The FREQUENCY of an alternating current or voltage is the number of complete cycles occurring in each second of time; hence, the speed of rotation of the loop determines the frequency.

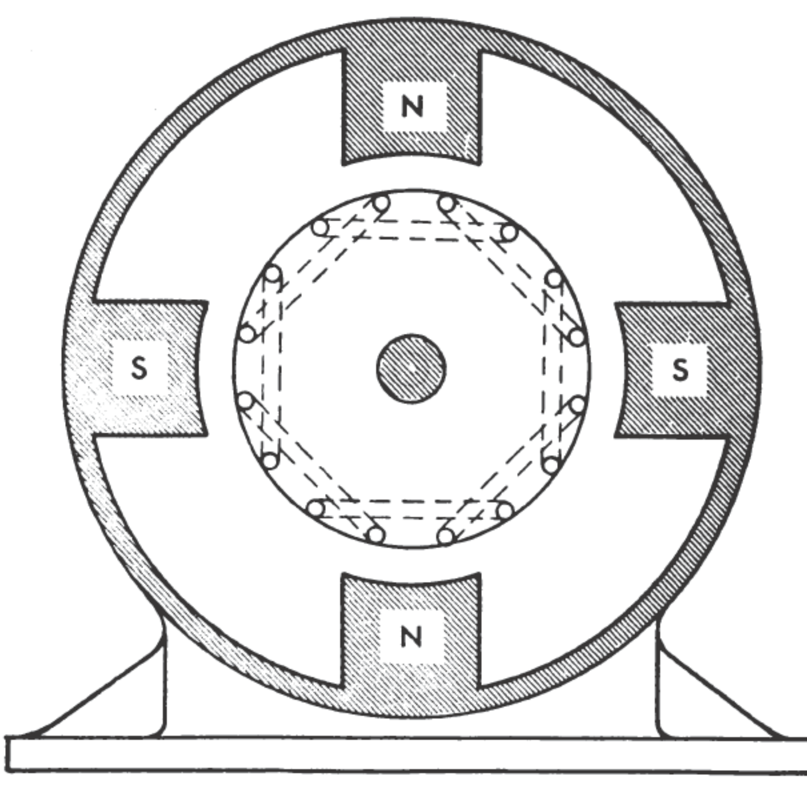


Figure 12–2.—Four-pole a-c generator.

A rapid rotation produces a high frequency while a slow rotation produces a low frequency.

A rapid rotation of the loop also results in the generation of a higher voltage than a slow rotation because the conductors cut the magnetic lines of force at a higher rate. The generated voltage can also be increased by increasing the field strength. A coil having many turns of wire produces a greater output voltage than the single turn loop because the voltage generated in each turn aids the voltage generated in the others.

If the field is stationary, the armature turns 360° in a 2-pole field in order to complete one cycle. In a 4-pole field (fig. 12–2) each armature coil generates 2 cycles of voltage for each complete turn of the armature. In other words, for each revolution of the armature there are always as many cycles completed as there are pairs of poles in the field.

The frequency, f, in cycles per second is related to the pairs of poles, P, and the speed, S, in revolutions per second as indicated in the following equation:

$$f = PS.$$

For example, the frequency of a 4-pole a-c generator having a speed of 30 revolutions per second is 2×30, or 60 cycles per second (3,600 revolutions per minute). If the speed is decreased to 25 revolutions per second, the frequency will be decreased to 2×25, or 50 cycles per second.

Period

The PERIOD of an a-c voltage or current of sine waveform is the time for one complete cycle, or $\frac{1}{f}$. For example, the period of a 60-cycle voltage is $1/60$ of a second; that of a 50-cycle voltage, $1/50$ second.

Generated Voltage

A-c generators are usually constant-potential machines because they are driven at constant speed and have a fixed field strength for a given load. The effective voltage, E, generated by a single winding (phase) a-c generator is related to the total field strength per pole, Φ; the frequency, f; and the total number of active con-

ductors, N, in the armature winding; as indicated in the following equation:

$$E = 2.22\Phi f N 10^{-8}.$$

For example, if $\Phi = 2.5 \times 10^6$, $f = 60$ cycles per second, and $N = 96$ conductors, the voltage generated is

$$\begin{aligned} E &= 2.22 \times 2.5 \times 10^6 \times 60 \times 96 \times 10^{-8} \\ &= 320 \text{ volts.} \end{aligned}$$

(The factors of poles and speed, which appear in the equation for generated voltage in multipolar series-wound d-c generators, do not appear in the formula for the voltage generated in each phase of the a-c generator because they are replaced by the equivalent factor of frequency ($f = PS$).)

The length of active conductor extending under a pole does not appear in the equation directly because it is included in the factor of total magnetic flux per pole. The longer the active conductor, the more flux there will be for each pole, since the pole length and conductor length are assumed to be the same. For example, if an active conductor length is doubled, the pole length is doubled, the flux per pole is doubled, and the generated voltage is doubled.

In commercial a-c generators the magnetic field is produced by an electromagnet which comprises the rotor. The field coils are energized with direct current supplied through slip rings and brushes. The armature winding consists of loops of wire mounted in slots in the steel laminations of the stationary member or stator. The armature coils are arranged so that the voltages induced in them are combined additively. All a-c generators consist fundamentally of a field and an armature, one of which rotates. In most a-c generators the armature windings are mounted on the stationary member, or stator, in order to supply the load current directly without going through sliding contacts, as in d-c generators. Large a-c generators have armature currents of several thousand amperes per terminal, and sliding contacts would not stand up under such heavy loads.

Alternating current is not suitable for field magnetization purposes. A-c generators require a magnetic field of fixed polarity, hence, d-c generators called EXCITERS are used as the source of d-c voltage for the field coils. Exciters are usually 110-volt to

440-volt compound generators with a kilowatt rating of from 1 percent to 10 percent of the rating of the associated a-c generator.

ANALYSIS OF SINE WAVE OF VOLTAGE

Vectors Defined

As mentioned previously, an alternating current or voltage is one in which the direction changes periodically. The electron movement is first in one direction, then in the other. The variation is of sine waveform. Straight lines drawn to scale, called VECTORS, are used in solving problems involving sine-wave currents and voltages.

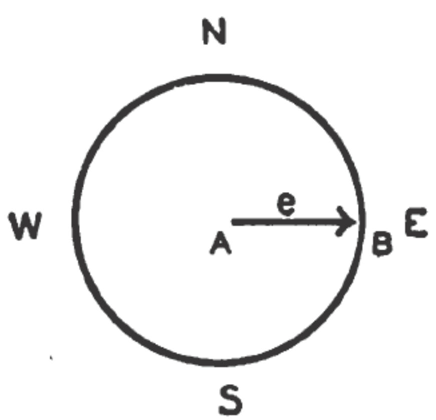


Figure 12–3.—Designating direction and distance by vectors.

A simple vector is a straight line used to denote the magnitude and direction of a given quantity. Magnitude is denoted by the length of the line and direction is indicated by an arrow at one end of the line, together with the angle that the vector makes with a horizontal reference vector.

For example, if a certain point, *B* (fig. 12–3) lies 1 mile east of point *A*, the direction and distance from *A* to *B* can be shown as vector *e* by using a scale of approximately ½ inch= mile.

Vectors may be rotated like the spokes of a wheel to generate angles. Positive rotation is counterclockwise and generates positive angles. Negative rotation is clockwise and generates negative angles.

The vertical projection of a ROTATING VECTOR may be used to represent the voltage at any instant (fig. 12–4). Vector E_m represents the maximum voltage induced in a conductor rotating

at uniform speed in a 2-pole field. The vector is rotated counter-clockwise through one complete revolution (360°). The point of the vector describes a circle. A line drawn from the point of the vector perpendicular to the horizontal diameter of the circle is the vertical projection of the vector.

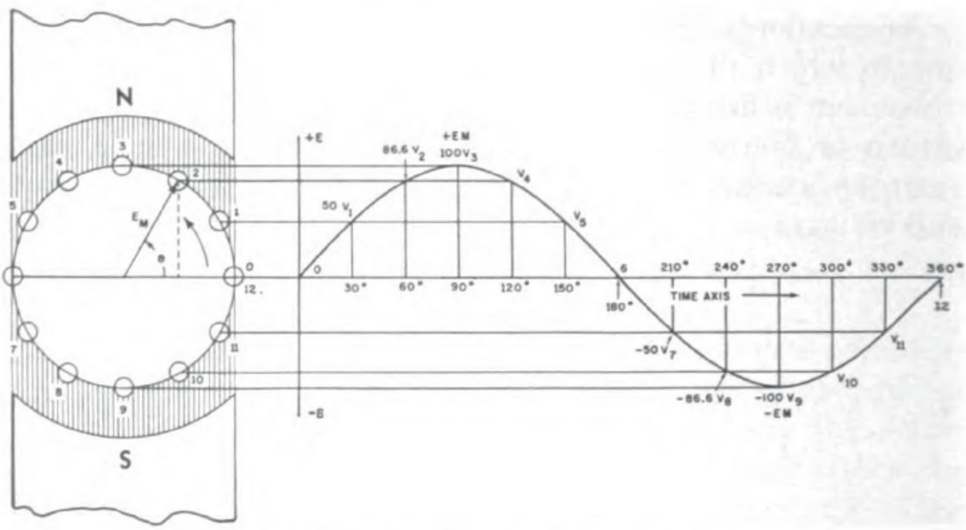


Figure 12–4.—Generation of sine-wave voltage.

The circle also described the path of the conductor rotating in the bipolar field. The vertical projection of the vector represents the voltage generated in the conductor at any instant corresponding to the position of the rotating vector as indicated by angle θ. Angle θ represents selected instants at which the generated voltage is plotted. The sine curve plotted at the right of the figure represents successive values of the a-c voltage induced in the conductor as it moves at uniform speed through the 2-pole field because the instantaneous values of rotationally induced voltage are proportional to the sine of the angle θ that the rotating vector makes with the horizontal.

Equation of Sine Wave of Voltage

The sine curve is the locus of all points, the ordinates of which are proportional to sin θ and the abscissas of which are proportional to θ. One complete cycle of 360 electrical degrees is indicated in the figure. The equation of the sine curve is

$$e = E_m \sin \theta,$$

where e is the instantaneous voltage, E_m the maximum voltage, and θ the angle in electrical degrees representing the instantaneous position of the rotating vector. Thus, when $\theta=60°$ and $E_m=100$ volts, $e=100 \sin 60°=86.6$ volts, and when $\theta=240°$, $e=100 \sin 240°=-86.6$ volts.

Another form of the equation for a sine wave of voltage includes the angular velocity, ω, of the rotating voltage vector—

$$e=E_m \sin \omega t,$$

where ω is the angular velocity in radians per second and t the time in seconds. There are 2π radians in 360°, or one complete cycle. In f cycles per second there are $2\pi f$ radians per second. Thus,

$$\omega=2\pi f;$$

therefore,

$$e=E_m \sin 2\pi ft.$$

This equation represents the graph of a sine wave of voltage having a fixed frequency and indicates the relation between the maximum voltage and the voltage corresponding to any other instant of time of operation.

For example, the equation $e=100 \sin 377t$ represents a sine-wave voltage having a frequency of 377 radians per second. The corresponding frequency in cycles per second is

$$f=\frac{\omega}{2\pi}=\frac{377}{6.28}=60 \text{ cycles per second.}$$

The maximum value of the voltage is 100 volts, as indicated by the value immediately preceding the sine function in the equation.

The voltage at any other instant may be determined by finding the angle corresponding to that instant and substituting in the voltage equation. For example, when $t=0.00139$, the corresponding angle is

$$\frac{0.00139}{0.0167}\times 360°=30°,$$

where 0.0167 is the time for one cycle, or $\frac{1}{60}=0.0167$ second. Therefore,

$$e=100 \sin 30=50 \text{ volts.}$$

There are four important values associated with sine waves of voltage or current: (1) instantaneous—designated as e or i, (2) maximum—designated as E_m or I_m, (3) average—designated as E_{avg} or I_{avg}, and (4) effective values—designated as E or I.

The INSTANTANEOUS value may be any value between zero and maximum depending on the instant chosen, as indicated by the equation $e=E_m \sin \omega t$.

The MAXIMUM value of voltage is reached twice each cycle and is the greatest value of instantaneous voltage generated during the cycle.

The ratio of the instantaneous value of voltage to the maximum value is equal to the sine of the angle corresponding to that instant.

Average Value of Voltage

The AVERAGE value of a sine wave of voltage or current is found by dividing the area under one alternation (half cycle) by the distance along the X axis between two successive instants (0° and 180°) corresponding to zero voltage. The average value is $\frac{2}{\pi}$, or 0.637, times the maximum. In the example in figure 12–4, the average voltage is

$$\frac{2}{\pi}\times 100, \text{ or } 63.7 \text{ volts.}$$

Thus,

$$E_{avg}=0.637E_{max}.$$

Effective or RMS Value

The EFFECTIVE value of an a-c voltage or current of sine waveform is defined in terms of the equivalent heating effect of a direct current.

Heating effect is INDEPENDENT of direction of electron flow and varies as the square of the instantaneous current. Thus, an alternating current of sine waveform having a maximum value of 14.14 amperes produces the same amount of heat in a circuit having a resistance of 1 ohm that a direct current of 10 amperes produces. The effective value of this alternating current is equal to 0.707 of the maximum value. Thus, $I_{eff}=0.707I_{max}$, or $0.707\times 14.14=10$ amperes. The effective value is also known

as the ROOT-MEAN-SQUARE) (rms) value because it is the square root of the average of the squared values between zero and maximum. Thus an effective, or rms, current of 10 amperes produces the same heating in a given resistance as a direct current of 10 amperes.

In practice, rms values of voltage and current are more important than instantaneous, maximum, or average values. Most a-c voltmeters and ammeters are calibrated in rms values. For example, suppose that an a-c voltmeter indicates that the effective voltage between the two conductors in a cable is 440 volts. Twice in each cycle the insulation is subject to a voltage stress that is $\frac{E_m}{E}=\frac{1}{0.707}$, or 1.41 times 440 volts, or 620 volts; where E_m is the maximum voltage and E is the rms value.

Suppose an a-c ammeter in the same circuit indicates an effective current of 10 amperes. Twice in each cycle the current exceeds this value by 41 percent but the average heating effect is that which a direct current of 10 amperes would produce. Thus, the conductor size is based on the ammeter indication (rms current) rather than on the maximum value, even though the insulation stress is always 41 percent higher than the voltmeter indication.

The ratio of the effective value to the average value of a wave is called the FORM FACTOR and for all sine waves is equal to $\frac{0.707}{0.637}$ or 1.11. The form factor for waveforms such as flat-top waves is less than 1.11. For waveforms such as peaked waves the form factor is greater than 1.11. If a waveform is not that of a sine wave the constants for effective and average values do not apply. Most electrical power and lighting circuits have sine-wave currents and voltages, and the effective values indicated by voltmeters and ammeters are appropriate.

COMBINING A-C VOLTAGES

Vectors may be used to combine a-c voltages of sine waveform and of the same frequency. The angle between the vectors indicates the time difference between their positive maximum values. The length of the vectors represents either the effective value or the positive maximum value, as desired.

90° Phase Difference

The sine-wave voltages generated in coils *a* and *b* of the simple generator shown in figure 12–5, A, are 90° out of phase because the coils are located 90° apart on the 2-pole armature. When coil *a* is cutting squarely across the field, coil *b* is moving parallel to the field and not cutting through it. Thus the voltage in coil *a* is maximum when the voltage in coil *b* is zero. If the frequency is 60 cycles per second, the time difference between the positive maximum values of these voltages is $\frac{90}{360}\times\frac{1}{60}=0.00416$ seconds.

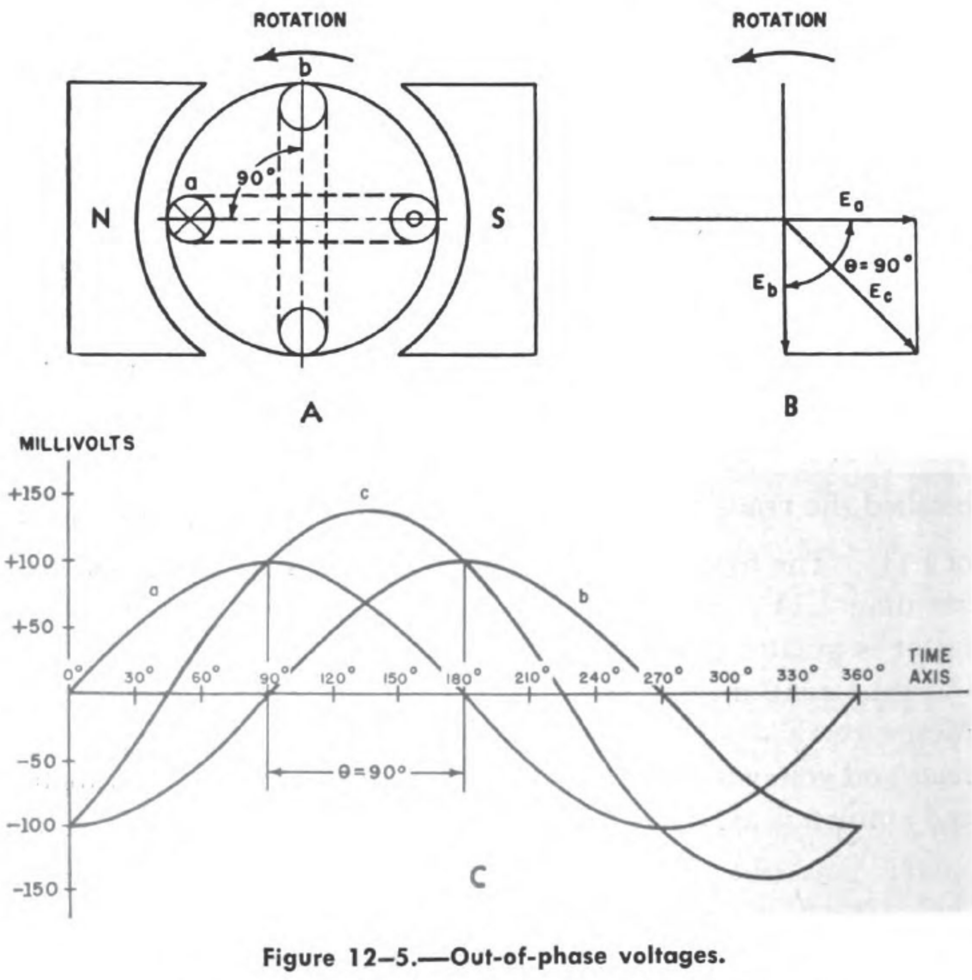


Figure 12–5.—Out-of-phase voltages.

Summation of Two Out-of-Phase Voltages

Vector E_a leads vector E_b by 90° in figure 12–5, B, and sine wave *a* leads sine wave *b* by 90° in figure 12–5, C. If coils *a* and

b are connected in series and the maximum voltage generated in each coil is 10 volts, the total voltage is not 20 volts because the two maximum values of voltage do not occur at the same instant, but are separated one-fourth of a cycle. The voltages cannot be added arithmetically because they are out of phase. These values, however, can be added vectorially.

E_c in figure 12–5, B, is the vector sum of E_a and E_b and is the diagonal of the parallelogram, the sides of which are E_a and E_b.

Curve *c* in figure 12–5, C, represents the sine-wave variations of the total voltage, E_c, developed in the series circuits connecting coils *a* and *b*. Voltage E_a leads E_b by 90°. Voltage E_c lags E_a by 45° and leads E_b by 45°. As mentioned previously, counterclockwise rotation of the vectors is considered positive rotation, thus giving the sense of lead or lag. Thus if E_a and E_b are rotated counterclockwise, and their movement is observed from a fixed position, E_a passes this position first, then 90° later E_b passes the position. Thus E_b lags E_a by 90°. If the maximum voltage in each coil is 10 volts, the maximum value of the combined voltage is $10\sqrt{2}$, or 14.14 volts. The effective voltage in each coil is 0.707×10, or 7.07 volts. The effective voltage of the series combination is $7.07 \times \sqrt{2}$, or 10 volts.

180° Phase Difference

Figure 12–6 represents two sine-wave voltages that are equal in magnitude and opposite in phase. Vectors *A* and *B* are 180°, or one-half cycle apart. Combining these two voltages in a series circuit gives a resultant of zero volts.

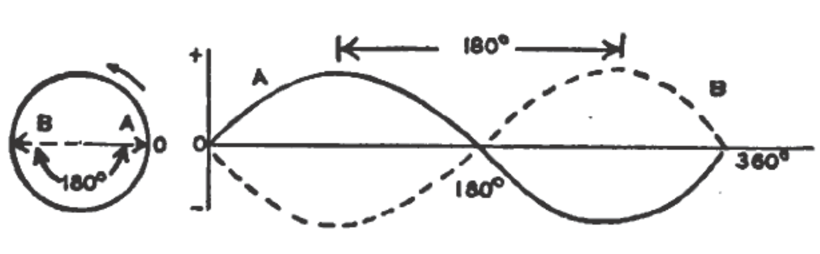


Figure 12–6.—Sine waves with a phase difference of 180°.

Summary of Use of Vectors

Vectors are used to:

1. Express the magnitude of voltages and currents and the phase angle between them.

2. Combine out-of-phase current, for example, in parallel a-c circuits (to be described in ch. 13).
3. Combine the impedances of series a-c circuits (to be described later in this chapter).

It is important to remember the two quantities that a vector represents: (1) The length of the vector indicates the MAGNITUDE of the quantity, and (2) the direction of the vector represents the TIME PHASE ANGLE between itself and a reference quantity, either voltage or current.

All positive vectors of voltage and current indicate the same (positive) direction through the electric circuit with which they are associated.

Vectors may be moved from one position to another provided their length and direction remain unchanged.

Topographic and Polar Vectors

Topographic vectors are arranged end to end. Polar vectors extend from a common center or pole.

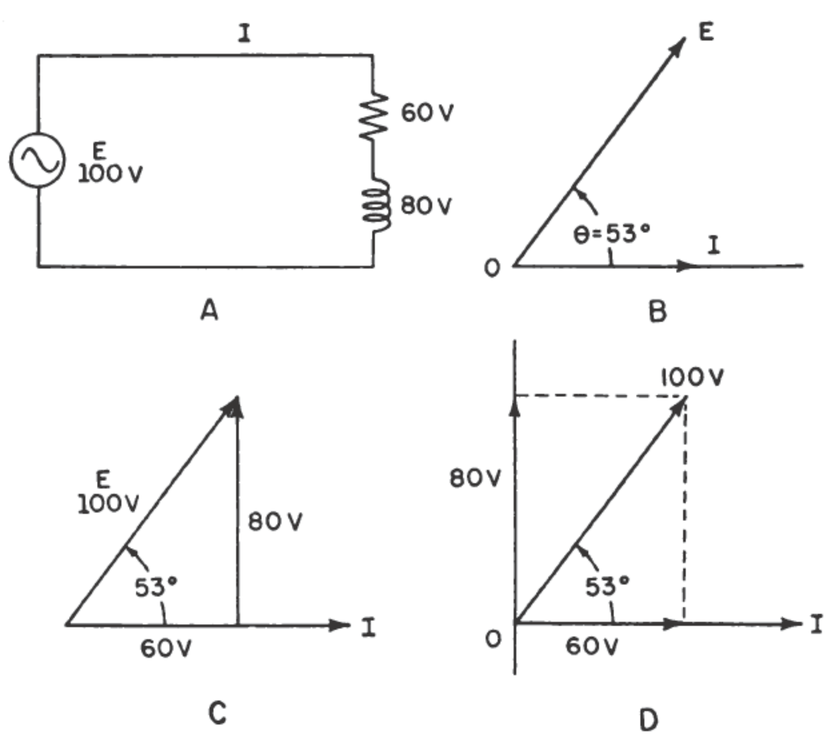


Figure 12–7.—Example of electrical vectors.

In figure 12–7, A, the resistor and coil are in series across a 100-volt a-c supply. In figure 12–7, B, the polar vector diagram shows that the circuit current, *I*, lags the applied voltage, *E*, by an angle of 53°. In the topographic vector diagram of figure 12–7, C, the applied voltage, *E*, is the hypotenuse of a right triangle. The base of the triangle represents the 60-volt drop across the resistor. The altitude of the triangle represents the 80-volt drop across the coil. Figure 12–7, D, represents the equivalent polar vector diagram.

The current is the same in all parts of a series circuit at any instant and is therefore assumed to be the reference vector for all other quantities in the circuit. The current reference vector is usually horizontal and points to the right.

In parallel circuits the voltage is common to all branches of the circuit and constitutes the reference vector for all other quantities in the circuit. The voltage reference vector is usually horizontal and points to the right.

If several current vectors are used in the same diagram they should all be drawn to the same scale. Similarly all voltage vectors in the same diagram should be drawn to the same scale.

INDUCTIVE REACTANCE

Inductance has been defined in chapter 8 as that property of a circuit that opposes any current change in the circuit. It is also defined as that property whereby energy may be stored in a magnetic field. Thus a coil of wire possesses the property of inductance because a magnetic field is established around the coil when current flows in the coil.

When the coil is energized with direct current, opposition is manifested only when the circuit is energized or when it is deenergized.

When the coil is supplied with alternating current, the opposition is continuous and much greater than when it is supplied with direct current. Thus, for equal applied voltages, the current through the coil is less when a-c is applied than when d-c is applied, as may be demonstrated by the circuits of figure 12–8. The alternating current is accompanied by an alternating mag-

netic field around the coil, which cuts through the turns of the coil. This action induces a voltage in the coil that always opposes the changing current. When the switch is in position ① the lamps burn brightly on direct current but in position ②, although the effective value of the applied a-c voltage is equal to the d-c value, the lamps burn dimly because of the opposition developed across the inductance. Most of the applied voltage appears across L with little remaining for the lamps.

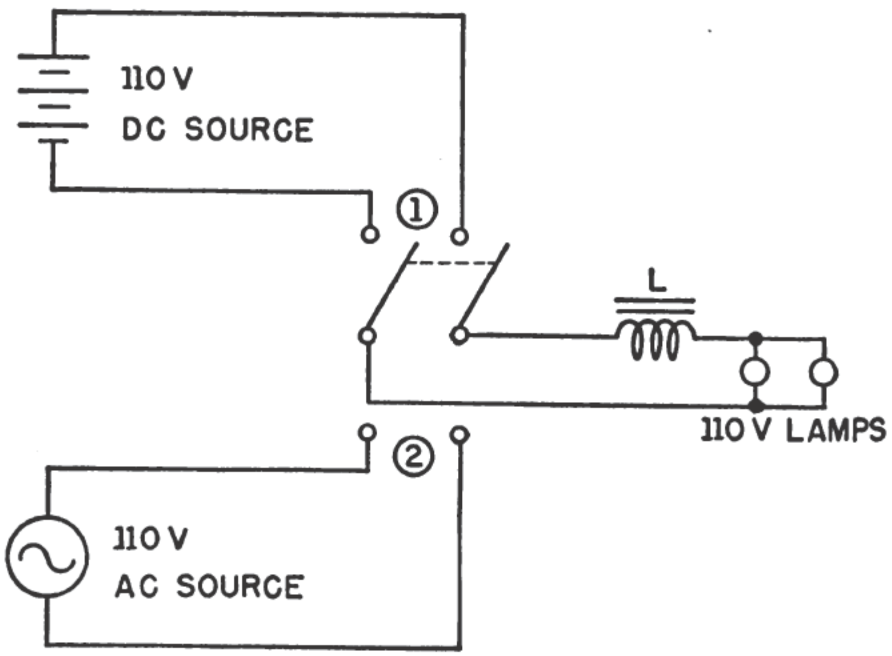


Figure 12–8.—Relative effects of inductance in d-c and in a-c circuits.

Relation Between Induced Voltage and Current

As stated previously in chapter 8, any change in current, either a rise or a fall, in a coil causes a corresponding change of the magnetic flux around the coil (fig. 12–9, A). If the current is sinusoidal, the induced voltage will have the form of a sine wave. Because the current changes at its maximum rate when it is going through its zero value at 0°, 180°, and 360° (fig. 12–9, B), the voltage induced in the coil is at its maximum value at these instants. According to Lenz's law, the induced voltage always opposes the change in current. Thus when the current is rising in a positive direction at 0° the induced emf is

of opposite polarity and opposes the rise in current. Later when the current is falling toward its zero value at 180° the induced voltage is of the same polarity as the current and tends to keep the current from falling. Thus the induced voltage can be seen to lag the current by 90°. The resistance of the coil is small and the principal opposition to the current flow through the coil is the induced voltage, E_{ind}. The applied voltage, E, is slightly larger than E_{ind} and diametrically opposed to it, as indicated in the vector diagram (fig. 12–9, C).

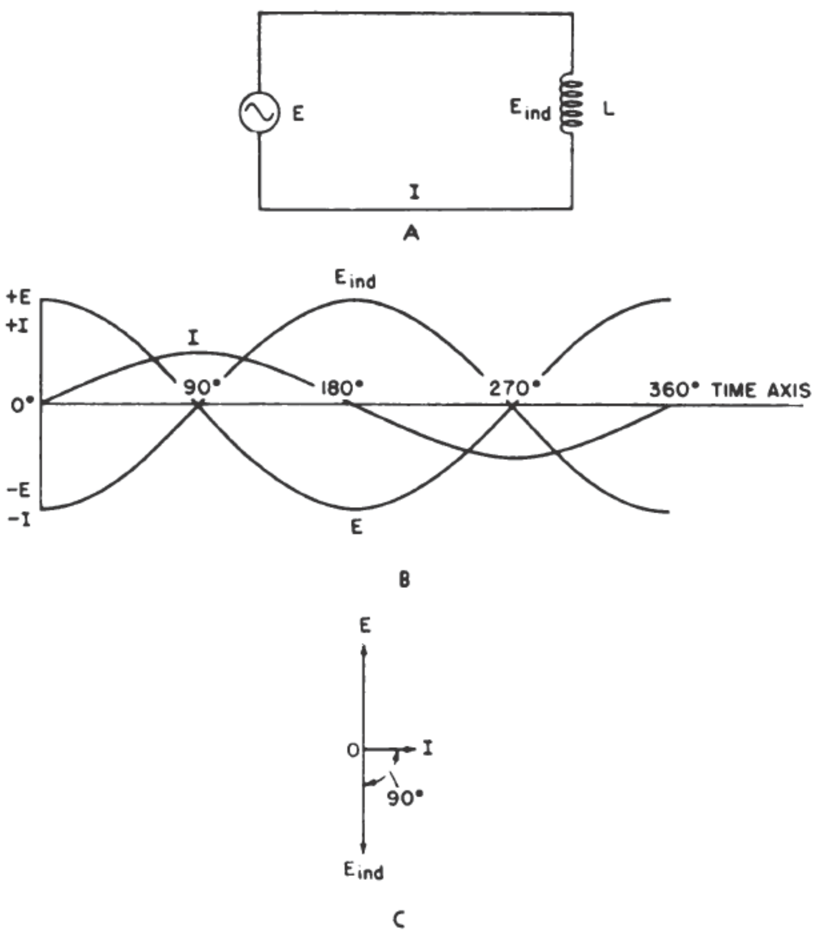


Figure 12–9.—Relation between induced voltage and current.

The current lags the applied voltage in an inductive circuit by an angle of 90° and leads the induced voltage by 90°. The induced voltage is always of opposite polarity to the applied voltage and is called a COUNTER EMF or a BACK EMF because it always opposes the change of current.

Formula for Inductive Reactance

The magnitude of the effective value of the counter emf of self-induction, E_{ind}, in volts in an inductive circuit is

$$E_{ind} = \omega LI = 2\pi fLI,$$

where $\omega = 2\pi f$, I is in effective amperes, L is in henrys, and f is in cycles per second. The inducted voltage varies directly with the frequency, the inductance, and the current.

The ratio of the effective value of the counter emf to the effective value of the current is called INDUCTIVE REACTANCE. When the counter emf is in volts and the current in amperes, the inductive reactance will be in ohms. Assuming the current and the counter emf have sine waveforms, the inductive reactance, X_L, is

$$X_L = 2\pi fL = \omega L.$$

The inductive reactance varies directly with the frequency and the inductance.

Power in an Inductive Circuit

The power in a d-c circuit is equal to the product of volts and amperes, but in an a-c circuit this is true only in resistive loads in which there is no reactance present.

In a circuit possessing inductance only, the true power is zero (fig. 12–10). The current lags the applied voltage by 90°. The TRUE POWER is the average power actually consumed by the circuit, the average being taken over one complete cycle of alternat-

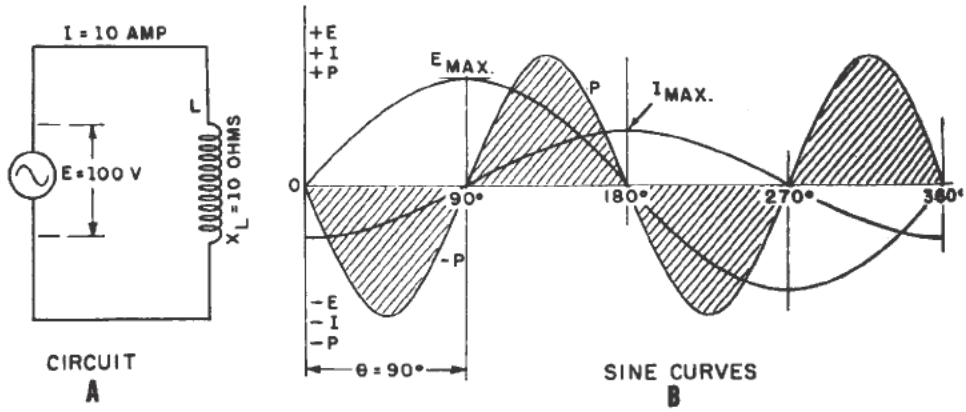


Figure 12–10.—Power in an inductive circuit.

ing current. The APPARENT POWER is the product of rms volts and rms amperes. Thus, in figure 12–10, A, the apparent power is $100 \times 10 = 1{,}000$ volt-amperes. However, the power absorbed by the coil during the time the current is rising (fig. 12–10, B) is returned to the source during the time the current is falling so that the average power is zero.

The ratio of the true power to the apparent power in an a-c circuit is called the POWER FACTOR. It may be expressed as a percent or as a decimal. In the inductor of figure 12–10, A, the power factor is $\frac{0}{1{,}000} = 0$. The power factor is also equal to the $\cos \theta$, where θ is the phase angle between the current and voltage. The phase angle between E and I in the inductor is $\theta = 90°$. Thus the power factor of an inductor of negligible losses is $\cos 90° = 0$. The apparent power in a purely inductive circuit is called REACTIVE POWER and the unit of reactive power is called the VAR. This unit is derived from the first letters of the words volt-ampere-reactive.

RESISTANCE IN AN A-C CIRCUIT

Relation Between *E*, *I*, and *P*

In an a-c circuit containing only resistance, the current and voltage are always in phase ($\theta = 0°$). (See fig. 12–11.) The true power dissipated in heat in a resistor in an a-c circuit when sine waveforms of voltage and current are applied is equal to the product of the rms volts and the rms amperes. In the circuit of

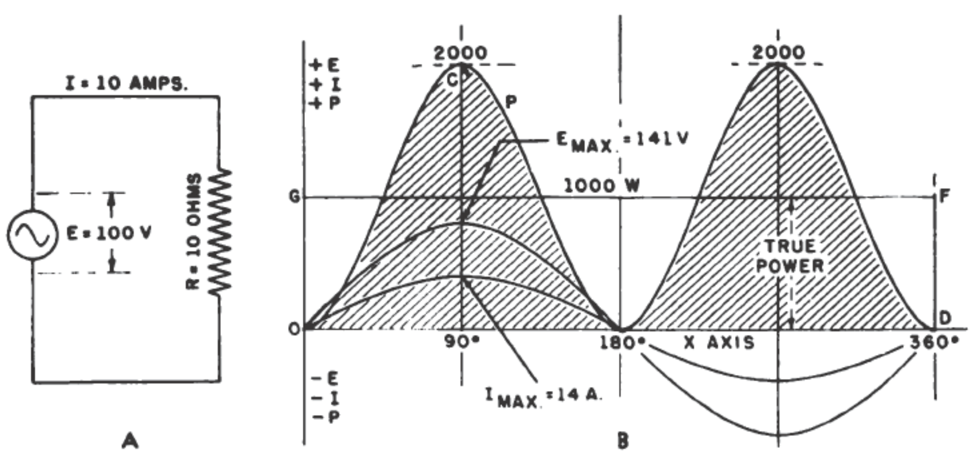


Figure 12–11.—Relation between *E*, *I*, and *P* in a resistive circuit.

figure 12–11, A, the power absorbed by the resistor is $P=EI$ $=100\times10=1{,}000$ watts. The product of the instantaneous values of current and voltage (fig. 12–11, B) gives the power curve, P, the axis of which is displaced above the X axis by an amount that is proportional to 1,000 watts.

The true average power is

$$P=\frac{E_{max}\times I_{max}}{2}=E_{eff}\times I_{eff}.$$

The power factor of a resistive circuit is cos 0° = 1, or 100 percent. The apparent power in the resistor is also equal to the true power. The reactive power in the resistive circuit is zero.

RESISTANCE AND INDUCTIVE REACTANCE IN SERIES

Because any practical inductor must be wound with wire that has resistance, it is not possible to obtain a coil without it. The

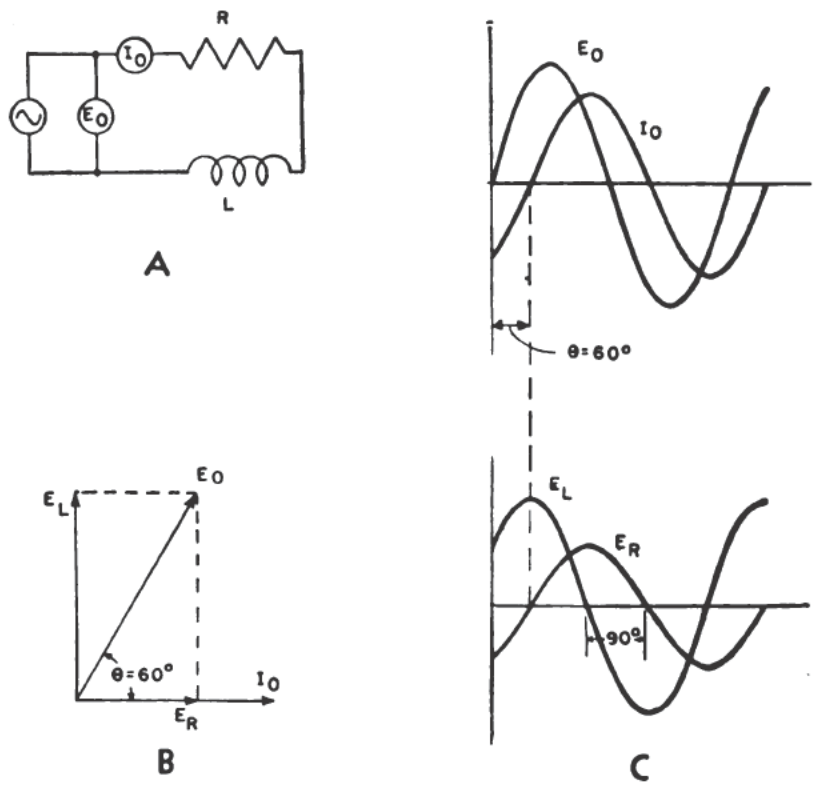


Figure 12–12.—Resistance and inductive reactance in series.

resistance associated with a coil may be considered as a separate resistor, R, in series with an inductor, L, which is purely inductive and contains only inductive reactance (fig. 12–12, A). The resistance has been exaggerated in this example to be of the same order of magnitude as the inductive reactance of the inductor in order to simplify the trigonometric solution.

If an alternating current, I_o, flows through the inductor a voltage drop occurs across both the resistor and the inductor. The voltage, E_r, across the resistor is in phase with the current, and the voltage drop, E_L, across the inductor leads the current by 90° (fig. 12–12, B). The voltage across the resistor is assumed to be 50 volts and the voltage across the inductor, 86.6 volts. These two voltages are 90° out of phase, as indicated in the figure. The applied voltage, E_o, is the hypotenuse of a right triangle, the sides of which are 50 and 86.6 respectively. Thus,

$$E_o=\sqrt{50^2+86.6^2}=100 \text{ volts}.$$

The sine curves of circuit current, I_o, applied voltage, E_o, the voltage across the coil, E_L, and the voltage across the resistor, E_r, are shown in figure 12–12, C.

Impedance

Impedance is the total opposition to the flow of alternating current in a circuit that contains resistance and reactance. In the case of pure inductance, inductive reactance, X_L is the total opposition to the flow of current through it. In the case of pure resistance, R represents the total opposition. The combined opposition of R and X_L in series or in parallel to current flow is called IMPEDANCE. The symbol for impedance is Z.

The impedance of resistance in series with inductance is

$$Z=\sqrt{R^2+X_L^2},$$

where Z, R, and X_L are the hypotenuse, base, and altitude respectively of a right triangle in which $\cos\theta=\frac{R}{Z}$, $\sin\theta=\frac{X_L}{Z}$, and $\tan\theta=\frac{X_L}{R}$. As mentioned before, $\cos\theta$ is equal to the circuit power factor; $\sin\theta$ is sometimes referred to as the reactive factor; and $\tan\theta$ is referred to as the quality, or Q, of a circuit or a circuit

component. The trigonometric functions including the sine, cosine, and tangent of angles between 0° and 90° are given in appendix VI at the end of this training manual.

Power in a Series Circuit Containing R and X_L

The true power in any circuit is the product of the applied voltage, the circuit current, and the cosine of the phase angle between them. Thus, in figure 12–13, the true power is

$$P=EI\cos\theta=100\times 7.07(\cos 45^\circ=0.707)=500\text{ watts.}$$

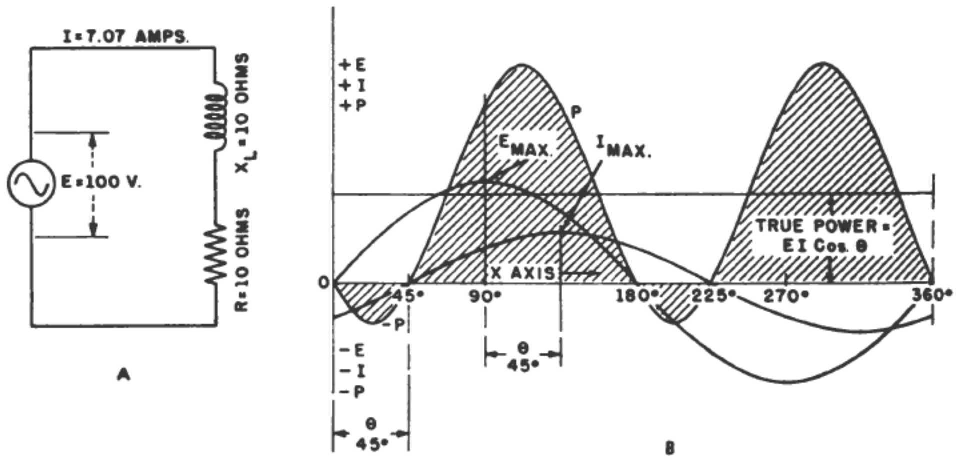


Figure 12–13.—Power in a circuit containing L and R in series.

The power curve is partly above the X axis (fig. 12–13, B) and partly below it. The axis of the power curve is displaced above the X axis an amount proportional to the true power, $EI\cos\theta$. The apparent power in this circuit is

$$100\times 7.07=707\text{ volt-amperes.}$$

The power factor is

$$\cos\theta=\frac{\text{true power}}{\text{apparent power}}=\frac{500}{707}=0.707,\text{ or }70.7\text{ percent.}$$

The reactive power in the L-R circuit is the product of $EI\sin\theta$ where $\sin\theta$ is the reactive factor. Thus the reactive power is

$$100\times 7.07\ (\sin 45^\circ=0.707)=500\text{ vars (lagging).}$$

The term LAGGING refers to the action of the current with respect to the applied voltage.

Summary of *E*, *Z*, and *P* Relations in *L-R* Circuit

The relation between voltage, impedance, and power in a series *L-R* circuit with sine waveforms applied is summarized in figure 12–14. In this example the circuit contains 12 ohms of resistance in series with 16 ohms of inductive reactance (fig. 12–14, A). The phase relations between the applied voltage; the voltage across the resistance, R; and the voltage across the inductance, L; are shown in figure 12–14, B. The IR drop across R is $5 \times 12 = 60$ volts and forms the base of the right triangle. The altitude is the IX_L drop, or $5 \times 16 = 80$ volts, across L. The applied voltage represents the vector sum of the IR drop and the IX_L drop and is the hypotenuse of the right triangle. Its magnitude is 100 volts. All voltages are effective values.

The relation between resistance, inductive reactance, and impedance is shown by the vectors of figure 12–14, C. The re-

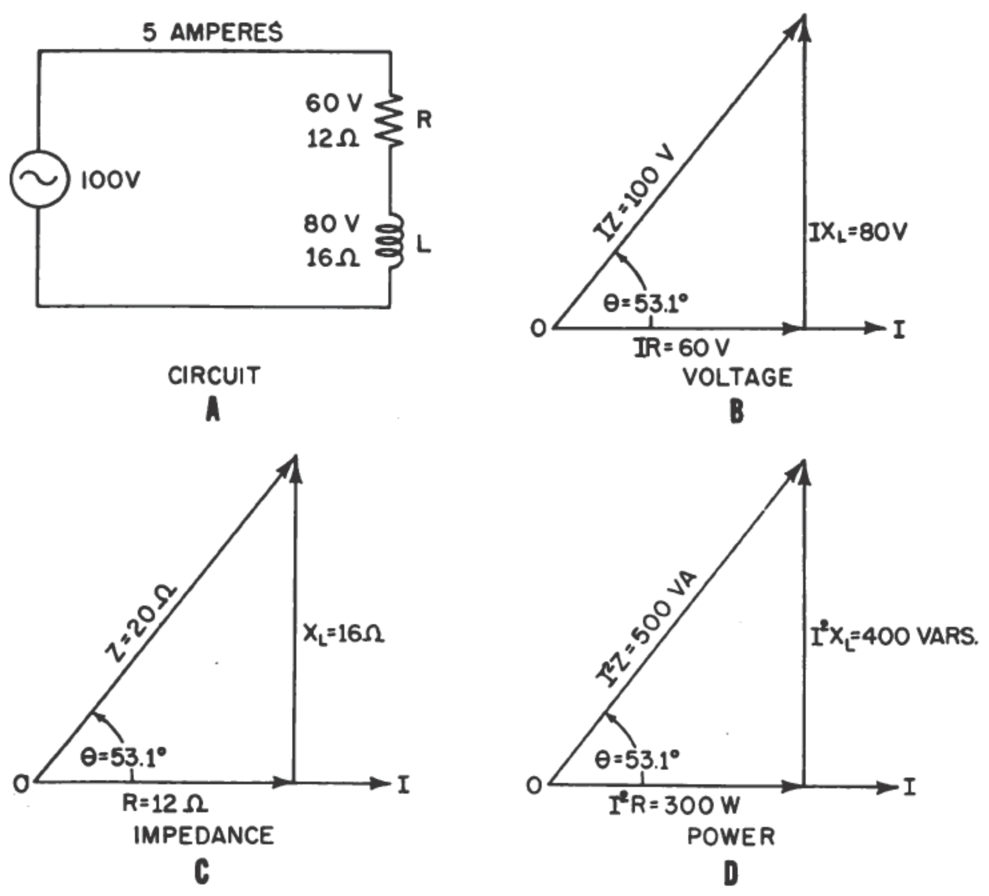


Figure 12–14.—Summary of relation between *E*, *Z*, and *P* in a series *L-R* circuit.

sistance of 12 ohms forms the base of the right triangle. The altitude of this triangle represents the inductive reactance of the coil and has a magnitude of 16 ohms. The combined impedance of the circuit is $Z=\sqrt{12^2+16^2}=20$ ohms. In this triangle cos $\theta=\frac{R}{Z}$. Thus cos $\theta=\frac{12}{20}=0.6$, from which $\theta=53.1°$.

The relation between apparent power, true power, and reactive power is shown in figure 12–14, D. The hypotenuse represents the apparent power and is equal to EI, or $100\times 5=500$ volt-amperes. The base represents true power and is equal to $EI\cos\theta$, or $100\times 5\times 0.6=300$ watts, where $0.6=\cos 53.1°$. The altitude represents reactive power and is equal to $EI\sin\theta$, or $100\times 5\times 0.8=400$ vars, where $0.8=\sin 53.1°$.

In all three vector diagrams the right triangles are similar. The common factor of current, I, makes their corresponding sides proportional. Thus the voltage triangle of figure 12–14, B, is obtained by multiplying the corresponding sides of the impedance triangle of figure 12–14, C, by I. The hypotenuse is equal to the applied voltage, or $IZ=5\times 20=100$ volts. The base is equal to the voltage across R, or $IR=5\times 12=60$ volts. The altitude is equal to the voltage across L, or $IX_L=5\times 16=80$ volts.

Multiplying corresponding sides of the voltage triangle by I gives the power triangle of figure 12–14, D. The hypotenuse is IZ times I, or $I^2Z=5^2\times 20=500$ volt-amperes of apparent power. The base is IR times I, or $I^2R=5^2\times 12=300$ watts of true power. The altitude is IX_L times I, or $I^2X_L=5^2\times 16=400$ vars of reactive power.

In the three vector diagrams, the circuit power factor is equal to the following ratios:

In the voltage diagram,

$$\cos\theta=\frac{IR}{IZ}=\frac{60}{100}=0.6.$$

In the impedance diagram,

$$\cos\theta=\frac{R}{Z}=\frac{12}{20}=0.6.$$

In the power diagram,

$$\cos\theta=\frac{I^2R}{I^2Z}=\frac{300}{500}=0.6.$$

In all three diagrams $\theta=53.1°$ and the power factor is 60 percent.

CAPACITIVE REACTANCE

Capacitance was defined in chapter 8 as that quality of a circuit that enables energy to be stored in an electric field. A capacitor is a device that possesses the quality of capacitance. In simple form it has been shown to consist of two parallel metal plates separated by an insulator, called a DIELECTRIC. The electric field consists of parallel lines of electric force which terminate in a positive charge on one plate and a negative charge on the other. The positive and negative charges on the plates establish the electric field in the dielectric because the dielectric prevents these charges from neutralizing each other.

Current Flow in a Capacitive Circuit

A capacitor that is initially uncharged tends to draw a large current when a d-c voltage is first applied. During the charging period, the capacitor voltage rises. After the capacitor has received sufficient charge the capacitor voltage equals the applied voltage and the current flow ceases. If a sine-wave a-c voltage is applied to a pure capacitance the current is a maximum when the voltage begins to rise from zero, and the current is zero when the voltage across the capacitor is a maximum (fig. 12–15, A). The current leads the applied voltage by 90°, as indicated in the vector diagram of figure 12–15, B.

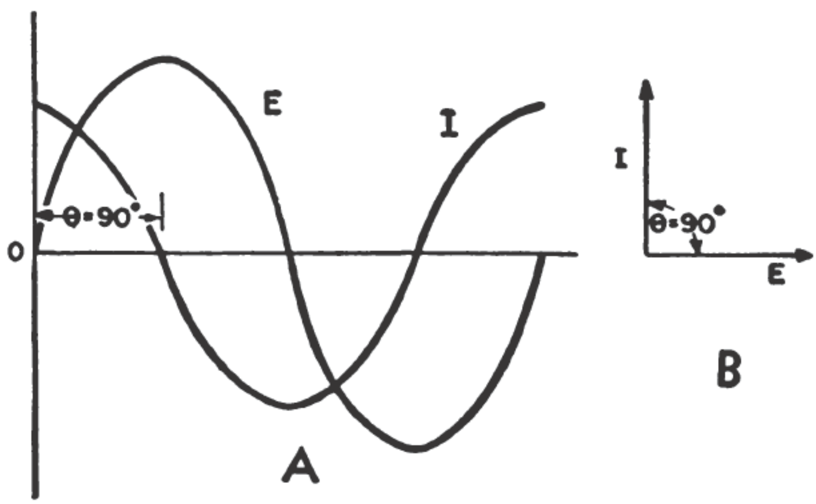


Figure 12–15.—Phase relation between *E* and *I* in a capacitive circuit.

Factors That Control Charging Current

Because a capacitor of large capacitance can store more energy than one of small capacitance, a larger current must flow to charge a large capacitor than to charge a small one, assuming the same time interval in both cases. Also, because the current flow depends on the rate of charge and discharge, the higher the frequency, the greater is the current flow per unit time.

The charging current in a purely capacitive circuit varies directly with the capacitance, voltage, and frequency—

$$I = 2\pi fCE = \omega CE,$$

where I is effective current in amperes, f is in cycles per second, C is the capacitance in farads, E is in effective volts, and ω is in radians per second.

Formula for Capacitive Reactance

The ratio of the effective voltage across the capacitor to the effective current is called the CAPACITIVE REACTANCE, X_c, and represents the opposition to current flow in a capacitive circuit of zero losses—

$$X_C = \frac{1}{2\pi fC} = \frac{1}{\omega C}.$$

When f is in cycles per second, C is in farads, and ω is in radians per second, then X_C is in ohms.

Example: What is the capacitive reactance of a capacitor operating at a frequency of 60 cycles per second and having a capacitance of 133 microfarads, or 1.33×10^{-4} farads?

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 60 \times 1.33 \times 10^{-4}} = 20 \text{ ohms}.$$

Power in a Capacitive Circuit

With no voltage or charge, the electrons in the dielectric between the capacitor plates rotate around their respective nuclei in normally circular orbits. When the capacitor receives a charge the positive plate repels the positive nuclei and at the same time the electrons in the dielectric are strained toward the positive plate and repelled away from the negative plate. This distorts

the orbits of the electrons in the direction of the positive charge. During the time the electron orbits are changing from normal to the strained position there is a movement of electrons in the direction of the positive charge. This movement constitutes the DISPLACEMENT CURRENT in the dielectric. When the polarity of the plates reverses, the electron strain is reversed. If a sine-wave voltage is applied across the capacitor plates the electrons will oscillate back and forth in a direction parallel to the electrostatic lines of force. DISPLACEMENT CURRENT is a result of the movement of bound electrons, whereas CONDUCTION CURRENT represents the movement of free electrons. One component of the displacement current leads the applied voltage by 90° and expends no energy in the dielectric.

Figure 12–16, A, shows a capacitive circuit and figure 12–16, B, indicates the sine waveform of charging current, applied voltage, and instantaneous power. The effective voltage is 70.7 volts. The effective current is 7.07 amperes. Because the losses are neglected, the phase angle between current and voltage is assumed to be 90°. The true power is zero, as indicated by the expression

$$
\begin{aligned}
P &= EI \cos \theta \\
&= 70.7 \times 7.07 (\cos 90° = 0) = 0 \text{ watt.}
\end{aligned}
$$

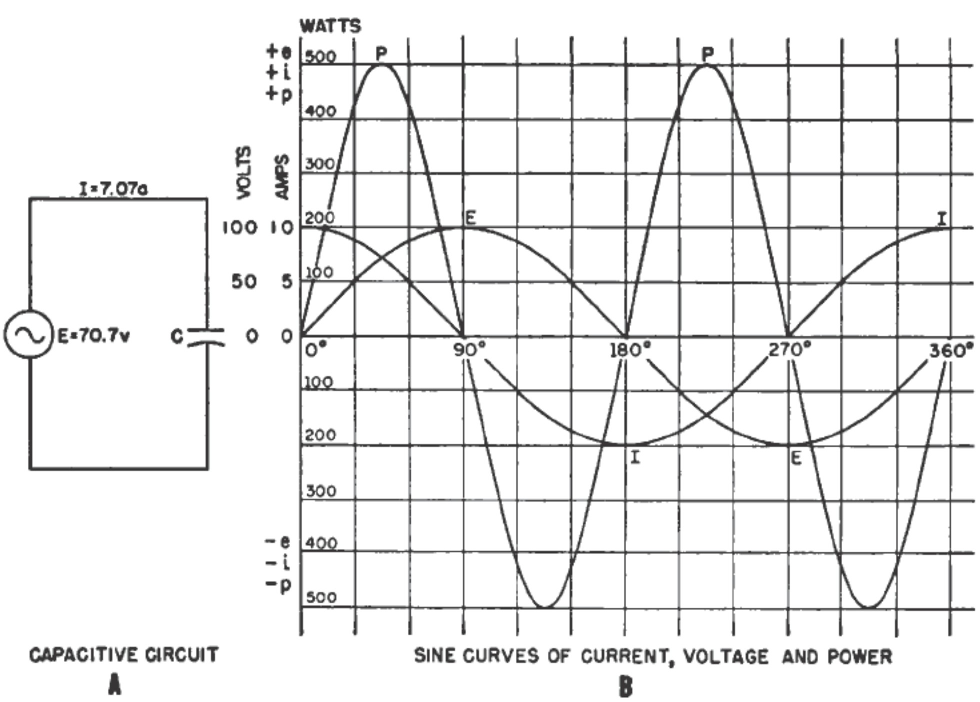


Figure 12–16.—Power in a capacitive circuit.

Multiplying instantaneous values of current and voltage over one cycle, or 360°, gives the power curve, *P*. During the first quarter cycle (from 0° to 90°) the applied voltage rises from zero to a maximum and the capacitor is receiving a charge. The power curve is positive during this period and represents energy stored in the capacitor. From 90° to 180° the applied voltage is falling from maximum to zero and the capacitor is discharging. The corresponding power curve is negative and represents energy returned to the circuit during this interval. The third quarter cycle represents a period of charging the capacitor and the fourth quarter cycle represents a discharge period. Thus, the average power absorbed by the capacitor is zero. The action is like the elasticity of a spring. Storing a charge in the capacitor is like compressing the spring. Discharging the capacitor is like releasing the pressure on the spring, thus allowing it to return the energy that was stored within it on compression.

The apparent power in the capacitor is $EI=70.7\times 7.07=500$ volt-amperes. The reactive power in the capacitor is

$$EI \sin \theta=70.7\times 7.07\times(\sin 90°=1)=500 \text{ vars (leading).}$$

CAPACITIVE REACTANCE AND RESISTANCE IN SERIES

Losses that appear in capacitive circuits may be lumped in a resistor connected in series with the capacitor, as indicated in figure 12–17. In this example a 39.8-microfarad capacitor is connected in series with a 20-ohm resistor (fig. 12–17, A). The applied voltage across the *R-C* series circuit is 134 volts and the frequency is 100 cycles per second.

The capacitive reactance at this frequency is

$$X_C=\frac{1}{2\pi fC}=\frac{1}{6.28\times 100\times 39.8\times 10^{-6}}=40 \text{ ohms.}$$

Voltage Relation

The voltages across *R* and *C* are 90° out of phase and equal to 60 volts and 120 volts respectively, as shown in the vector diagram of figure 12–17, B. The voltage across *C* is represented as IX_C and is plotted vertically downward from the horizontal in order to indicate that the current leads the voltage across the

capacitor by 90°. Angle θ between the voltage across the capacitor and the circuit current is represented as $-90°$ because it is measured clockwise from the horizontal reference vector, OI. The total voltage is equal to the vector sum of IR and IX_C and is represented in the figure as the hypotenuse of a right triangle

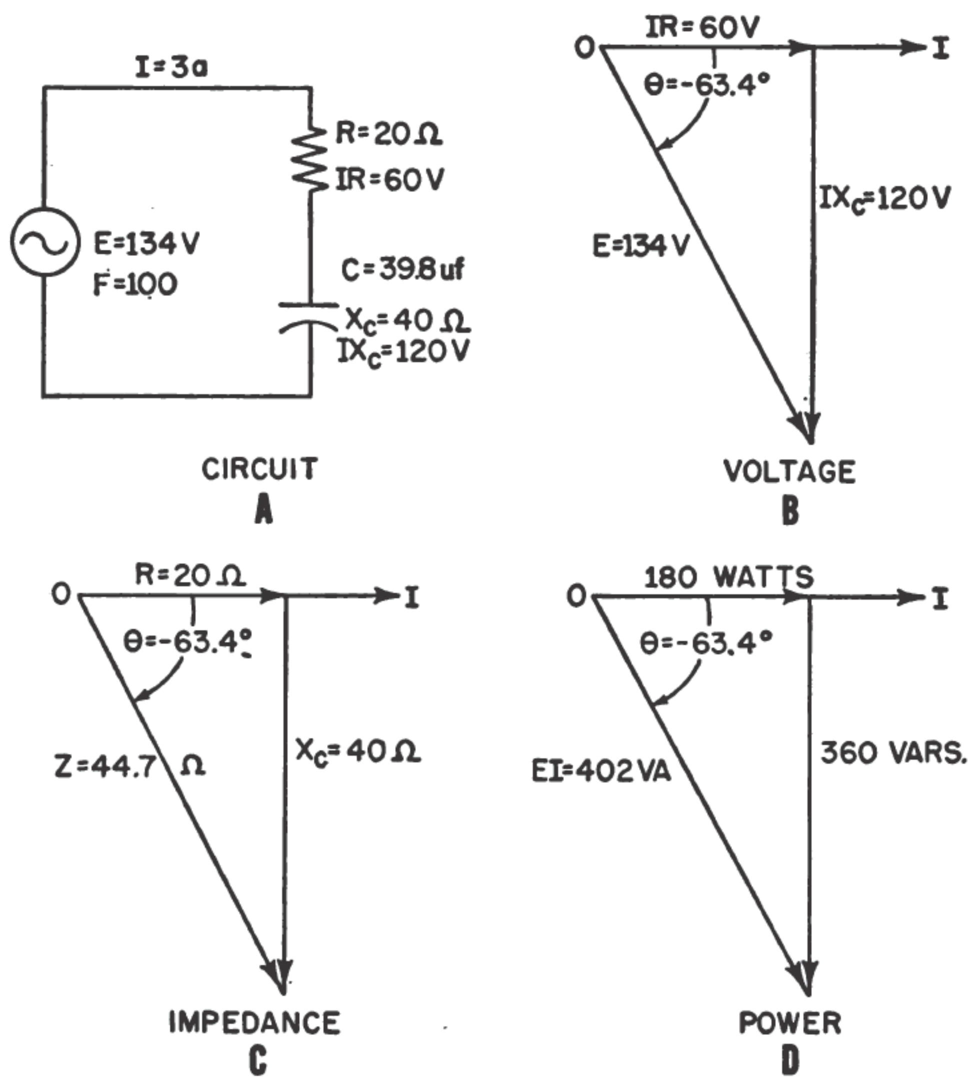


Figure 12–17.—Capacitive reactance and resistance in series.

the base of which represents the voltage across R, having an effective value of 60 volts and the voltage drop across C, having an effective value of 120 volts. The total (applied) voltage is $\sqrt{60^2+120^2}=134$ volts.

Impedance

The total impedance, Z, of the series circuit is

$$Z=\frac{E}{I}=\frac{134}{3}=44.7 \text{ ohms.}$$

The impedance diagram is indicated in figure 12–17, C. The base of the triangle represents the resistance, R, of the circuit and has a magnitude of 20 ohms. The capacitive reactance, X_C, is represented as the altitude of the triangle and has a magnitude of 40 ohms. The total impedance may be represented as the hypotenuse of the triangle and has a magnitude of $\sqrt{20^2+40^2}=$ 44.7 ohms, which is also the ratio of the applied voltage to the circuit current. The impedance diagram is a right triangle that is similar to the triangle representing the voltage relation in figure 12–17, B.

Power

The power relations are indicated in the vector diagram of figure 12–17, D. This diagram is also a right triangle similar to the other two. The base represents the true power absorbed by the circuit resistance and has a magnitude of $I^2R=3^2\times 20=180$ watts. The altitude represents the reactive volt-ampere (leading) and has a magnitude of $I^2X_C=3^2\times 40=360$ vars. The total apparent power is

$$EI=134\times 3=402 \text{ volt-amperes.}$$

Summary of *E*, *Z*, and *P* Relations in *R-C* Circuit

As mentioned previously, all three triangles are similar. The common factor is the circuit current; and the phase angle, θ, between E and I is equal to the same value in all three diagrams. Thus, the circuit power factor is equal to

$$\cos\theta=\frac{IR}{IZ}=\frac{60}{134}=0.446 \text{ (fig. 12–17, B)}$$

$$\cos\theta=\frac{R}{Z}=\frac{20}{44.7}=0.446 \text{ (fig. 12–17, C)}$$

$$\cos\theta=\frac{I^2R}{I^2Z}=\frac{180}{402}=0.446 \text{ (fig. 12–17, D)}$$

and angle $\theta=63.4°$ in all three triangles.

The true power is

$$EI \cos\theta=134\times3\times(\cos 63.4°=0.446)=180 \text{ watts.}$$

The reactive power is

$$EI \sin\theta=134\times3\times(\sin 63.4°=0.894)=360 \text{ vars.}$$

The apparent power is

$$EI=134\times3=402 \text{ volt-amperes.}$$

QUIZ

1. What principle does the a-c generator use in converting mechanical energy into electrical energy?
2. How many cycles of a-c voltage are produced in a 6-pole alternator of the revolving field type for each revolution of the rotor?
3. What is the frequency of an a-c generator having 8 poles and a speed of 900 rpm?
4. In a 60-cycle alternator:
   (a) How much time does it take to complete one cycle?
   (b) What is this interval called?
5. What is the effective voltage generated by an alternator having a field strength of 9×10^8 lines of magnetic flux per pole, a frequency of 60 cycles per second, and 100 active conductors?
6. What normally supplies the field current for a-c generators?
7. What two quantities does a simple vector represent?
8. At what angle in figure 12–4 is the vertical projection of the rotating vector at its maximum positive value?
9. If the maximum value of an a-c voltage of sine waveform is 170 volts, what is the average value?
10. If the maximum value of an a-c voltage of sine waveform is 1,100 volts, what is the effective value?
11. What is the effective value of an alternating current that has the same heating effect in a given resistor as a direct current of 7.07 amperes?
12. What is the maximum value of an alternating current of sine waveform that has the same heating effect in a given resistor as a direct current of 10 amperes?
13. What is the ratio of the effective value to the average value of a wave called?

14. When representing two a-c voltages as vectors:
    (a) To what quantities are the lengths of the vectors proportional?
    (b) To what quantity is the angle between the vectors proportional?
15. If the rms voltage between two insulated conductors is 1,000 volts and the voltage is of sine waveform, what is the maximum voltage stress on the insulation?
16. In the generator represented schematically in figure 12–5:
    (a) Why cannot the voltages generated in coils *a* and *b* be added arithmetically?
    (b) If the phase angle between two voltages in a series circuit is 90° and their effective values are each 115 volts, what is the total voltage?
    (c) If the frequency is 60 cycles per second, what is the time difference in seconds, between the total voltage and either component?
17. If the phase angle between two a-c voltages is 180° and they are of equal magnitude and act in the same circuit, what is the total voltage?
18. Distinguish between topographic and polar vectors.
19. For equal applied voltages, why is the current through a coil less when a-c is used than when d-c is used?
20. If the resistance of a coil is small, what is the principal opposition to the flow of alternating current through the coil?
21. Inductive reactance represents the ratio of what two effective values?
22. States the formula for inductive reactance in terms of:
    (a) Inductance and frequency in cycles per second.
    (b) Inductance and angular velocity in radians per second.
23. What is the unit of reactive power?
24. What is the true power in a circuit containing only resistance, if the maximum voltage is 120 volts and the maximum current is 10 amperes?
25. (a) What is the total impedance of a series circuit in which R is 3 ohms and X_L is 4 ohms?
    (b) What is the phase angle between R and Z?
26. What is the true power absorbed by a circuit in which the applied voltage is 120 volts, rms; the current is 10 amperes, rms; and the phase angle is 60°?
27. State the formula for charging current in a purely capacitive circuit in terms of the frequency, capacitance, and applied voltage.
28. What is the capacitive reactance of an 8 microfarad capacitor, if the frequency is 60 cycles per second?

29. What is the essential difference between dielectric displacement currents and conduction currents?
30. In a capacitor circuit of negligible losses, what is the relation between the energy stored during charge and the energy returned to the circuit during discharge?
31. In the circuit of figure 12–17, if E is 134 volts, rms; R is 40 ohms; and X_C is 40 ohms:
    (a) What is the circuit impedance?
    (b) What is the circuit current?
    (c) What is the voltage drop across R?
    (d) What is the voltage drop across C?
    (e) What is the circuit power factor?
    (f) What is the apparent power of the circuit?
    (g) What is the true power of the circuit?
    (h) What is the reactive power of the circuit?

CHAPTER 13

BASIC ALTERNATING-CURRENT CIRCUIT THEORY

INTRODUCTION

In chapter 12 the student was introduced to alternating-current electricity. Some of the common terms employed in describing the operation of a-c equipment were defined. Likewise, circuit characteristics such as resistance, inductance, capacitance, and impedance, as well as the effects of various SERIES combinations, were described. In this chapter the effects produced by the various possible PARALLEL combinations of resistance, inductance, and capacitance are analyzed by means of vectors. The energy losses in a-c circuits are also considered; and finally, a brief treatment of 3-phase circuits is included.

All electrical equipments, as well as individual components, have the property of resistance, inductance, and capacitance, one or more of which may be negligible under certain operating conditions.

Electrical equipment (lights, heating elements, motors, relays, and so forth) is usually connected in parallel across the a-c power source. Not only are individual equipments connected in parallel, but also the individual components of equipments are often connected in parallel. The effects of the electrical characteristics of these components on the operation of the equipment and on the source voltage, current, and power output is of great importance to the technician dealing with power problems.

The parallel circuit combinations treated in this chapter are:

1. A resistor in parallel with an inductor that is assumed to have negligible resistance.
2. A resistor in parallel with a capacitor that is assumed to have negligible resistance.

3. An inductor in parallel with a capacitor each of whose resistance is assumed to be lumped in a resistor connected in series with the element.
4. A resistor, a capacitor of negligible resistance, and an inductor whose equivalent resistance is connected in series with it—connected in parallel.

As in the series-circuit calculations of chapter 12, a working knowledge of the trigonometric functions of the sine, cosine, and tangent of the angles in a right triangle is desirable in obtaining a better understanding of the behavior of the current, voltage, and power relations in parallel circuits. The series circuits described in chapter 12 contain resistance in series with inductance or capacitance. To properly combine R and X_L, or R and X_C, they are represented as the base and altitude, respectively, of a right triangle, the hypotenuse of which constitutes the total opposition to current flow in the series circuit. The angle θ is the angle between the base and hypotenuse, or between R and Z. The three trigonometric functions of θ that are most useful in solving the impedance triangle are

$$\cos\theta=\frac{R}{Z},$$

$$\sin\theta=\frac{X_L}{Z},$$

and

$$\tan\theta=\frac{X_L}{R}.$$

Each of these may be transferred, as desired, to solve for any one of the three quantities in terms of the remaining two. Thus,

$$R=Z\cos\theta,$$

$$X_L=Z\sin\theta,$$

and

$$X_L=R\tan\theta.$$

Similar triangles were developed in chapter 12 for the relation between true power, apparent power, and reactive power, and for the distribution of the voltages around a simple series circuit containing R and X_L, or R and X_C. The application of right tri-

angles is extended in this chapter to represent the distribution of currents in the resistive, inductive, and capacitive branches of parallel circuits.

INDUCTANCE AND RESISTANCE IN PARALLEL

If the voltage applied across a resistor and inductor in parallel has a sine waveform, the currents in the branches will also have sine waveforms. In the parallel circuit of figure 13–1, A, the applied voltage, E, has a magnitude of 100 volts (rms). Branch ① contains a 20-ohm resistor, and the current, I_1, is equal to $\frac{100}{20}$, or 5 amperes (rms). This current is in phase with E. Branch ② contains an inductance of 0.053 henry. The line frequency is 60 cycles and the INDUCTIVE REACTANCE is

$$X_L = 2\pi f L = 6.28 \times 60 \times 0.053 = 20 \text{ ohms.}$$

The true power losses associated with the inductor in this example are considered negligible. Therefore, current I_2 is equal to $\frac{100}{20}$, or 5 amperes. This current lags E by an angle of 90°. (In an inductive circuit the current always lags the voltage by some angle.)

The sine waveforms of applied voltage and current are shown in figure 13–1, B. I_1 is in phase with E. I_2 lags E by an angle of 90°. The total current, I_t, lags E by an angle of 45°.

Current Vectors

A polar-vector diagram representing the three currents and the applied voltage is shown in figure 13–1, C. Vector OE represents the effective value of the applied voltage and is the horizontal reference vector for both branches because it is common to both branches. Vector I_1 is the effective current of 5 amperes in branch ①. This vector is in the same line as vector OE because the current and voltage in the resistive circuit are in phase. Vector I_2 is the effective current of 5 amperes in branch ② and lags vector OE by an angle of 90°. I_1 is called the ENERGY COMPONENT of the circuit current. It flows in the resistive branch in which true power is absorbed from the source and is dissipated as heat. I_2 is called the NONENERGY COMPONENT of the circuit

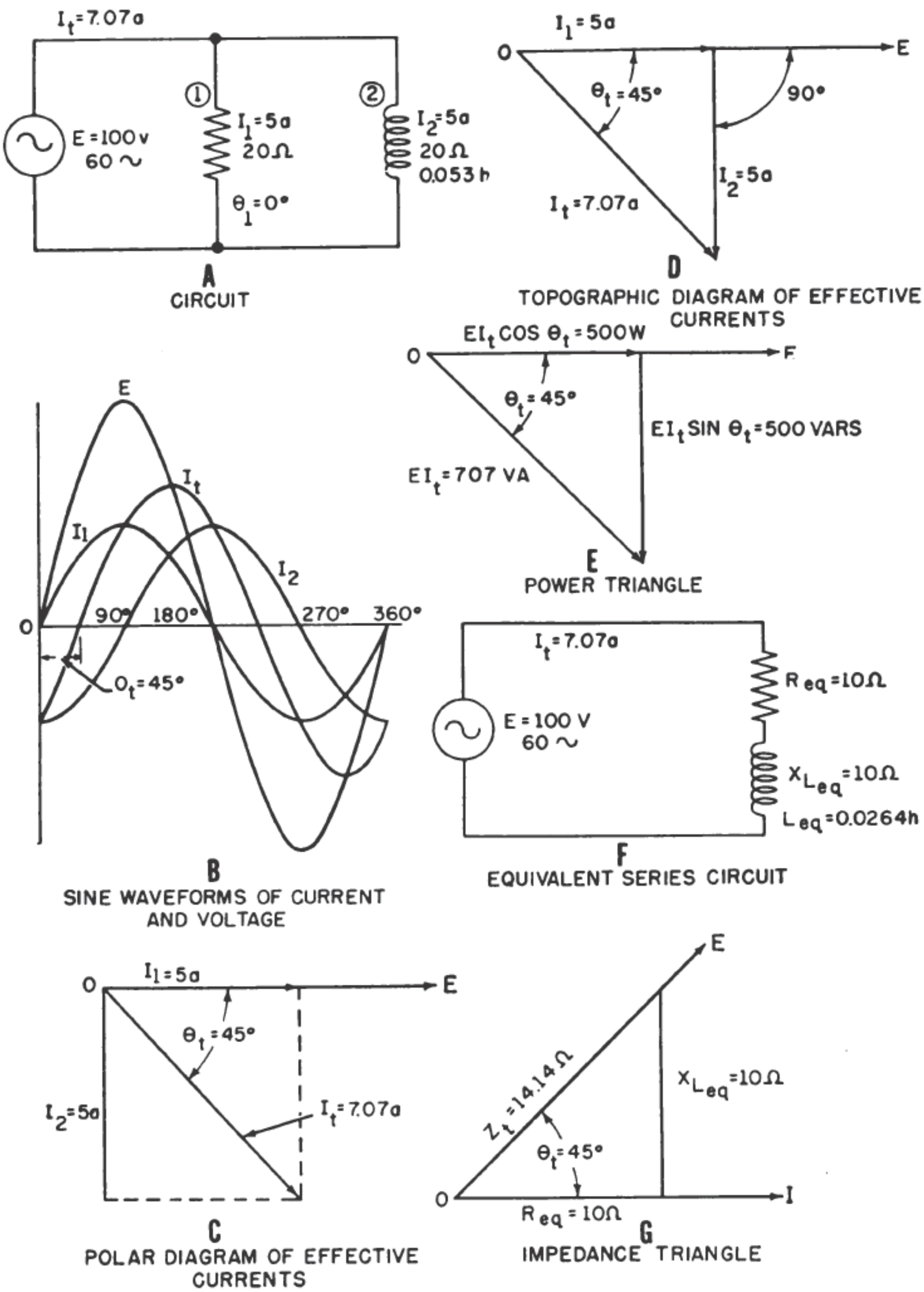


Figure 13–1.—Resistance and inductance in parallel.

current. This current flows in the inductive branch where the true power is zero, and the reactive power is exchanged with the source twice during each cycle of applied voltage.

The total circuit current, I_t, is represented as the diagonal of the parallelogram, the sides of which are I_1 and I_2 (fig. 13–1, C). In this example, the sides are 5 amperes (rms) each and the diagonal is 7.07 amperes (rms).

A topographic-vector diagram representing the three currents and the applied voltage is shown in figure 13–1, D. As in the polar diagram, OE is the reference vector. I_1, in phase with OE, is the base of the triangle. I_2, 90° out of phase with OE, is the altitude of the triangle, and is plotted downward to indicate lag. The resultant current, I_t, is the hypotenuse of the triangle. The hypotenuse is equal to the square root of the sum of the squares of the other two sides. Thus,

$$I_t=\sqrt{I_1^2+I_2^2}=\sqrt{5^2+5^2}=7.07 \text{ amperes.}$$

The phase angle between I_t and E is the angle whose cosine is

$$\frac{I_1}{I_t}=\frac{5}{7.07}=0.707.$$

This angle is 45°.

Power and Power Factor

The apparent power, true power, and reactive power in the parallel circuit are related as the hypotenuse, base, and altitude, respectively, of a right triangle in a similar manner to that described in the series circuits of chapter 12.

The relation between apparent power, true power, and reactive power in the example of figure 13–1 is shown in figure 13–1, E. The hypotenuse of the right triangle represents the apparent power and is equal to EI_t, or $100\times7.07=707$ volt-amperes. The base of the triangle represents the true power and is equal to $EI_t \cos \theta_t$, or $100\times7.07\times0.707=500$ watts, where $0.707=\cos 45°$. The altitude represents the reactive power and is equal to $EI_t \sin \theta_t$, or $100\times7.07\times0.707=500$ vars, where $0.707=\sin 45°$. The power triangle is similar to the current triangle and is related to it by the common factor of voltage (the voltage is the same across both branches of the parallel circuit).

Because branch ① (fig. 13–1, A) is purely resistive and branch

② is purely inductive, the true power of the circuit is absorbed in branch ① and the reactive power is developed in branch ②. In branch ① the true power is $EI_1 \cos \theta_1$, or $100\times 5\times 1=500$ watts, where $1= \cos 0°$. In branch ② the reactive power is $EI_2 \sin \theta_2$, or $100\times 5\times 1=500$ vars, where $1=\sin 90°$.

The true power in branch ① may also be calculated as $I_1^2R_1$, or $5^2\times 20=500$ watts. The reactive power in branch ② may be calculated as $I_2^2X_{L2}$, or $5^2\times 20=500$ vars. The total circuit power factor is $\cos \theta_t=\frac{\text{true power}}{\text{apparent power}}=\frac{500}{707}=0.707$. The power factor of branch ① is $\cos \theta_1=\cos 0°=1$, and the power factor of branch ② is $\cos \theta_2=\cos 90°=0$.

Equivalent Series-Circuit Impedance

The combined impedance of the parallel circuit is

$$Z_t=\frac{E}{I_t}=\frac{100}{7.07}=14.14 \text{ ohms.}$$

The combined impedance is also called the impedance of the equivalent series circuit. The equivalent series circuit (fig. 13–1, F) contains a resistor and an inductor in series that combine to give the same impedance as the total impedance of the given parallel circuit. Thus, the current in the equivalent series circuit is equal to the total circuit current in the parallel circuit when rated voltage and frequency are applied to the circuits.

In figure 13–1, G, the hypotenuse, Z_t, of the impedance triangle is 14.14 ohms and the phase angle, θ_t, between total current and line voltage, is 45°. The equivalent series RESISTANCE, R_{eq}, is the base of the impedance triangle and is equal to $Z_t \cos \theta$, or $14.14\times \cos 45°=10$ ohms. The equivalent series REACTANCE, $X_{L_{eq}}$, is the altitude of the impedance triangle and is equal to $Z_t \sin \theta_t$, or $14.14\times \sin 45°=10$ ohms.

The inductance of the equivalent series circuit is

$$L_{eq}=\frac{X_L}{2\pi f}=\frac{10}{6.28\times 60}=0.0264 \text{ henry.}$$

Thus, in this example, the 20-ohm resistor in branch ① shunts the 0.053-henry inductor in branch ②, and the source "sees" an equivalent resistor of 10 ohms in series with an equivalent inductor of 0.0264 henry.

Equivalent Circuit of a Low-Loss Inductor

The losses in an air-core inductor occur in the wire with which the inductor is wound. If the losses are small, the resistance of the wire will be small compared with the inductive reactance developed at the operating frequency. The coil resistance acts in series with the coil reactance.

An equivalent parallel circuit for a low-loss coil can be established by substituting an equivalent coil of zero losses, but having the same inductance as the given coil for one branch and a resistor for the other branch. The resistor is of such a magnitude that the same losses will occur in this branch as occur in the given coil when rated voltage and frequency are applied.

In the following example a low-loss circuit is established and then the equivalent parallel circuit is derived. A low-loss coil is indicated in figure 13–2, A. It has an inductance of 1.59 henry and a d-c resistance of 10 ohms at an operating frequency of 100 cps. The INDUCTIVE REACTANCE is

$$X_L = 2\pi fL = 6.28 \times 100 \times 1.59 = 1{,}000 \text{ ohms.}$$

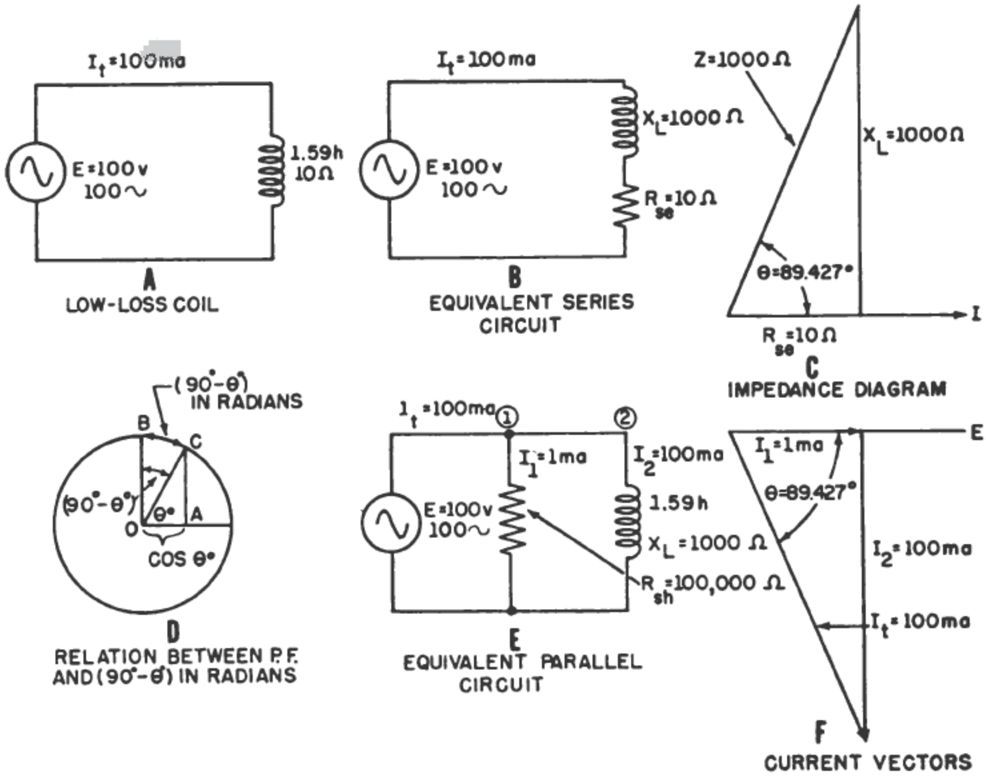


Figure 13–2.—Equivalent circuits of a low-loss coil.

The equivalent series circuit (fig. 13–2, B) is shown with 10 ohms of resistance acting in series with 1,000 ohms of inductive reactance. The impedance triangle is shown in figure 13–2, C.

A low-loss coil has a low power factor. Thus, in the impedance triangle, $\cos\theta=\frac{R_{se}}{Z}=\frac{10}{1{,}000}=0.01$, or 1 percent. As can be seen in this example, Z is approximately equal to X_L, and $\cos\theta=\frac{R_{se}}{X_L}$. Since $\tan\theta=\frac{X_L}{R_{se}}$, it follows that in this case $\cos\theta=\frac{1}{\tan\theta}$. Thus $\tan\theta=\frac{1{,}000}{10}=100$, and the POWER FACTOR$=\frac{10}{1{,}000}$, or 0.01, or 1 percent.

A useful relationship for finding angle θ in low-loss circuits is shown in figure 13–2, D. This concept involves the relation between $\cos\theta$, expressed decimally, and the complementary angle, $90-\theta$, expressed in radians. The figure indicates a unit-radius circle in which any length of arc BC measures the corresponding angle $(90-\theta)$ which it subtends. The length of arc is equal to the angle in radians. Since 2π radians are equal to 360° it follows that 1 radian is approximately equal to $\frac{360°}{2\pi}$, or 57.3°. Thus, from a knowledge of the angle in radians, the equivalent angle θ, in degrees, is $\theta=57.3\times$angle in radians.

For angles greater than 84.3°, $\tan\theta$ is greater than 10 and $\cos\theta$—which equals OA, in figure 13–2, D—is approximately equal to arc BC. Arc BC is a measure of the complementary angle $90-\theta$ and is expressed in radians. Thus, $\cos\theta$ is approximately equal to the angle in radians by which the current fails to be exactly 90° out of phase with the voltage. In this example, the power factor is 0.01 and therefore the angle by which the current fails to be 90° out of phase with the voltage (complementary angle) is 0.01 radian. This angle, in degrees, becomes $57.3°\times0.01$, or 0.573°. Therefore, θ is equal to $90°-0.573°$, or 89.427°, and $\cos 89.427°=0.01$.

The relationship described in the preceding example is expressed in general terms as follows: The complementary angle, $(90-\theta)$ in radians, is equal to the power factor, $\cos\theta$, expressed

decimally, where $\tan \theta$ is numerically equal to or greater than 10. From this relationship it is possible to estimate quickly the phase angle between current and voltage in low power-factor (low-loss) circuits.

The equivalent parallel circuit for the 1.59-henry coil discussed in this example is shown in figure 13–2, E. The equivalent shunt resistor, R_{sh}, has a resistance of 100,000 ohms (to be derived later), and this resistor is connected in parallel with a 1.59-henry inductor having zero losses. With rated voltage and frequency applied, the input current to the parallel circuit has the same magnitude and phase with respect to the applied voltage as in the original coil.

The current in the resistive branch is $\frac{100}{100{,}000}=0.001$ ampere, or 1 milliampere, and is the base of the current triangle (fig. 13–2, F). The current in the inductive branch is $\frac{100}{1{,}000}=0.1$ ampere, or 100 ma, and is the altitude of the current triangle. The total current in the parallel circuit is equal to $\sqrt{1^2+100^2}=100$ milliamperes (approx.), and is the hypotenuse of the current triangle. This current lags the line voltage by an angle of $90°-(57.3°\times 0.01)$, or 89.427°.

The equivalent series-circuit impedance triangle (fig. 13–2, C) is similar to the current triangle (fig. 13–2, F). From the impedance triangle, $\cos \theta=\frac{R_{se}}{X_L}$; and from the current triangle, $\cos \theta=\frac{I_1}{I_2}$, where I_1 is the energy current and I_2 is the nonenergy current. Therefore,

$$\frac{I_1}{I_2}=\frac{R_{se}}{X_L}. \tag{13–1}$$

For the resistive branch,

$$I_1=\frac{E}{R_{sh}}, \tag{13–2}$$

and for the inductive branch,

$$I_2=\frac{E}{X_L}. \tag{13–3}$$

Substituting equations 13–2 and 13–3 in equation 13–1,

$$\frac{\frac{E}{R_{sh}}}{\frac{E}{X_L}}=\frac{R_{se}}{X_L}. \tag{13–4}$$

Canceling E and transposing equation 13–4,

$$R_{sh}=\frac{X_L^2}{R_{se}}. \tag{13–5}$$

In the example of figure 13–2, $R_{se}=10$ ohms, $X_L=1{,}000$ ohms, and $R_{sh}=\frac{(1{,}000)^2}{10}=100{,}000$ ohms.

The preceding relations are sufficiently accurate for low-loss inductive circuits in which θ is 84.3° or higher, and $\tan\theta$ is 10 or higher.

Combining Currents at Acute Angles

Figure 13–3, A, represents a 2-branch parallel circuit in which branch ① has a power factor of 0.50 and branch ② has a power factor of 0.866. The current in branch ① lags the applied voltage by an angle of 60° and the current in branch ② lags the applied voltage by an angle of 30°. The series resistance, R_1, of branch ① is 10 ohms, the series inductive reactance, X_{L1}, is 17.32 ohms, and the impedance, Z_1, is $\sqrt{10^2+(17.32)^2}=20$ ohms. The series resistance, R_2, of branch ② is 17.32 ohms, the series inductive reactance, X_{L2}, is 10 ohms, and the impedance, Z_2, is $\sqrt{(17.32)^2+10^2}=20$ ohms.

A topographic-vector diagram of the currents in the two branches and the total circuit current is shown in figure 13–3, B. The current in branch ① is $I_1=\frac{100}{20}=5$ amperes and is the hypotenuse of the right triangle of which the base (energy component) is $5\cos 60°=2.5$ amperes and the altitude (nonenergy component) is $5\sin 60°=4.33$ amperes. The current in branch ② is $I_2=\frac{100}{20}=5$ amperes and is the hypotenuse of the right angle of which the base (energy component) is $5\cos 30°=4.33$ amperes

and the altitude (nonenergy component) is 5 sin 30°=2.5 amperes. The total circuit current is the vector sum of I_1 and I_2, and is the hypotenuse of the resultant right triangle, the base of which is the sum of the energy components of both branches, 2.5+4.33=6.83 amperes, and the altitude of which is the sum of the nonenergy components of both branches, 4.33+2.5=6.83 amperes. Thus,

$$I_t=\sqrt{(2.5+4.33)^2+(4.33+2.5)^2}=9.66 \text{ amperes.}$$

The circuit power factor is $\cos\theta_t=\dfrac{2.5+4.33}{9.66}=0.707$, and $\theta_t=45°$.

The power relations for this circuit are shown in figure 13–3, C. The apparent power in branch ① is $EI_1=100\times5=500$ volt-

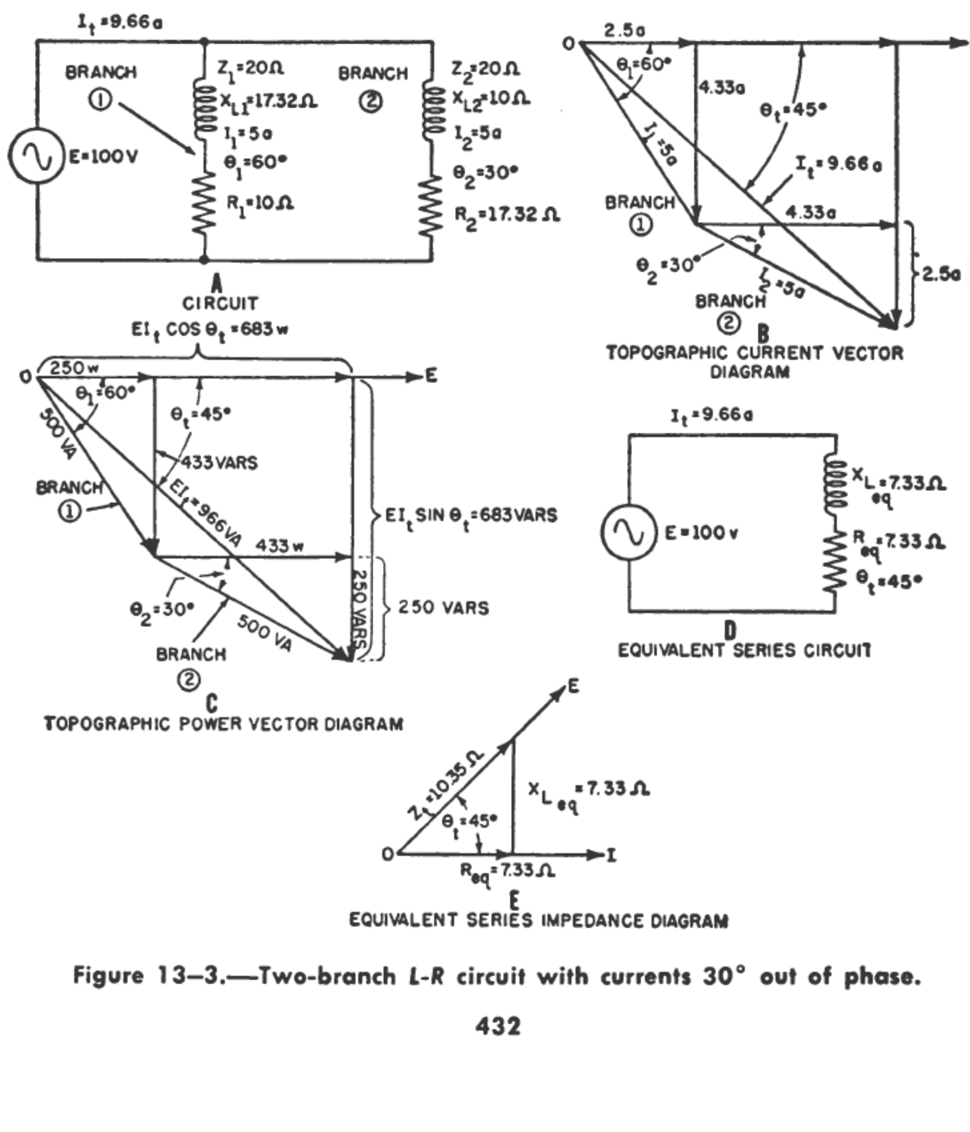


Figure 13–3.—Two-branch *L-R* circuit with currents 30° out of phase.

amperes and is the hypotenuse of the right triangle, the base of which is $EI_1 \cos \theta_1 = 100 \times 5 \times \cos 60° = 250$ watts of true power, and the altitude of which is $EI_1 \sin \theta_1 = 100 \times 5 \times \sin 60° = 433$ vars of reactive power.

The apparent power in branch ② is $EI_2 = 100 \times 5 = 500$ volt-amperes and is the hypotenuse of the right triangle, the base of which is $EI_2 \cos \theta_2 = 100 \times 5 \times \cos 30° = 433$ watts of true power, and the altitude of which is $EI_2 \sin \theta_2 = 100 \times 5 \times \sin 30° = 250$ vars of reactive power.

The total apparent power of the combined parallel circuit is the hypotenuse of the resultant triangle, the base of which is equal to the sum of the true power components in both branches, and the altitude of which is equal to the sum of the reactive power components in both branches. Thus, the total apparent power $= \sqrt{(250+433)^2 + (433+250)^2} = 966$ volt-amperes.

The equivalent series circuit (fig. 13–3, D), representing the combined impedance of the given parallel circuit, has a total impedance, Z_t, that is determined by dividing the applied voltage, E, by the total circuit current, I_t.

$$Z_t = \frac{E}{I_t} = \frac{100}{9.66} = 10.35 \text{ ohms.}$$

The hypotenuse of the impedance triangle (fig. 13–3, E) is 10.35 ohms. The impedance triangle is similar to the resultant-current right triangle (fig. 13–3, B), and angle θ_t has the same magnitude in both triangles. This angle is equal to 45°, as determined in the current-triangle calculations.

The equivalent series resistance, R_{eq}, is the base of the impedance triangle and is equal to $Z_t \cos \theta_t$, or $10.35 \times \cos 45° = 7.33$ ohms.

The equivalent series reactance, $X_{L_{eq}}$, is the altitude of the impedance triangle and is equal to $Z_t \sin \theta_t$ or $10.35 \times \sin 45° = 7.33$ ohms.

The impedance triangle is also similar to the resultant power triangle for the combined parallel circuit (fig. 13–3, C). The common factor between the two triangles is the square of the total circuit current, I_t^2. The base of the resultant power triangle is $I_t^2 R_{eq} = (9.66)^2 \times 7.33 = 683$ watts of true power. This product

may be checked by adding the true power components in branches ① and ②. Thus, $250+433=683$ watts of true power.

The altitude of the resultant power triangle is $I_t^2 X_{L_{eq}} = (9.66)^2 \times 7.33 = 683$ vars of reactive power. This product may be checked by adding the reactive var components in branches ① and ②. Thus, $433+250=683$ vars of reactive power.

The hypotenuse of the resultant power triangle is $I_t^2 Z_t = (9.66)^2 \times 10.35 = 966$ volt-amperes of apparent power. This product may be checked by adding vectorially the apparent power in branch ① and the apparent power in branch ②. This product is equal to the hypotenuse of the resultant power triangle (fig. 13–3, C). This product CANNOT be checked accurately by adding arithmetically the apparent power in branch ① to the apparent power in branch ② because these quantities are not in phase with each other.

CAPACITANCE AND RESISTANCE IN PARALLEL

The action of a capacitor in an a-c series circuit was described in chapter 12 and is amplified further at this time as an introduction to the a-c parallel R-C circuit. In figure 13–4, A, an a-c

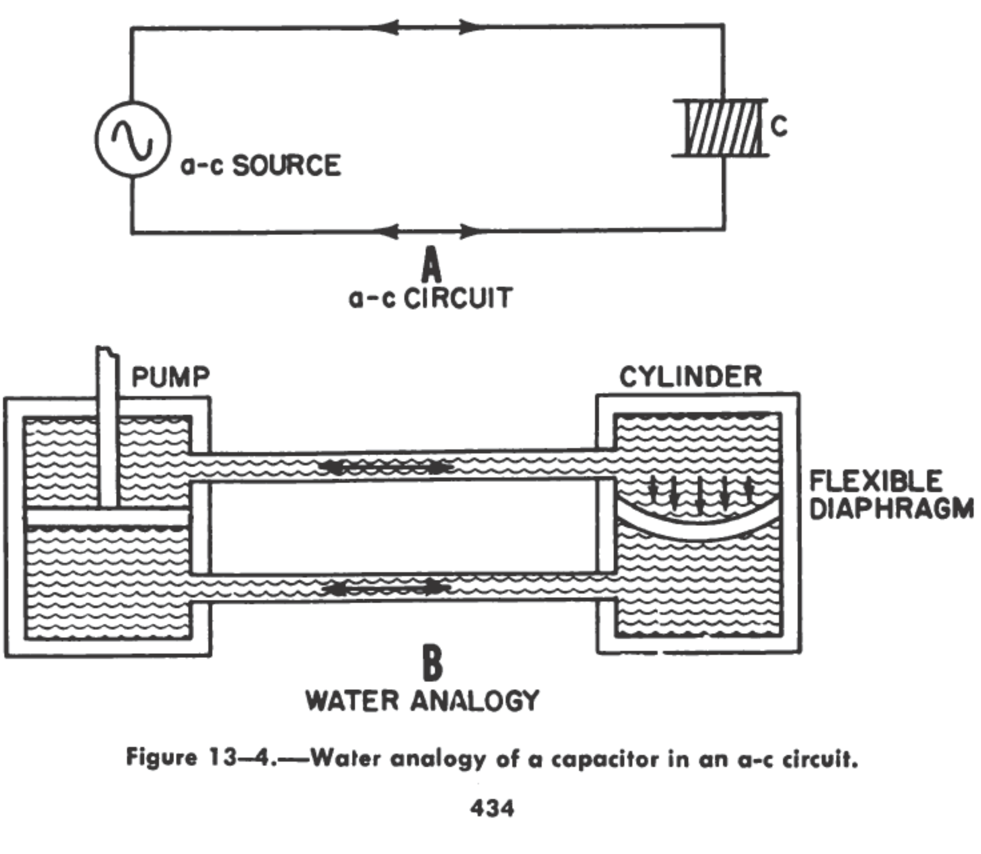


Figure 13–4.—Water analogy of a capacitor in an a-c circuit.

voltage of sine waveform is applied across a capacitor and a charging current of sine waveform flows around the circuit in a manner something like that of the flow of water in the hydraulic analogy shown in figure 13–4, B. The pump at the left corresponds to the a-c source and the cylinder at the right corresponds to the capacitor. If the pump piston is driven by means of a crank turning at uniform speed, the resulting motion of the water will be sinusoidal. The motion of the water is transmitted through the flexible diaphragm in the cylinder and the resulting motion, first in one direction and then in the other, corresponds to the electron flow in the wires connecting the capacitor to the a-c source.

The mechanical stress in the diaphragm in the cylinder corresponds to the electric stress in the dielectric between the plates of the capacitor. Electron flow does not occur through the dielectric in the same way that water would flow through the flexible diaphragm if a hole were punctured in it. Instead, the electrons flow around the capacitor circuit on one alternation causing a negative charge to build up on one plate, and a corresponding positive charge on the other, and on the next alternation causing a reversal of the polarity of the charges on the plates. Thus, the effective impedance which the capacitor offers to the flow of ALTERNATING CURRENT can be relatively low at the same time that the insulation resistance which the dielectric offers to the flow of DIRECT CURRENT is extremely high.

In the example of figure 13–5, a 2-branch circuit consists of a 100-ohm resistor and a 15.9-microfarad capacitor of negligible losses in parallel with a 100-volt a-c source. The frequency of the source voltage is 100 cps and the voltage is assumed to have a sine waveform. The current in branch ① is $\frac{100}{100}=1$ ampere (rms). The impedance of branch ② is composed of capacitive reactance; its resistance component is neglected. In branch ② $X_C=\frac{1}{2\pi fC}=\frac{1}{6.28\times 100\times 15.9\times 10^{-6}}=100$ ohms. The current in branch ② is $\frac{100}{100}=1$ ampere (rms.) The sine waveform of the branch currents and the applied voltage, together with the resultant line current, are shown in figure 13–5, B.

The current in the resistive branch is in phase with the applied voltage and has a peak value of $\frac{1}{0.707}=1.41$ amperes. The current in the capacitor branch leads the applied voltage by an angle of 90° and has a peak value of $\frac{1}{0.707}=1.41$ amperes. The resultant line current is the algebraic sum of the instantaneous

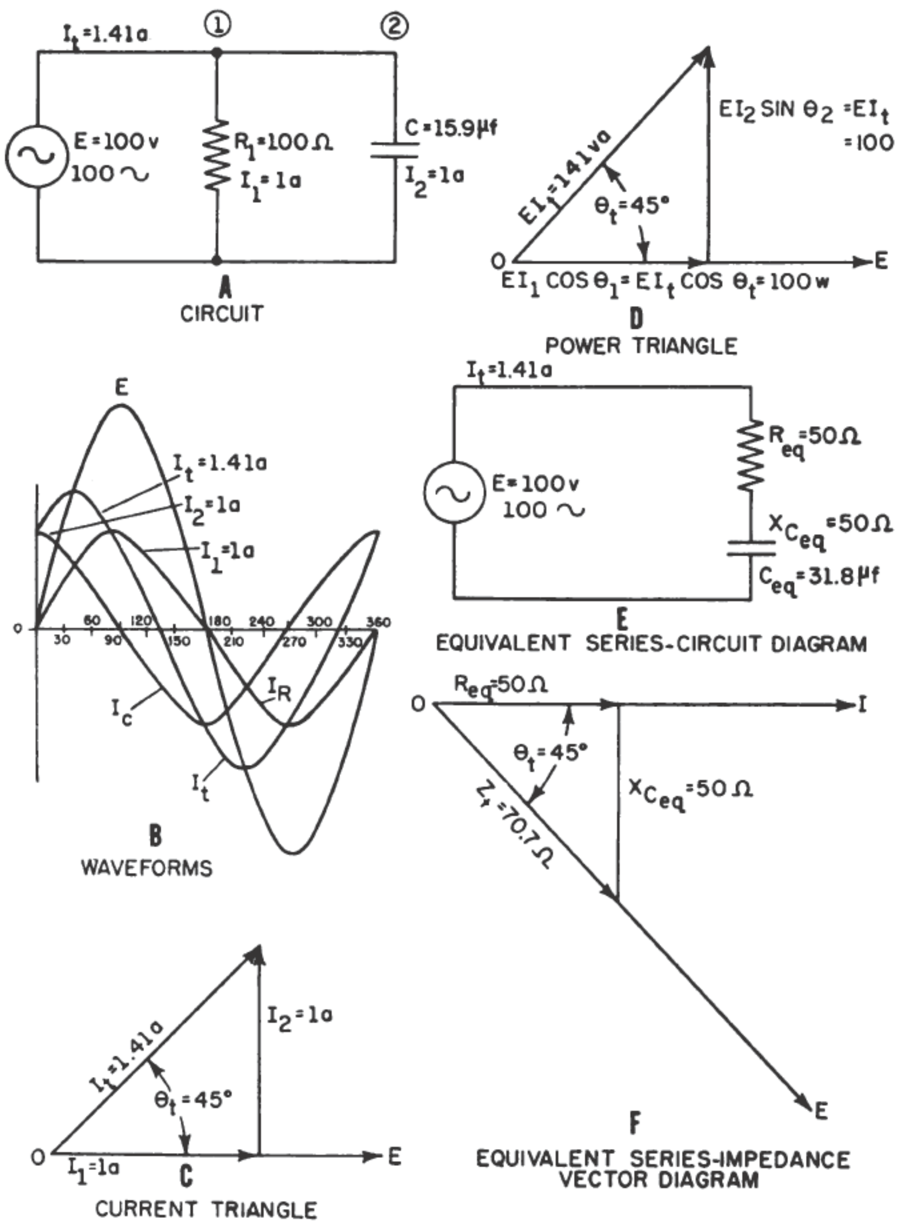


Figure 13–5.—Parallel *R-C* circuit analysis.

values of the currents in the two branches and has a peak value of 2 amperes. The resultant line current, I_t, leads the applied voltage by an angle of 45°.

Current Vectors

A topographic-vector diagram of the EFFECTIVE VALUES of these currents is shown in figure 13–5, C. The base of the current triangle is 1 ampere and represents the current in branch ①. This current is in phase with the applied voltage and represents the energy component of the total line current.

The altitude of the current triangle is 1 ampere and represents the current in branch ②. This current leads the applied voltage by 90° and represents the nonenergy component of the total line current.

The hypotenuse of the triangle is 1.41 amperes and represents the total line current.

The reference vector, OE, for the current triangle is the line voltage, and in the R-C parallel circuit, the altitude extends above the reference vector to indicate the sense of lead. This direction is opposite to that of the altitude of the current triangle for the R-L parallel circuit in figure 13–1, D. In both figures the vectors are assumed to rotate in a counterclockwise direction to indicate the sense of lead or lag of the currents with respect to the line voltage. In all single-phase circuits such as these, the current vectors are considered in their relative phase by angles that never exceed 90° with respect to their common reference voltage vector.

In figure 13–1, D, the line current lags the line voltage by an angle of 45°, and in figure 13–5, C, the line current leads the line voltage by an angle of 45°. If both inductance and capacitance exist in the same parallel circuit as separate branches, the currents in these branches will be 180° out of phase with each other, but the current in the inductive branch will never lag the line voltage by an angle in excess of 90°, and the current in the capacitive branch will never lead the line voltage by an angle in excess of 90°.

Power and Power Factor

The power triangle for the parallel R-C circuit is shown in figure 13–5, D. The TRUE power in branch ① is

$$EI_1 \cos \theta_1 = 100 \times 1 \times (\cos 0° = 1) = 100 \text{ watts},$$

and forms the base of the triangle in line with the voltage vector, OE. The REACTIVE power in branch ② is

$$EI_2 \sin \theta_2 = 100 \times 1 (\sin 90° = 1) = 100 \text{ vars},$$

and is the altitude of the power triangle, perpendicular to OE. The REACTIVE power in branch ① is

$$EI_1 \sin \theta_1 = 100 \times 1 (\sin 0° = 0) = 0 \text{ var},$$

and the TRUE power in branch ② is

$$EI_2 \cos \theta_2 = 100 \times 1 (\cos 90° = 0) = 0 \text{ watt}.$$

The APPARENT power of the parallel R-C circuit is

$$EI_t = 100 \times 1.41 = 141 \text{ volt-amperes}.$$

and is the hypotenuse of the power triangle.

The power triangle (fig. 13–5, D) is similar to the current triangle (fig. 13–5, C), since θ_t has the same magnitude in both triangles. The total circuit power factor, as determined from the current triangle, is

$$\cos \theta_t = \frac{I_1}{I_t} = \frac{1}{1.41} = 0.707.$$

The total circuit power factor, as determined from the power triangle, is

$$\cos \theta_t = \frac{\text{true power}}{\text{apparent power}} = \frac{100}{141} = 0.707.$$

The TRUE power of the total circuit, as determined from the power triangle, is

$$EI_t \cos \theta_t = 100 \times 1.41 (\cos 45° = 0.707) = 100 \text{ watts}.$$

This value is equal to the true power in branch ①.

The REACTIVE power of the total circuit, as determined from the power triangle, is

$$EI_t \sin \theta_t = 100 \times 1.41 (\sin 45° = 0.707) = 100 \text{ vars}.$$

This value is equal to the reactive power in branch ②.

Equivalent Series Impedance

The total impedance of the parallel *R-C* circuit is

$$Z_t=\frac{E}{I_t}=\frac{100}{1.41}=70.7 \text{ ohms.}$$

As mentioned previously, the total impedance is also the impedance of the equivalent series circuit (fig. 13–5, E). In figure 13–5, F, the hypotenuse, Z_t, of the impedance triangle is 70.7 ohms and the phase angle, θ_t, between total current and line voltage is 45°. The equivalent series resistance, R_{eq}, is the base of the impedance triangle and has a magnitude of

$$R_{eq}=Z_t \cos \theta_t=70.7(\cos 45°=0.707)=50 \text{ ohms.}$$

The equivalent series reactance, $X_{C_{eq}}$, is the altitude of the impedance triangle and has a magnitude of $Z_t \sin \theta_t=70.7$ $(\sin 45° =0.707)=50$ ohms. The altitude is extended downward, in contrast to the upward direction of the altitude of the current triangle in figure 13–5, C, to maintain the sense of current lead for counterclockwise vector rotation. The line voltage is the horizontal reference vector, *OE* (fig. 13–5, C), and the line current is the horizontal reference vector, *OI* (fig. 13–5, F).

The capacitance in microfarads of the equivalent series impedance is

$$C_{eq}=\frac{10^6}{2\pi f X_{C_{eq}}}=\frac{10^6}{6.28\times 100\times 50}=31.8 \text{ microfarads.}$$

Thus, in this example, the 100-ohm resistor in branch 1 shunts the 15.9-microfarad capacitor in branch 2 and the source "sees" an equivalent resistor of 50 ohms in series with an equivalent capacitor of 31.8 microfarads (fig. 13–5, E).

Equivalent Circuit of a Low-Loss Capacitor

The action of a capacitor in an a-c circuit was described as being like the flow of water in a cylinder having a flexible diaphragm. The diaphragm corresponds to the dielectric in a capacitor and the mechanical stresses in the diaphragm correspond to the electric stress in the dielectric.

Most of the heating produced in a capacitor is due to the loss in the dielectric. The movement of the electrons in the atoms of

a solid dielectric is pictured in figure 13–6. The capacitor is connected to a source of a-c voltage of sine waveform and the conditions are indicated for three successive instants in the cycle of applied a-c voltage.

In figure 13–6, A, the voltage across the capacitor plates is positive maximum and the capacitor is charged. The atoms in the dielectric are subjected to electric stress that causes their orbital electrons to move in paths that are changed from cir-

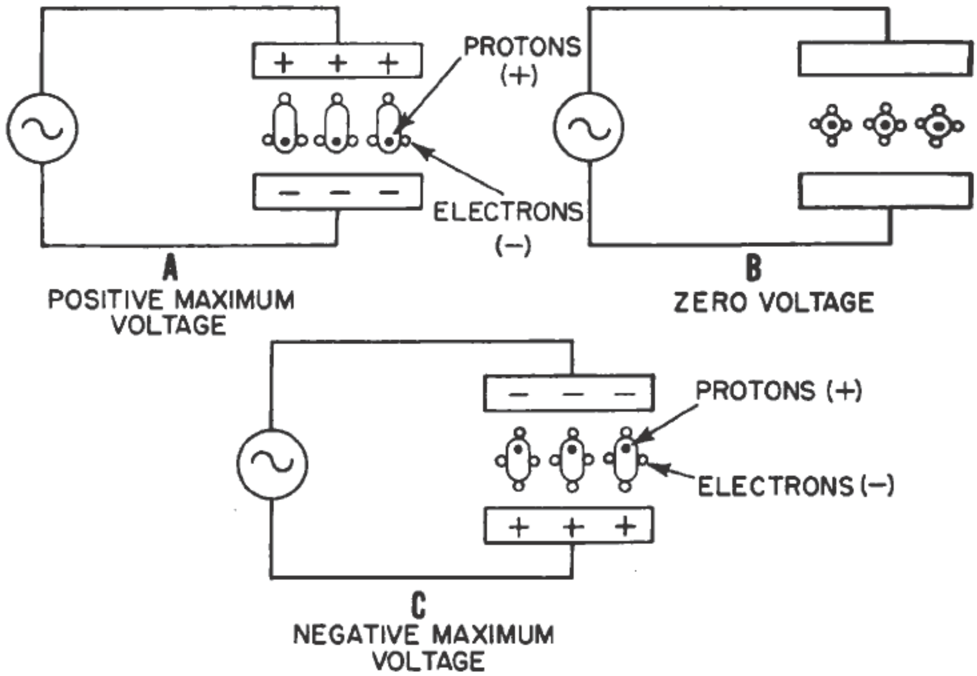


Figure 13–6.—Capacitor dielectric losses.

cular to elliptical patterns. The elliptical pattern forms as a result of the attractive force of the negatively charged plate on the positive nucleus and the simultaneous repulsion of the positively charged plate on the nucleus. At the same time the positive plate attracts the orbital electrons the negative plate repels them.

In figure 13–6, B, the charge on the plates is zero and the electric stress is removed from the dielectric. In this case the orbital electrons travel in circular paths about the nucleus with no external forces applied to them.

In figure 13–6, C, the charges on the plates are reversed and the orbital electrons are again caused to move in elliptical paths.

In this case the protons (entire nucleus) are moved toward the upper plate, the forces being opposite to those developed in figure 13–6, A.

The rapid reversals of applied voltage accompanied by the change of the orbital electron paths from circular to elliptical and back to circular patterns cause heat to be developed in the dielectric. The change in pattern of the electron orbits constitutes a dielectric displacement current. This current is considered to be made up of two components, one leading the voltage by 90°, the other an energy component in phase with the applied voltage. Factors that determine the dielectric loss are the applied voltage, the dielectric constant, the capacitor power factor, and the frequency of the applied voltage. An air-dielectric capacitor has no appreciable loss, and the power factor is 0. A mica capacitor, having a dielectric constant of 7 and a power factor of 0.0001, has relatively low loss and low dielectric heating at high voltages and high frequencies.

The product of the dielectric constant and the power factor is called the LOSS FACTOR. The loss factor is low for good dielectrics that operate without much dielectric heating. The loss factor is the best indication of the ability of a material to withstand high voltages at high frequencies. The loss factor for the previously mentioned mica capacitor is $7 \times 0.0001 = 0.0007$. The loss factor for air dielectric is zero because the power factor is zero.

The equivalent circuits of a low power-factor capacitor are represented in figure 13–7. The circuits are derived by assuming a capacitor of the same capacitance as that of the original capacitor, but having no losses (zero power factor), to be connected (1) in SERIES with a resistor that develops the same true power loss as in the original capacitor; and (2) a capacitor of the same capacitance as that of the original capacitor, but having no losses (zero power factor), to be connected in SHUNT with a resistor that develops the same true power loss as in the original capacitor. The capacitance of the capacitor in this example is 66.4 microfarads. The power factor of the capacitor is $\cos \theta = 0.05$. The effective voltage is 200 volts and the capacitor current is 5 amperes.

The equivalent series circuit is shown in figure 13–7, B. At

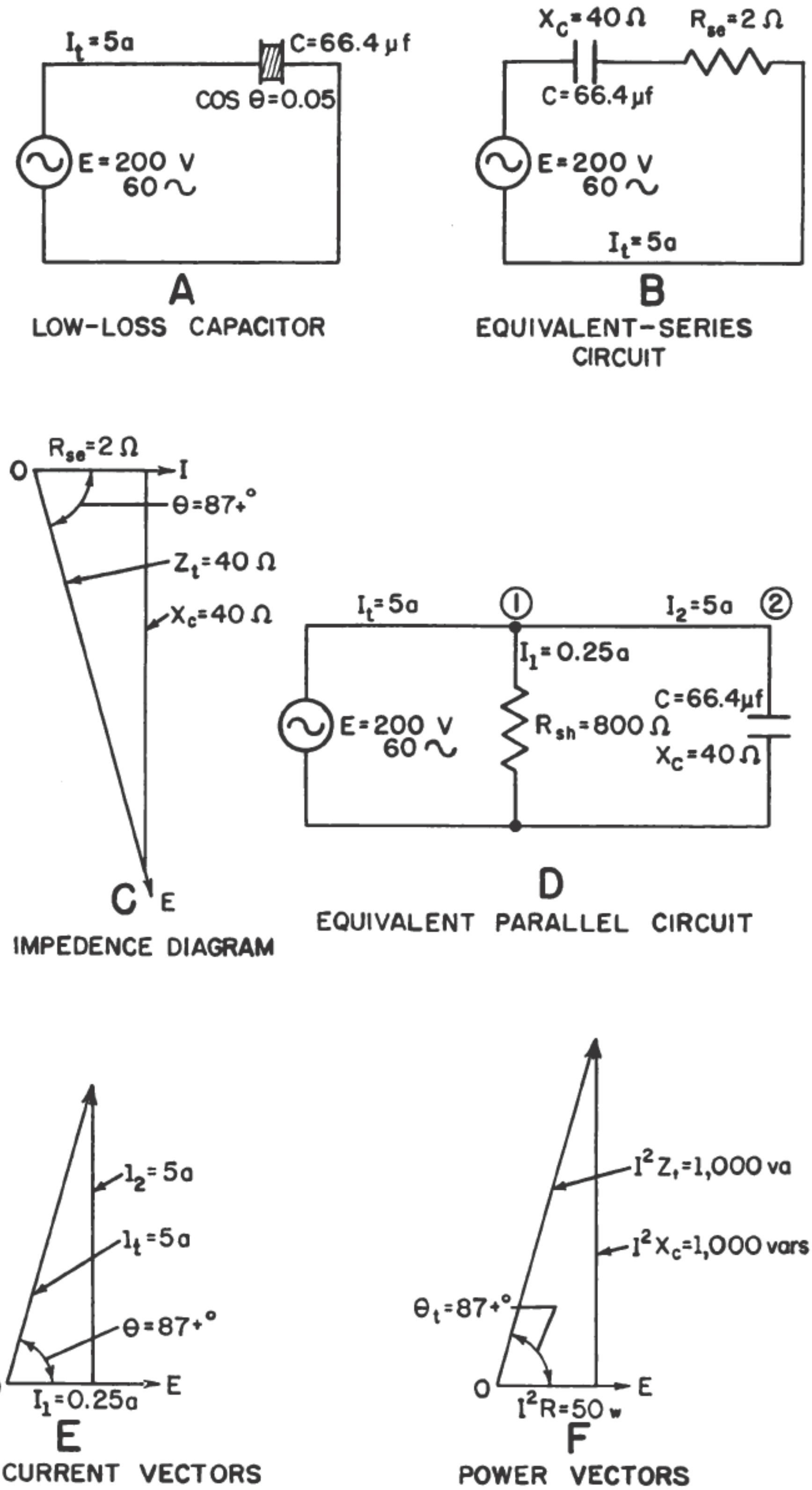


Figure 13–7.—Equivalent circuits of a low-loss capacitor.

the operating frequency of 60 cycles per second the capacitive reactance of the capacitor is

$$X_C=\frac{1}{2\pi fC}=\frac{1}{6.28\times 60\times 66.4\times 10^{-6}}=40 \text{ ohms approximately.}$$

The impedance of the series circuit is

$$Z_t=\frac{E}{I_t}=\frac{200}{5}=40 \text{ ohms.}$$

The equivalent series resistance is

$$R_{se}=Z_t \cos \theta_t=40\times 0.05=2 \text{ ohms,}$$

(fig. 13–7, B). The base of the impedance triangle is 2 ohms of resistance (fig. 13–7, C), the altitude of the impedance triangle is 40 ohms of capacitive reactance, and the hypotenuse is approximately 40 ohms of impedance.

The power factor of this circuit is $\cos \theta_t=\frac{R_{se}}{Z_t}=\frac{2}{40}=0.05$, or 5 percent. In this example, as in the low-loss inductor of figure 13–2, Z_t is approximately equal to X_C, and $\cos \theta_t=\frac{R_{se}}{X_C}$. Since $\tan \theta_t=\frac{X_C}{R_{se}}=\frac{40}{2}=20$, it follows that $\cos \theta_t=\frac{1}{\tan \theta}$, and the power factor of the capacitor is $\frac{2}{40}$, or 0.05.

The angle whose cosine is 0.05 may be closely approximated by the previously described relation between the power factor and the complementary angle in radians. In this example, the complementary angle by which the current fails to be 90° out of phase with the voltage is 0.05 radian, or $0.05\times 57.3=2.865°$, and $\theta=90°-2.865°=87.135°$. This angle is indicated in all of the vector diagrams of figure 13–7 as 87+°.

The equivalent parallel circuit for the 66.4-microfarad capacitor discussed in this example is shown in figure 13–7, D. The equivalent series resistance and the equivalent shunt resistance are related in the equivalent capacitor circuits in the same way that R_{se} and R_{sh} are related in the low-loss inductor circuits of

figure 13–2. This relation was stated in equation 13–5. Thus, in the capacitor circuits,

$$R_{sh}=\frac{(X_C)^2}{R_{se}}=\frac{(40)^2}{2}=800 \text{ ohms.}$$

The equivalent shunt resistance of 800 ohms is connected in parallel with a 66.4-microfarad capacitor of zero losses. With rated voltage and frequency applied, the input current to the parallel circuit will have the same magnitude and phase with respect to the applied voltage as in the original capacitor.

The energy current in the resistive branch is $I_1=\frac{E}{R_{sh}}=\frac{200}{800}$ $=0.25$ ampere and is the base of the current triangle (fig. 13–7, E). The nonenergy current in the capacitive branch is $I_2=\frac{E}{X_C}=$ $\frac{200}{40}=5$ amperes and is the altitude of the current triangle. The total current in the parallel circuit is equal to $\sqrt{(0.25)^2+(5)^2}=5$ amperes (approx.) and is the hypotenuse of the current triangle. This current leads the voltage by an angle of 87.135°.

The power relations are shown in figure 13–7, F. The true power of the circuit may be found in a number of ways. Three methods are listed as follows:

1. $P=EI_t \cos \theta_t=200\times5\times0.05=50$ watts (fig. 13–7, A).

2. $P=I_t^2R_{se}=5^2\times2=50$ watts (fig. 13–7, B).

3. $P=I_1^2R_{sh}=(0.25)^2\times800=50$ watts (fig. 13–7, D).

The true power of 50 watts is the base of the power triangle (fig. 13–7, F).

The reactive power may be calculated in a number of ways. Three methods are indicated as follows:

1. Vars$=E_tI_t \sin \theta_t=200\times5\times(\sin 87+°=1$ approx.)
$=1{,}000$ vars (fig. 13–7, A).

2. Vars$=I_t^2X_C=5^2\times40=1{,}000$ vars (fig. 13–7, B).

3. Vars$=I_2^2X_C=5^2\times40=1{,}000$ vars (fig. 13–7, D).

The reactive power of 1,000 vars is the altitude of the power triangle (fig. 13–7, F).

The apparent power of the capacitor may be calculated as:

1. Apparent power$=EI_t=200\times 5=1{,}000$ va (fig. 13–7, A).

2. Apparent power$=I_t^2Z_t=5^2\times 40=1{,}000$ va (fig. 13–7, B).

The apparent power of 1,000 volt-amperes is the hypotenuse of the power triangle (fig. 13–7, F).

Combining Currents at Acute Angles

A 2-branch parallel circuit containing a 75-ohm resistor in branch ① and a series combination of a 79.6-microfarad capacitor and a 30-ohm resistor in branch ② is shown in figure 13–8, A. The capacitor losses are assumed to be included with those of the 30-ohm resistor so that the power factor of that portion of branch ② represented by the capacitor is zero.

The capacitive reactance at the operating frequency of 50 cycles per second is

$$X_C=\frac{1}{2\pi fC}=\frac{1}{6.28\times 50\times 79.6\times 10^{-6}}=40 \text{ ohms.}$$

The impedance, Z_2, of branch ② is the combined opposition of 30 ohms of resistance in series with 40 ohms of capacitive reactance and is the hypotenuse of the right triangle of which 30 ohms is the base and 40 ohms is the altitude (not shown in the figure). Thus,

$$Z_2=\sqrt{30^2+40^2}=50 \text{ ohms.}$$

The power factor of branch ② is $\cos\theta_2=\frac{R_2}{Z_2}=\frac{30}{50}=0.60$ and the phase angle is 53.1°.

The current vectors for both branches are shown in figure 13–8, B. The current in branch ① is $I_1=\frac{E}{Z_1}=\frac{150}{75}=2$ amperes, and is a portion of the base of the equivalent right triangle in line with the horizontal reference voltage, OE. This current is in phase with the applied voltage because there is no reactance present in branch ①.

The current in branch ② is

$$I_2=\frac{E}{Z_2}=\frac{150}{50}=3 \text{ amperes,}$$

and is the hypotenuse of the right triangle, the base of which is $I_2 \cos \theta_2=3(\cos 53.1=0.6)=1.8$ amperes (energy current) and the altitude of which is $I_2 \sin \theta_2=3(\sin 53.1=0.8)=2.4$ amperes (nonenergy current). The total current is the vector sum of I_1 and I_2 and is the hypotenuse of the equivalent right triangle, the base of which is the arithmetic sum of the current in branch ① and the energy component of current in branch ②, or $2+1.8=3.8$

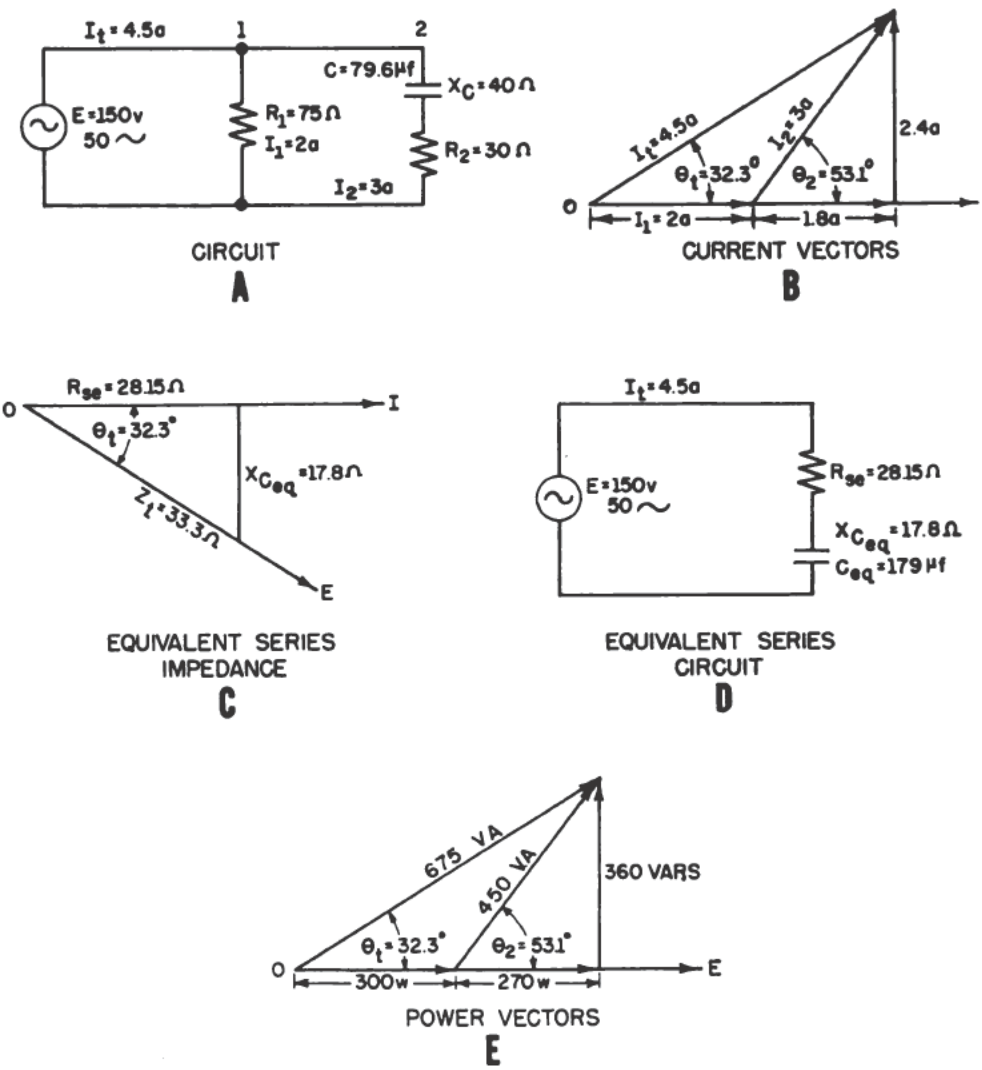


Figure 13–8.—Two-branch *R-C* circuit with currents out of phase by an angle of 53.1°.

amperes. The altitude of the equivalent right triangle is equal to the nonenergy current in branch ②, or 2.4 amperes. The total current in the parallel circuit is

$$I_t=\sqrt{(3.8)^2+(2.4)^2}=4.5 \text{ amperes.}$$

The phase angle, θ_t, between the total circuit current and the applied voltage is the angle whose cosine is $\frac{3.8}{4.5}=0.845$. This angle is approximately 32.3°. Thus, the total current leads the applied voltage by an angle of 32.3°.

The total impedance of the parallel circuit is

$$Z_t=\frac{E}{I_t}=\frac{150}{4.5}=33.3 \text{ ohms}$$

and is the hypotenuse of the impedance triangle (fig. 13–8, C). The base of the triangle is

$$R_{se}=Z_t \cos \theta_t=33.3\ (\cos 32.3°=0.845)=28.15 \text{ ohms}$$

of resistance and is in line with the horizontal current reference vector for the equivalent series circuit (fig. 13–8, D). The altitude of the triangle is

$$X_{C_{eq}}=Z_t \sin \theta_t=33.3\ (\sin 32.3°=0.534)=17.8 \text{ ohms}$$

of capacitive reactance. The capacitance of the equivalent series circuit at the operating frequency of 50 cycles per second is

$$C_{eq}=\frac{10^6}{2\pi f X_C}=\frac{10^6}{6.28\times 50\times 17.8}=179 \text{ microfarads.}$$

The power relations are indicated in the triangles of figure 13–8, E. The apparent power of the total circuit is $EI_t=150\times 4.5=675$ volt-amperes and is the hypotenuse of the equivalent right triangle representing the total parallel circuit.

The true power in branch ① is

$$I_1^2R_1=2^2\times 75=300 \text{ watts.}$$

The true power in branch ② is

$$I_2^2R_2=3^2\times 30=270 \text{ watts.}$$

The total true power is

$$300+270=570 \text{ watts},$$

and is the base of the equivalent right triangle for the entire circuit. The reactive power in the total circuit—that is, the reactive power of branch ②—is

$$I_2^2X_C=3^2\times40=360 \text{ vars},$$

and is the altitude of the equivalent triangle.

The apparent power in branch ① is equal to the true power because the power factor is unity. The apparent power in branch ② is $EI_2=150\times3=450$ volt-amperes and is the hypotenuse of the power triangle for this branch. The true power of 270 watts in branch ② is the base of this triangle. The reactive power of 360 vars in branch ② is the altitude of the triangle. The power factor for branch ② is $\cos\theta_2=\frac{\text{true power}}{\text{apparent power}}=\frac{270}{450}=0.60$, and the phase angle for branch ② is $\theta_2=53.1°$.

PARELLEL CIRCUITS CONTAINING *L*, *R*, AND *C*

Inductance in an a-c circuit causes the current to lag behind the applied voltage. Transformers and induction motors are essentially inductive in nature and the power factor, especially on light loads (as contrasted with full loads), is relatively low.

Most circuits that suppy electric power from the source to the consumer transmit the power at relatively high voltage and low current in order to keep the I^2R loss in the transmission lines satisfactorily low. Transformers at the point of utilization reduce the voltage to the proper value to operate the equipment.

Capacitance in a-c circuits causes the current to lead the applied voltage and when placed in parallel with inductive components can produce a neutralizing effect so that lagging currents can be brought into phase with the applied voltage or may be made to lead the applied voltage, depending on the relative magnitude of the capacitance and inductance in parallel.

The true power of a circuit is $P=EI\cos\theta$; and for any given amount of power to be transmitted, the current, I, varies inversely with the power factor, $\cos\theta$. Thus, the addition of capacitance in parallel with inductance will, under the proper condi-

tions, improve the power factor (make nearer unity power factor) of the circuit and make possible the transmission of electric power with reduced line loss and improved voltage regulation.

Current Vectors

In the example of figure 13–9, the parallel circuit contains three branches. Branch ① contains a 30-ohm resistor. Branch ② consists of an inductor of 0.0612 henry and a resistor of 6.6 ohms. Branch ③ contains a capacitance of 44.3 microfarads having negligible losses. The parallel circuit is shown in figure 13–9, A. The current vectors are shown in figure 13–9, B.

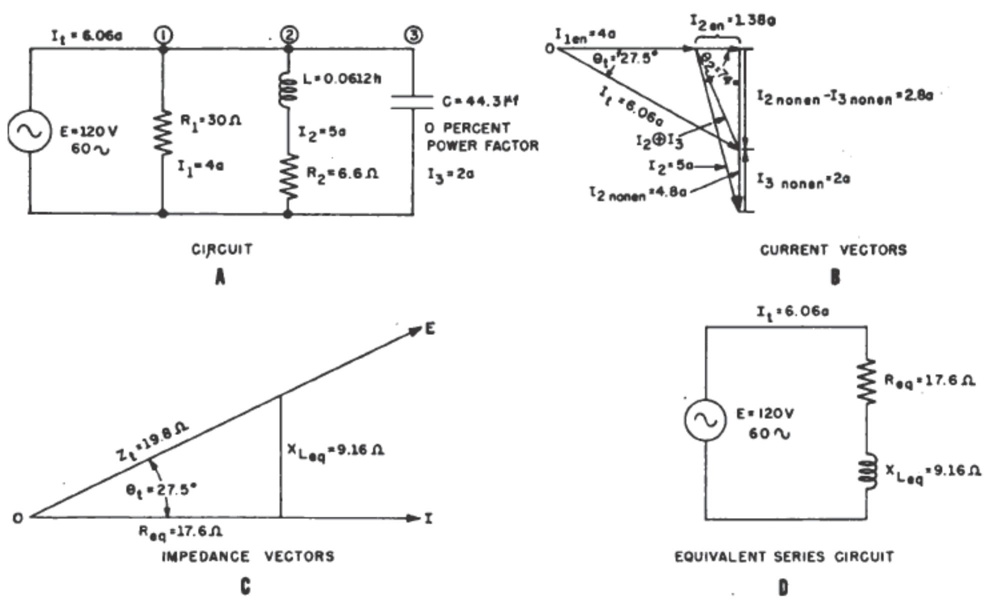


Figure 13–9.—Parallel circuit containing *L*, *R*, and C.

The current in branch ① is

$$I_{1\,\mathrm{en}}=\frac{E}{R_1}=\frac{120}{30}=4 \text{ amperes.}$$

The corresponding 4-ampere vector in figure 13–9, B, is drawn in the same horizontal reference line as the voltage vector because the phase angle between the current and voltage in this branch is zero.

The inductive reactance of branch ② at the operating frequency of 60 cycles per second is

$$X_L = 2\pi fL = 6.28 \times 60 \times 0.0612 = 23.1 \text{ ohms.}$$

The impedance of branch ② is

$$Z_2 = \sqrt{R_2^2 + X_L^2} = \sqrt{(6.6)^2 + (23.1)^2} = 24 \text{ ohms.}$$

The current in branch ②, at the operating voltage of 120 volts, is $I_2 = \frac{E}{Z_2} = \frac{120}{24} = 5$ amperes and is the hypotenuse of the current triangle for branch ②. The angle by which the current in branch ② lags the applied voltage is the angle whose cosine is

$$\frac{R_2}{Z_2} = \frac{6.6}{24} = 0.275.$$

This angle is $\theta_2 = 74°$. The base of the current triangle for branch ② is

$$I_{2\,\text{en}} = I_2 \cos 74° = 5(\cos 74° = 0.275) = 1.38 \text{ amperes.}$$

The altitude of the current triangle for branch ② is

$$I_{2\,\text{nonen}} = I_2 \sin 74° = 5(\sin 74° = 0.962) = 4.8 \text{ (approx.) amperes}$$

and extends below the horizontal voltage reference vector to indicate current lag.

The capacitive reactance in branch ③ is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 60 \times 44.3 \times 10^{-6}} = 60 \text{ ohms,}$$

and with negligible losses, the impedance, Z_3, is also 60 ohms. The current in branch ③ is

$$I_{3\,\text{nonen}} = \frac{E}{Z_3} = \frac{120}{60} = 2 \text{ amperes.}$$

The current in the capacitor branch leads the applied voltage by an angle of 90°. The nonenergy component of the current in the inductive branch lags the applied voltage by an angle of 90°. Therefore, these two currents are 180° out of phase with each other, and the capacity current vector, I_3, extends upward from the lower extremity of the vector representing the nonenergy component of current in branch ②.

The total current, I_t, in the parallel circuit is the vector sum of the currents in the three branches and is the hypotenuse of the equivalent current triangle, the base of which is the arithmetic sum of the energy components of the currents and the altitude of which is the algebraic sum of the nonenergy components of the currents. Thus, the total current is

$$I_t=\sqrt{(I_{1en}+I_{2en})^2+(I_{2nonen}-I_{3nonen})^2}$$
$$=\sqrt{(4+1.38)^2+(4.8-2)^2}$$
$$=6.06 \text{ amperes.}$$

The phase angle, θ_t, between the total current and the applied voltage is the angle whose cosine is the ratio of the sum of the energy components of the currents in all the branches to the total current. Thus,

$$\cos\theta_t=\frac{4+1.38}{6.06}=0.888.$$
$$\theta_t=27.5°,$$

and the total circuit current lags the applied voltage by an angle of 27.5°.

Equivalent Series Impedance

The total impedance of the parallel circuit (fig. 13–9, A) is equal to the total circuit voltage divided by the total circuit current, or

$$Z_t=\frac{E}{I_t}=\frac{120}{6.06}=19.8 \text{ ohms,}$$

and is the hypotenuse of the impedance triangle (fig. 13–9, C). This triangle represents the impedance relations existing in the equivalent series circuit (fig. 13–9, D). The phase angle, θ_t, of the equivalent series circuit has the same magnitude as the phase angle between the total circuit current and the applied voltage across the parallel circuit. The impedance of the equivalent series circuit has the same magnitude as the total impedance of the parallel circuit. The base of the triangle is

$$R_{eq}=Z_t\cos\theta_t=19.8\ (\cos 27.5°=0.889)=17.6 \text{ ohms,}$$

and represents the resistive component of the equivalent series circuit. The altitude of the triangle is

$$X_{L_{eq}}=Z_t \sin \theta_t=19.8 \ (\sin 27.5°=0.462)=9.16 \text{ ohms},$$

and represents the inductive reactance component of the equivalent series circuit.

Power and Power Factor

The true power of the parallel circuit (fig. 13–9, A) is the arithmetic sum of the power absorbed in each branch. The true power in branch ① is

$$P_1=I_1^2R_1=4^2\times30=480 \text{ watts.}$$

The true power in branch ② is

$$P_2=I_2^2R^2=5^2\times6.6=165 \text{ watts.}$$

The true power in branch ③ is negligible since the power factor of the capacitor is assumed to be zero. The total true power is

$$480+165=645 \text{ watts.}$$

The total apparent power of the parallel circuit is the product of the total circuit current and the applied voltage. Thus, the apparent power is

$$EI_t=120\times6.06=727.2 \text{ volt-amperes.}$$

The parallel circuit power factor is the ratio of the total true power to the total apparent power. Thus, the circuit power factor is

$$\cos \theta_t=\frac{\text{true power}}{\text{apparent power}}=\frac{645}{727.2}=0.888.$$

Power-Factor Correction

Power-factor correction in parallel circuits is accomplished by placing a capacitor of the proper size in parallel with the circuit at the point where the power-factor correction is to be effected. The leading current of the capacitor branch supplies the lagging component of current in the inductive portion of the parallel circuit and reduces the line current accordingly. As mentioned

before, this action improves the efficiency of transmission by reducing the line current and I^2R losses.

In the example of figure 13–10, the load is rated at 10 amperes, 1,000 volts, and a power factor of 50 percent lagging (the current lags the voltage). The true power absorbed by the load (fig. 13–10, A) is

$$P = EI \cos \theta = 1{,}000 \times 10 \times 0.5 = 5{,}000 \text{ watts.}$$

The load is supplied through a line having a resistance of 20 ohms and negligible reactance. The power loss in the line is

$$I^2R = 10^2 \times 20 = 2{,}000 \text{ watts,}$$

and the efficiency of transmission is

$$\frac{\text{output}}{\text{output} + \text{losses}} = \frac{5{,}000}{5{,}000 + 2{,}000} = 71.4 \text{ percent.}$$

If a 1,000-volt capacitor of negligible losses supplying a leading current of 8.66 amperes is placed in parallel with the inductive load (fig. 13–10, B), the total line current will be reduced from 10 amperes to 5 amperes.

The current vectors are shown in figure 13–10, C. The nonenergy component of the current in the inductive branch is $I_{nonen} = I \sin \theta = 10(\sin 60° = 0.866) = 8.66$ amperes (lagging) and is 180° out of phase with the capacitor current of 8.66 amperes (leading). These currents circulate between the capacitor and the inductive load and do not enter the line. The vector sum of the capacitor current and the total inductive load current is equal to the line current ($I_t = 5$ amperes), and in the vector diagram is represented as the diagonal of the parallelogram of which the 2 branch currents are the sides. The line current vector, I_t, is in the same horizontal reference as the load voltage vector, indicating that the line current is in phase with the voltage applied to the parallel combination of the inductive load and the capacitor.

The reduction in line current from 10 to 5 amperes reduces the line loss from 2,000 watts to $5^2 \times 20 = 500$ watts and increases the efficiency of transmission from 71.4 percent to $\frac{5{,}000}{5{,}000 + 500} = 91$ percent. Thus, the improvement in efficiency of operation of the line and load is demonstrated. The condition represented

involves an interchange of energy between the inductive and capacitive branches known as parallel resonance and is described in the training manual, *Basic Electronics,* NavPers 10087, chapter 1, on the subject of resonance in *L-R-C* circuits.

VOLTAGE REDUCTION WITH RESISTANCE.—The example of figure 13–11 illustrates the advantages and disadvantages of controlling the voltage on a load by means of resistance and induct-

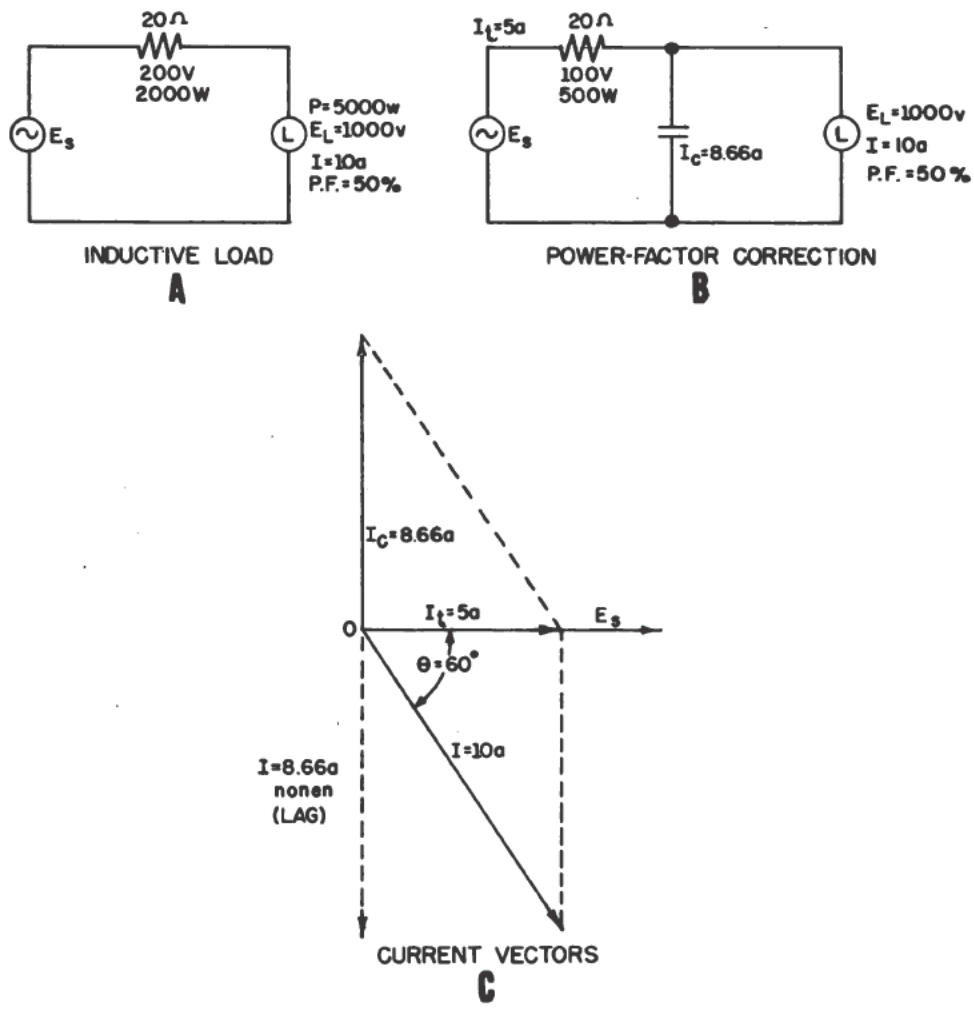


Figure 13–10.—Power-factor correction.

ance and also the effect of power-factor correction on the circuit efficiency.

In figure 13–11, A, the voltage applied to the load is reduced from 100 volts to 50 volts through the action of the dropping resistor. The circuit current is 10 amperes, and the voltage across

the dropping resistor is 50 volts. With this arrangement, the input power to the circuit is divided equally between the resistor and the load, and the circuit efficiency $\left(\frac{\text{output}}{\text{output}+\text{losses}}\times 100\right)$ is $\frac{500}{500+500}\times 100=50$ percent. This arrangement represents an inefficient method of voltage reduction.

VOLTAGE REDUCTION WITH INDUCTANCE.—In figure 13–11, B, the voltage applied to the load is reduced to 50 volts through the action of the series inductor. Neglecting the relatively small losses in the inductor, the circuit efficiency remains high compared to the efficiency of the previous circuit, but the series inductor lowers the power factor from unity (100 percent) to 50 percent. The circuit current is still 10 amperes, and the accompanying line loss between the source and the inductor unnecessarily high.

The voltage vectors for the *L-R* circuit are shown in figure 13–11, C. The load voltage of 50 volts is the base of the right triangle and is in phase with the load current. The voltage drop across the inductor is 86.6 volts and is the altitude of the voltage triangle. The source voltage, $E_s=100$ volts, is the vector sum of the load voltage and the inductor voltage and is the hypotenuse of the voltage triangle. The circuit power factor is

$$\cos\theta=\frac{E_{\text{load}}}{E_{\text{source}}}=\frac{50}{100}=0.5,$$

and

$$\theta=60°.$$

In figure 13–11, D, the circuit power factor is improved to unity by the addition of a capacitor of negligible losses which supplies a leading current of 8.66 amperes. This current supplies the nonenergy component of the current in the branch containing the inductor and load, and reduces the line current from 10 amperes to 5 amperes.

The current vectors are shown in figure 13–11, E. The capacitor current of 8.66 amperes is represented by the vector extending above the horizontal voltage reference vector to indicate a lead of 90°. The inductor branch (load) current extends below the horizontal at an angle of 60° to indicate lag. The vector sum of these currents is the diagonal of the parallelogram of which the branch currents are the sides, and is a horizontal vector of 5

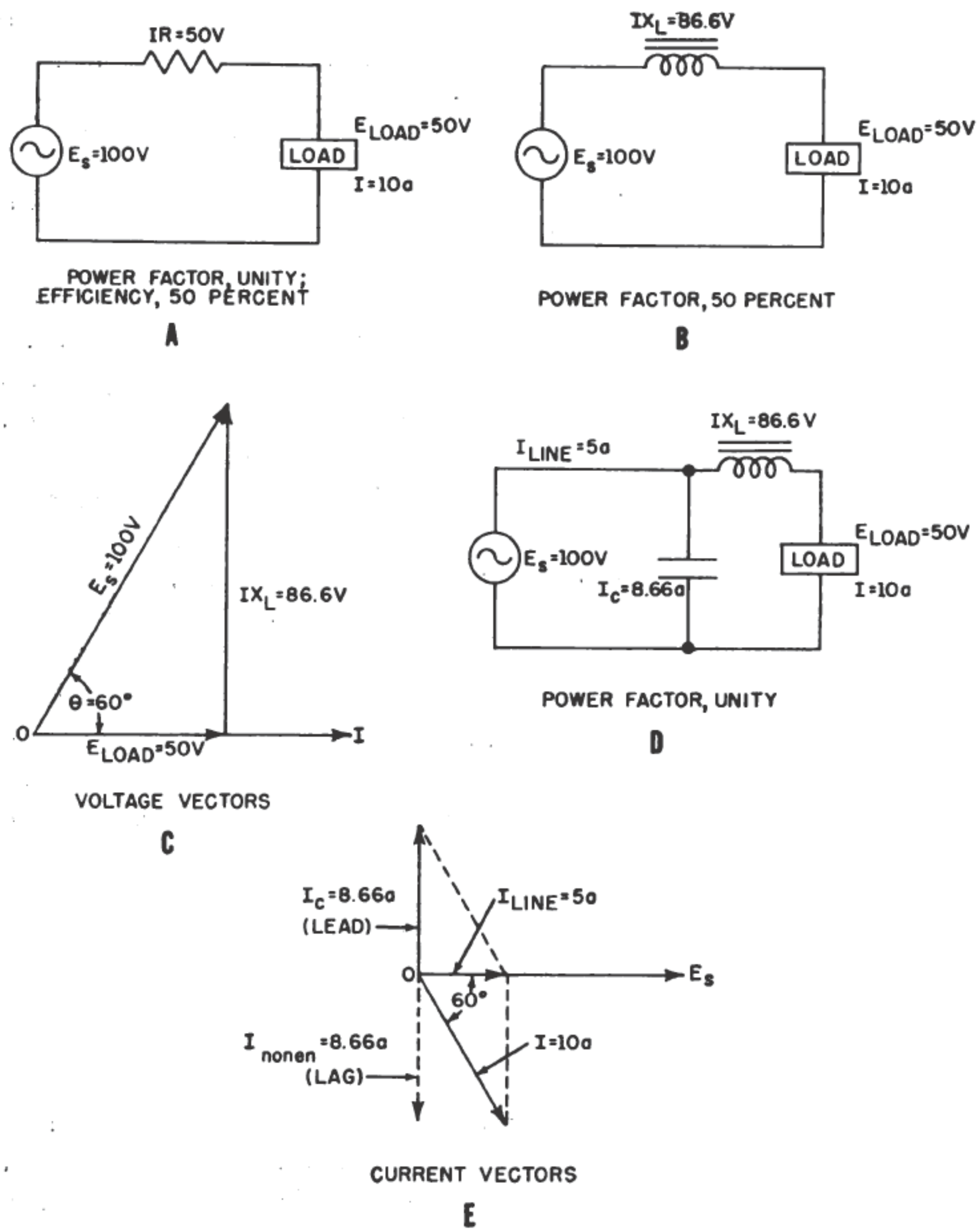


Figure 13–11.—Voltage reduction and power-factor correction.

amperes in phase with the source voltage. Thus, the power factor of the parallel circuit is unity and the line current is reduced from 10 amperes to 5 amperes.

ADVANTAGES OF INDUCTIVE ARRANGEMENT WITH POWER-FACTOR CORRECTION.—In the example under consideration, voltage reduction with a series inductor and a shunt capacitor provides a means of supplying a 50-volt 10-ampere load from a 100-volt

source with only the small losses associated with the reactive components. The line current is kept to the minimum value required to supply the 500-watt load, and the circuit effieciency is high. Inductive control alone provides a means of reducing the load voltage by changing the phase of the applied voltage with respect to the load voltage. The addition of the capacitor reduces the current in the line without altering the load current, and thus reduces the line losses; at the same time the circuit power factor is increased to unity. In most circuits it is not economical to improve the power factor to unity, but to improve it, for example, from 50 percent lagging to 85 percent lagging. The reduction in line losses is most pronounced in this range and any further reduction in losses may not justify the added expense of the capacitance required to further improve the power factor in the range from 85 percent lagging to unity.

EFFECTIVE RESISTANCE—NONUSEFUL ENERGY LOSSES IN A-C CIRCUITS

Energy Concept of Resistance

The energy stored in the magnetic field of a pure inductance, as the result of a rise in current through the coil, is returned to the circuit when the current decreases and the field collapses. Similarly, the energy stored in the electric field of a pure capacitance, as a result of the rise in voltage across the capacitor, is returned to the circuit when the voltage falls and the field collapses. Hence, in a pure inductance and a pure capacitance there is no loss or expenditure of energy.

When current flows through a conductor having appreciable resistance the flow is accompanied by the generation of heat. Work is done in moving the electrons through the conductor resistance. The energy converted into heat is not returned to the circuit when the current falls, but is expended rather than stored. Thus, energy is stored periodically in inductance and capacitance but always expended in resistance.

Because resistance is the only circuit quality capable of expending electrical energy, all energy expended in any circuit can be identified in electrical terms, one factor of which is effective resistance. The effective resistance, R_{ac}, of any circuit may be defined as the ratio of the true power absorbed by the circuit to

the square of the effective current flowing in the circuit, or $R_{ac} = \frac{P}{I^2}$. When the power is expressed in watts and the current is expressed in amperes, the effective resistance will be in ohms. The d-c circuit resistance as measured by an ohmmeter or d-c bridge may be considerably lower than the effective a-c resistance as calculated from the readings on a wattmeter and an ammeter.

For example, assume that a motor draws 1 kilowatt from a 110-volt source. The input current is 10 amperes. The effective resistance between the motor terminals is

$$R_{ac} = \frac{P}{I^2} = \frac{1{,}000}{(10)^2} = 10 \text{ ohms.}$$

The d-c resistance measured between the motor terminals, for example with an ohmmeter, is 0.5 ohm. Thus, in this example the effective a-c resistance is $\frac{10}{0.5}$, or 20 times the d-c resistance. Most of the energy taken from the line is converted into mechanical energy and is not returnable to the electric circuit; hence, it is represented electrically as being expended in an effective a-c resistance of 10 ohms.

The source of power for the motor is unaware of the manner in which the motor expends the electrical energy. To the source, the motor appears as an impedance, $Z = \frac{E}{I} = \frac{110}{10} = 11$ ohms, having a resistive component of 10 ohms. The nature of the various energy conversions taking place inside the motor is important only when the motor itself is being analyzed. From this point of view a motor, electric light, loudspeaker, electron tube, or any other electrical device can be pictured as an equivalent electric circuit containing the fundamental components of inductance, capacitance, and resistance. The energy expended in the circuit is always interpreted in terms of the effective resistance component.

The energy expended in any electrical device may be divided into two parts: (1) That which is converted into useful form; and (2) that which is not useful. No machine has been built that is capable of perfect conversion—that is, one in which there

are no losses. In the motor, for example, there are friction losses in the bearings and heat losses in the windings as a result of the current flow through the resistance which they possess.

The number of possible nonuseful losses in a-c circuits is much greater than in d-c circuits. These include: (1) ohmic-resistance loss, (2) skin-effect loss, (3) eddy-current loss, (4) dielectric loss, (5) magnetic-hysteresis loss, (6) corona loss, and (7) radiation loss.

Effective Resistance of Conductors

The effective (a-c) resistance of electrical conductors is frequently higher than their d-c resistance especially when they are embedded in iron slots, as in the case of motor and generator armatures; and when they are being used in high-frequency circuits, as in radio transmitters and receivers.

Direct current is distributed uniformly throughout the cross-sectional area of a homogeneous conductor. For example, if a conductor having a cross-sectional area of 1,000 circular mils is carrying one ampere of direct current then one-thousandth of an ampere (one milliampere) is flowing in each circular mil of cross-sectional area. However, when the current in the conductor varies in amplitude, this uniform distribution throughout the conductor cross section is no longer obtained. The accompanying magnetic field is strongest near the center of the conductor and weaker at the circumference. The varying field induces a voltage in the conductor that opposes the change in current. The voltage induced in that portion of the conductor near the center is greater than the voltage induced in the outer surface of the conductor. The total opposition to the current flow includes the effect of this induced emf and is greater near the center of the conductor than at the surface. Therefore, the current divides inversely with the opposition—more of the current flowing near the circumference, and less near the center of the conductor.

The over-all result of this action is a decrease in the available area of cross section to conduct the current and an increase in conductor resistance. The decrease in area and increase in resistance become pronounced at high frequencies, at high current densities, and at high magnetic flux densities. This action is called SKIN EFFECT. It represents the tendency of a-c conductors

to carry the circuit current on the surface, or skin, of the conductors rather than uniformly throughout their cross section. As a result of this tendency, many electrical conductors are made of hollow tubing in order to save the added weight and expense of the unused central portion of the solid conductor. The effective a-c resistance of an isolated circular conductor varies approximately as the product of the square root of the frenquency and the length of the conductor, and inversely as the conductor diameter.

Effective Resistance of Inductors

When a conductor is wound in the form of a coil, the current is concentrated on the inner sides of the turns and into an area much smaller than would be the case in an isolated straight conductor. This action results in a large increase in effective resistance. The area in which the current is concentrated decreases as the frequency increases, hence, effective resistance will increase with frequency. When two or more conductors carrying alternating current are so placed that the magnetic field of one reacts with the field of the other, the resultant field around each conductor is no longer uniform. The change in current distribution in a conductor due to the action of an alternating current in a nearby conductor is called PROXIMITY EFFECT.

The proximity effect decreases as the separation between conductors increases. Thus, to lower the effective resistance of radio-frequency inductance coils, it is common practice to space the turns a distance equal to the diameter of the conductor. This decreases the reaction between magnetic fields of adjacent turns and permits the current to distribute itself over a larger area in the cross section of each turn.

The inductance of a hollow-core coil operating at a frequency of 60 cycles per second is increased many fold when a laminated core of soft silicon steel is inserted in the coil. This increase is due to the high permeability of the transformer-iron laminations. Thus, the skin effect is also increased due to the increased field strength. In addition to the increased skin effect in the coil, the effective a-c resistance is further increased because of the magnetic hysteresis loss in the iron. Thus, if the coil is connected to a constant-potential a-c source, the current in the coil will decrease when the iron core is inserted because of a small increase

in effective resistance and a large increase in the coil reactance. If the laminated steel core is removed and a piece of steel shafting is inserted in the coil, the effective resistance is further increased due to the eddy-current losses and the larger hysteresis losses in the steel shaft. A wattmeter inserted in the coil circuit will indicate this increase in effective resistance by an increased deflection when the solid steel core is inserted in place of the laminated core.

Powdered iron cores are used in certain types of coils on frequencies as high as 100 megacycles in order to limit the effective resistance of the coil to a satisfactorily low value. The iron particles are separated from each other by an insulated coating and when compressed into cylindrical form and inserted in the coil the induced voltage in each iron particle is so small in relation to the resistance to the path for eddy currents that the accompanying heat loss is negligible. Eddy-current losses are reduced in generator and motor armature conductors of large size by laminating the conductors and insulating the adjacent laminations in a manner similar to that in which the iron of the armature core itself is laminated. Thus, the effective resistance of the armature conductors is reduced.

Effective Resistance of Capacitors

The equivalent circuits of a low-loss capacitor were described earlier in this chapter and the factors affecting the equivalent series resistance noted. The effective a-c resistance of a capacitor is equal to its equivalent series resistance and represents the factor which when multiplied by the square of the effective capacitor charging current will equal the power expended in heat in the capacitor circuit.

As mentioned previously, most of the heating is produced in solid dielectrics, and only a negligible amount is produced in the capacitor plates themselves. The dielectric heating is produced by dielectric displacement currents described in connection with figure 13–6. In most electrical circuits, dielectric heating is a nonuseful loss. However, in one commercial application, dielectric heating has been put to good use—that of facilitating the gluing together of stacks of laminated plywood. The plywood laminations are stacked between the plates of a capacitor and a moderately high-frequency voltage is applied across the plates.

The resulting dielectric displacement currents heat the stack from the inside and the glue is quickly set—much more rapidly than in processes involving the external application of steam heat.

Corona Loss

Corona loss occurs as the result of the emission of electrons from the surface of electrical conductors at high potentials. It is dependent upon the curvature of the conductor surface, with most emission occuring from sharp points and the least emission occurring from surfaces having a large radius of curvature. Corona loss is frequently accompanied by a visual blue glow and an audible hissing sound as the electrons leak off the conductor surface into the atmosphere. Corona loss increases with voltage increase and decreases with increase in atmospheric pressure. This loss is held to a satisfactorily low value by (1) the use of large-diameter conductors, (2) not excessively high voltages, (3) smooth polished surfaces, (4) avoiding sharp points, bends, or turns, and (5) in some devices, for example, high-voltage capacitors, by the use of a compressed gas to retard the electron emission.

Radiation Loss

Radiation loss is not appreciable at power line frequencies, but in the field of communications this loss may become excessive. Power is radiated from transmitting antennas in the form of electric and magnetic fields, and its magnitude varies as the square of the antenna input current and as the so-called radiation resistance of the antenna. The transmission line that connects the transmitter and the antenna may, under certain circumstances. develop a radiation loss. This loss is discussed in chapter 10 of *Basic Electronics,* NavPers 10087, in connection with various types of transmission lines.

THREE-PHASE CIRCUITS

Most large central-station electric generators are three-phase machines, and the transmission and distribution circuits to which they are connected are 3-phase systems. Three-phase machines and systems have certain inherent advantages in economy and operating characteristics over their single-phase counterpart. For example, 3-phase motors are smaller, lighter in weight, and more

efficient than single-phase motors of the same horsepower. Three-phase transmission lines require less weight of electrical conductors than single-phase lines of the same kilowatt rating. Similarly, 3-phase generators require less copper and iron than single-phase generators of the same kilowatt rating.

Generation of Voltages

A 3-phase circuit is similar to three single-phase circuits having their voltages displaced by an angle of 120°, or one-third of a cycle. As previously described, a sinusoidal voltage is generated in a coil when the coil rotates at uniform speed in a magnetic field of uniform flux distribution (fig. 12–4).

In the basic 3-phase generator (fig. 13–12), three coil groups are spaced 120° apart on the armature. The armature revolves at constant speed in a 2-pole field and as each coil cuts through the field a voltage of sine waveform is generated in the coil. The positive maximum voltages of the three coils occur 120° apart due to the 120° spacing of the coils around the armature. The voltage of coil *A* leads that of coil *B* by 120°, the voltage of coil *B* leads that of coil *C* by 120°, and the sequence is *A, B, C*.

In a practical 3-phase generator the three windings are symmetrically placed in slots in the stator iron laminations, and the rotor comprises the magnetic revolving field. The stator arma-

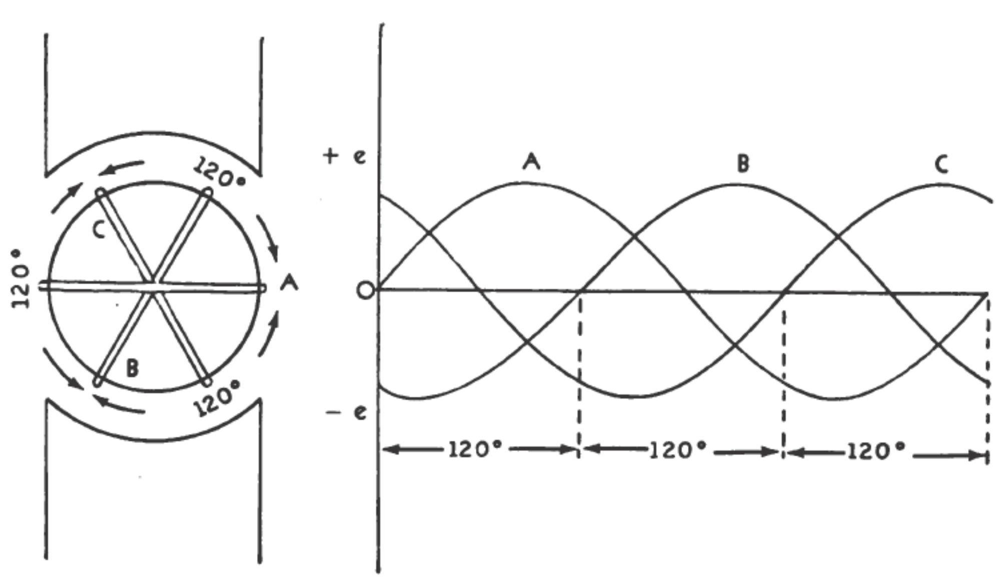


Figure 13–12.—Basic 3-phase generator and waveforms.

ture windings terminate in six free ends which are interconnected so that only three leads are brought out from the stator to the line. Three-phase alternators are either wye-connected or delta-connected.

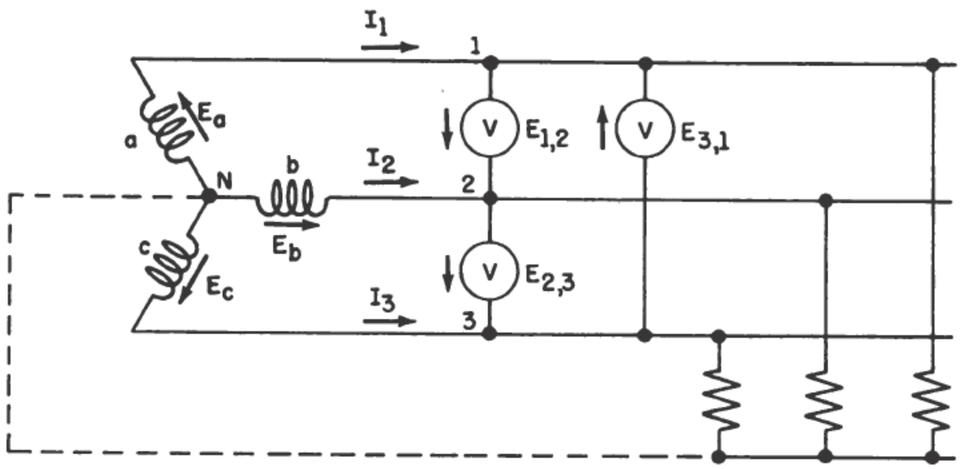
WYE CONNECTION.—In a wye-connected alternator the three start ends of each single-phase winding are connected together to a common neutral point and the opposite, or finish, ends are connected to the line terminals *A, B,* and *C*. These letters are always used to designate the three phases of a 3-phase system, or the three line wires to which the alternator phases connect. A 3-phase wye-connected alternator supplying three separate loads is shown in figure 13–13, A. When unbalanced loads are used, a neutral may be added as shown in the figure by the broken line between the common neutral point and the loads. The neutral wire serves as a common return circuit for all three phases and maintains a voltage balance across the loads. No current flows in the neutral wire when the loads are balanced. This system is a 3-phase 4-wire circuit and is used to distribute 3-phase power to shore-based installations. The 3-phase 4-wire system is not used aboard ship, but is widely used in industry.

The phase relations in a 3-wire 3-phase wye-connected system are shown in figure 13–13, B. In constructing vector diagrams of 3-phase circuits, a counterclockwise rotation is assumed in order to maintain the correct phase relation between line voltages and currents. Thus, the alternator is assumed to rotate in such a direction as to generate the three phase voltages in the order, E_a, E_b, E_c.

The voltage in phase *b,* or E_b, lags the voltage in phase *a,* or E_a, by 120°. Likewise, E_c lags E_b by 120°, and E_a lags E_c by 120°. In figure 13–13, A, the arrows E_a, E_b, and E_c represent the positive direction of generated voltage in the wye-connected alternator. The arrows I_1, I_2, and I_3 represent the positive direction of phase and line currents supplied to balanced unity-power-factor loads connected in wye. The three voltmeters connected between lines 1–2, 2–3, and 3–1, respectively, indicate effective values of line voltage. The line voltage is greater than the voltage of a phase in the wye-connected circuit because there are two phases connected in series between each pair of line wires, and their voltages combine. Line voltage is not twice the

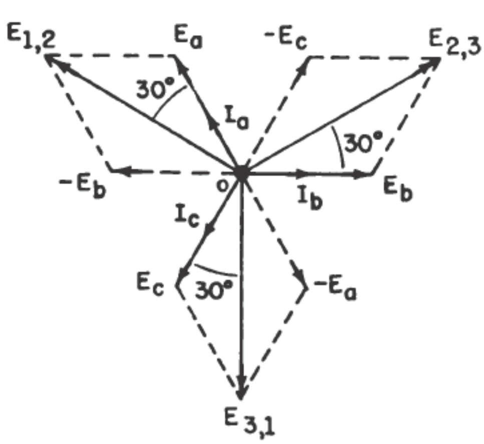
value of phase voltage, however, because the phase voltages are not in phase with each other.

The relation between phase and line voltages is shown in the vector diagram. Effective values of phase voltage are indicated by vectors, E_a, E_b, and E_c. Effective values of line and phase current are indicated by vectors I_a, I_b, and I_c. Because there is



CIRCUIT

A



$E_{1,2} = E_a \ominus E_b$
$E_{2,3} = E_b \ominus E_c$
$E_{3,1} = E_c \ominus E_a$
$I_a = I_1$
$I_b = I_2$
$I_c = I_3$
$E_{1,2} = \sqrt{3}\ E_a$
$E_{2,3} = \sqrt{3}\ E_b$
$E_{3,1} = \sqrt{3}\ E_c$

VECTORS

B

Figure 13–13.—Three-phase wye-connected system.

only one path for current between any given phase and the line wire to which it is connected, the phase current is equal to the line current. The respective phase currents have equal values because the load is assumed to be balanced. For the same reason,

the respective line currents have equal values. When the load has unity power factor, the phase currents are in phase with their respective phase voltages.

In combining a-c voltages it is necessary to know the DIRECTION in which the positive maximum values of the voltages act in the circuit as well as the MAGNITUDES of the voltages. For example,

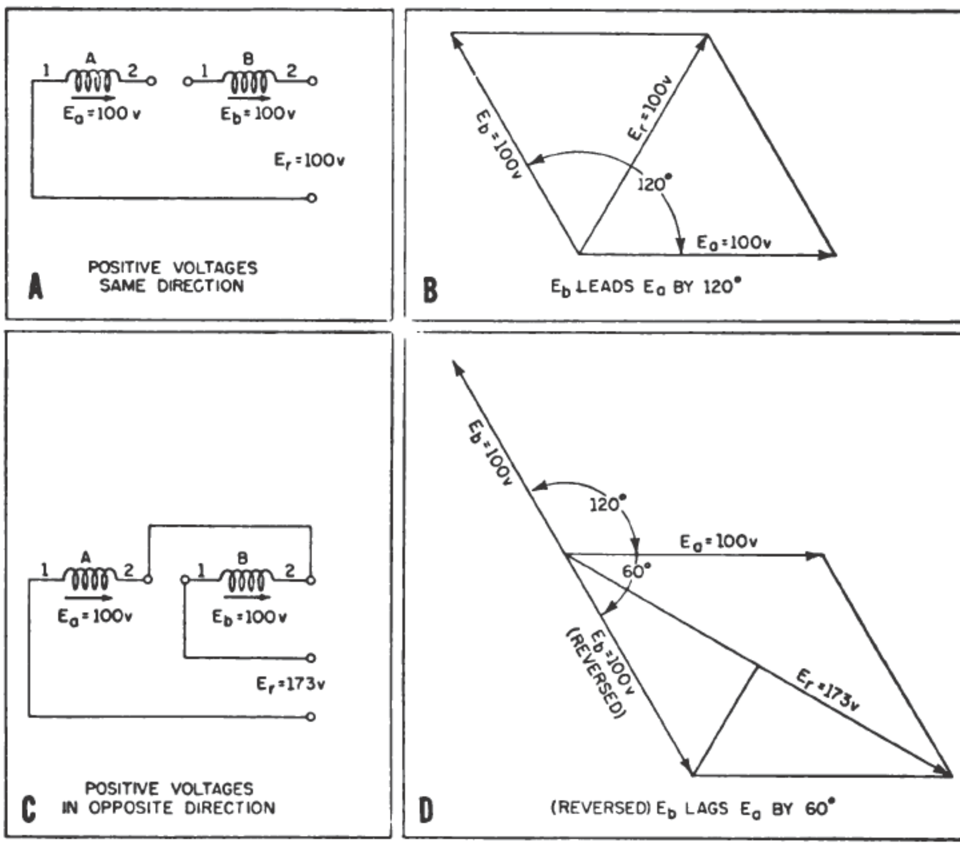


Figure 13–14.—Vector analysis of voltages in series aiding and opposing.

in figure 13–14, A, the positive maximum voltages generated in coils *A* and *B* act in the direction of the arrows, and *B* leads *A* by 120°. This arrangement may be obtained by assuming coils *A* and *B* to be two armature windings located 120° apart. If each voltage has an effective value of 100 volts, the total voltage is $E_r = 100$ volts, as shown by the polar vectors in figure 13–14, B.

If the connections of coil *B* are reversed (fig. 13–14, C) with respect to their original connections, the two voltages will be in opposition, as will be seen by tracing around the circuit in the direction of the arrow in coil *A*. The positive direction of the

voltage in coil B is opposite to the direction of the trace; the positive direction of the voltage generated in coil A is the same as that of the trace; hence, the two voltages are in opposition. This effect is the same as though the positive maximum value of E_b were 60° out of phase with that of E_a, and E_b acted in the same direction as E_a when the circuit trace was made (fig. 13–14, A). Adding the (reversed) vector E_b (fig. 13–14, D) to vector E_a is accomplished by reversing the position of E_b from that shown in figure 13–14, B, to the position shown in figure 13–14, D, and completing the parallelogram. If E_a and E_b are each 100 volts, $E_r = \sqrt{3} \times 100$, or 173 volts. The value of E_r may be derived in the following manner: Erecting a perpendicular to E_r divides the isosceles triangle into two equal right triangles each having a hypotenuse of 100 volts and a base of 100 cos 30°, or 86.6 volts. The total length of E_r is 2×86.6, or 173.2 volts.

To construct the line voltage vectors $E_{1,2}$, $E_{2,3}$, and $E_{3,1}$ in figure 13–13, it is first necessary to trace a path around the closed circuit which includes the line wires, alternator windings, and one of the three voltmeters. For example, in figure 13–13, A, consider the circuit that includes the upper and middle wires, the voltmeter connected across them, and the alternator phases a and b. The circuit trace is started at the center of the wye, proceeds through phase a of the alternator, out line 1, down through the voltmeter from line 1 to line 2, and through phase b of the alternator back to the starting point. Voltage drops along line wires are disregarded. The voltmeter indicates an effective value equal to the vector sum of the effective value of voltage in phases a and b. This value is the line voltage, $E_{1,2}$. According to Kirchhoff's law, the source voltage between lines 1 and 2 equals the voltage drop across the voltmeter connected to these lines.

If the direction of the path traced through the GENERATOR is the SAME as that of the arrow, the sign of the voltage is PLUS; if the direction of the trace is OPPOSITE to the arrow, the sign of the voltage is MINUS. If the direction of the path traced through the VOLTMETER is the SAME as that of the arrow, the sign of the voltage is MINUS; if the direction of the trace is OPPOSITE to that of the arrow, the sign of the voltage is PLUS.

The following equations for voltage are based on the preceding rules:

$$E_a \oplus (-E_b) = E_{1,2}, \text{ or } E_{1,2} = E_a \ominus E_b.$$

$$E_b \oplus (-E_c) = E_{2,3}, \text{ or } E_{2,3} = E_b \ominus E_c.$$

$$E_c \oplus (-E_a) = E_{3,1}, \text{ or } E_{3,1} = E_c \ominus E_a.$$

The signs $\oplus$ and $\ominus$ mean vector addition and vector subtraction, respectively. One vector is subtracted from another by reversing the position of the vector to be subtracted through an angle of 180° and constructing a parallelogram, the sides of which are the reversed vector and the other vector. The diagonal of the parallelogram is the difference vector.

These equations are applied to the vector diagram of figure 13–13, B, to derive the line voltages $E_{1,2}$, $E_{2,3}$, and $E_{3,1}$, as the diagonals of three parallelograms of which the sides are the phase voltages E_a, E_b, and E_c. From this vector diagram, the following facts are observed: (1) The line voltages are equal and 120° apart; (2) the line currents are equal and 120° apart; (3) the line currents are 30° out of phase with the line voltages when the power factor of the load is 100 percent; and (4) line voltage is the product of the phase voltage and $\sqrt{3}$.

DELTA CONNECTION.—In a delta-connected alternator, the start end of one phase winding is connected to the finish end of the second; the start end of the second, to the finish end of the third; and the start end of the third, to the finish end of the first. The three junction points are connected to the line wires leading to the load. A 3-phase delta-connected alternator is represented at the left (fig. 13–15, A). The alternator is connected to a 3-phase 3-wire circuit which supplies a 3-phase delta-connected load at the right-hand end of the 3-phase line. Because the phases are connected directly across the line wires, phase voltage is equal to line voltage. When the alternator phases are properly connected in delta, no appreciable current flows within the delta loop when there is no external load connected to the alternator. If any one of the phases is reversed with respect to its correct connection, a short-circuit current will flow within the windings on no load, causing damage to the windings.

To avoid connecting a phase in reverse it is necessary to test the circuit before closing the delta. This may be done by connecting a voltmeter or fuse wire between the two ends of the delta loop before closing the delta. The two ends of the delta

loop should never be connected together if there is an indication of any appreciable current or voltage between them when no load is connected to the alternator.

The three phase currents, I_a, I_b, and I_c, are indicated by accompanying arrows in the alternator phases at the left in figure 13–15, A. These arrows point in the direction of the positive current and voltage of each phase. The three voltmeters connected across lines 1–2, 2–3, and 3–1, respectively, indicate effective values of line and phase voltage. Line current I_1 is supplied by phases a and c, which are connected to line 1. Line current is greater than phase current, but not twice as great because the phase currents are not in phase with each other. The relation between line currents and phase currents is shown in figure 13–15,

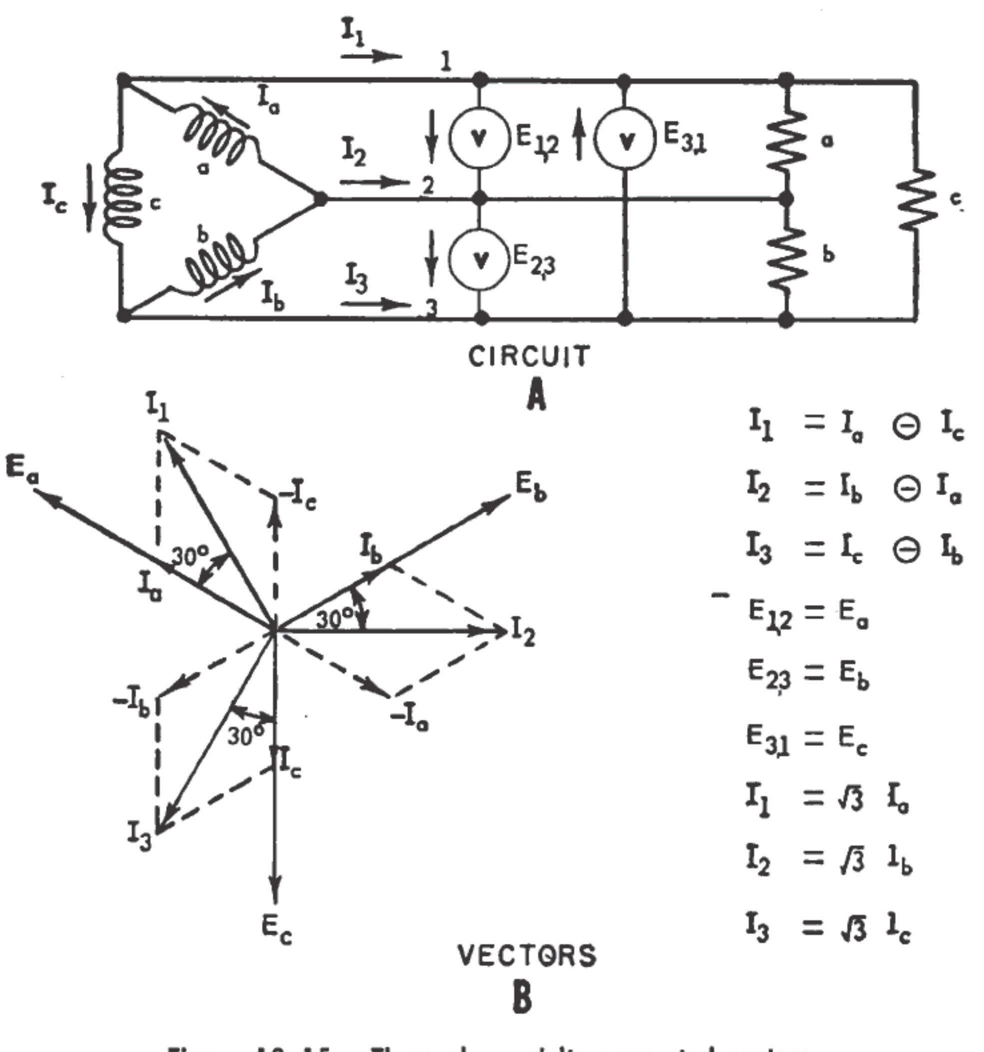


Figure 13–15.—Three-phase delta-connected system.

B. Effective values of line and phase voltages are indicated by vectors E_a, E_b, and E_c. Note that the vector sum of E_a, E_b, and E_c is zero. The phase currents are equal to each other because the loads are balanced. The line currents are equal to each other for the same reason. At unity-power-factor loads, the phase current and phase voltage have a 0° angle between them.

To construct the line current vectors it is first necessary to consider the direction of currents at the junctions of the line wires and the alternator phases. Consider the junction formed by phase *a*, phase *c*, and line 1. According to Kirchhoff's law, line current I_1 is equal to the vector difference between I_a and I_c. In figure 13–15, B, current vector I_a is in the same general direction as current vector I_1 (less than 90° displaced from I_1), and current vector I_c is in opposition to vector I_1 (more than 90° displaced from it). I_1 is the vector sum of I_a and $-I_c$. These are two equal values of phase current and they are 60° out of phase with each other. The vector sum of all the currents entering and leaving the junction is zero. Current arrows that point toward the junction are considered to represent positive currents. Current arrows that point away from the junction are considered to represent negative currents. The following equations for current are based on the preceding rules:

$$I_a \oplus (-I_c) = I_1, \text{ or } I_1 = I_a \ominus I_c.$$

$$I_b \oplus (-I_a) = I_2, \text{ or } I_2 = I_b \ominus I_a.$$

$$I_c \oplus (-I_b) = I_3, \text{ or } I_3 = I_c \ominus I_b.$$

Here also, the signs $\oplus$ and $\ominus$ mean vector addition and vector subtraction, respectively. Vector subtraction is accomplished again by reversing the vector being subtracted through an angle of 180° and combining it with the other vector by the parallelogram method to form the diagonal which is the vector difference.

If the current equations are applied to the vector diagram of figure 13–15, B, the line currents I_1, I_2, and I_3 are derived as diagonals of the three parallelograms in which the sides are the phase currents I_a, I_b, and I_c. From this diagram the following facts are observed: (1) The line currents are equal and 120° apart; (2) the line voltages are equal and 120° apart; (3) the line

currents are 30° out of phase with the line voltages when the power factor of the load is unity; and (4) the line current is equal to the product of the phase current and $\sqrt{3}$.

The power delivered by a balanced 3-phase wye-connected system is equal to three times the power delivered by each phase. The total true power is

$$P_t = 3E_{\text{phase}} I_{\text{phase}} \cos\theta.$$

Because $E_{\text{phase}} = \dfrac{E_{\text{line}}}{\sqrt{3}}$ and $I_{\text{phase}} = I_{\text{line}}$, the total true power is

$$P_t = 3\,\frac{E_{\text{line}}}{\sqrt{3}}\,I_{\text{line}} \cos\theta,$$

$$= \sqrt{3}\,E_{\text{line}} I_{\text{line}} \cos\theta.$$

The power delivered by a balanced 3-phase delta-connected system is also three times the power delivered by each phase. Because $E_{\text{phase}} = E_{\text{line}}$ and $I_{\text{phase}} = \dfrac{I_{\text{line}}}{\sqrt{3}}$, the total true power is

$$P_t = 3E_{\text{line}}\,\frac{I_{\text{line}}}{\sqrt{3}} \cos\theta,$$

$$= \sqrt{3}\,E_{\text{line}} I_{\text{line}} \cos\theta.$$

Thus, the expression for 3-phase power delivered by a balanced delta-connected system is the same as the expression for 3-phase power delivered by a balanced wye-connected system. Two examples are given to illustrate the phase relations between current, voltage, and power in (1) a 3-phase wye-connected system and (2) a 3-phase delta-connected system.

Example 1: A 3-phase wye-connected alternator has a terminal voltage of 450 volts and delivers a full-load current of 300 amperes per terminal at a power factor of 80 percent. Find (a) the phase voltage, (b) the full-load current per phase, (c) the kilovolt-ampere, or apparent power, rating, and (d) the true power output.

(a) $E_{\text{phase}} = \dfrac{E_{\text{line}}}{\sqrt{3}} = \dfrac{450}{\sqrt{3}} = 260$ volts.

(b) $I_{\text{phase}} = I_{\text{line}} = 300$ amperes.

(c) Apparent power $= \sqrt{3}\, E_{\text{line}} I_{\text{line}}$
$= \sqrt{3} \times 450 \times 300 = 233{,}600$ va, or 233.6 kva.

(d) True power $= \sqrt{3}\, E_{\text{line}} I_{\text{line}} \cos\theta$
$= \sqrt{3} \times 450 \times 300 \times 0.8$
$= 186{,}800$ watts, or 186.8 kw.

Example 2: A 3-phase delta-connected alternator has a terminal voltage of 450 volts and the current in each phase is 200 amperes. The power factor of the load is 75 percent. Find (a) the line voltage, (b) the line current, (c) the apparent power, and (d) the true power.

(a) $E_{\text{phase}} = E_{\text{line}} = 450$ volts.

(b) $I_{\text{line}} = \sqrt{3}\, I_{\text{phase}} = 1.732 \times 200 = 346$ amperes.

(c) Apparent power $= \sqrt{3}\, E_{\text{line}} I_{\text{line}}$
$= 1.732 \times 450 \times 346 = 269{,}000$ va, or 269 kva.

(d) True power $= \sqrt{3}\, E_{\text{line}} I_{\text{line}} \cos\theta$
$= 1.732 \times 450 \times 346 \times 0.75$
$= 201{,}700$ watts, or 201.7 kw.

Measurement of Power

The wattmeter connections for measuring the true power in a 3-phase system are shown in figure 13–16. The method shown in figure 13–16, A, uses three wattmeters with their current coils inserted in series with the line wires and their potential coils connected between line and neutral wires. The total true power is equal to the arithmetic sum of the three wattmeter readings.

The method shown in figure 13–16, B, uses two wattmeters with their current coils connected in series with two line wires and their potential coils connected between these line wires and the common, or third, wire that does not contain the current coils. The total true power is equal to the algebraic sum of the two watt-

meter readings. If one meter reads backward, its potential coil connections are first reversed to make the meter read up-scale and the total true power is then equal to the difference in the two wattmeter readings. If the load power factor is less than 0.5 and

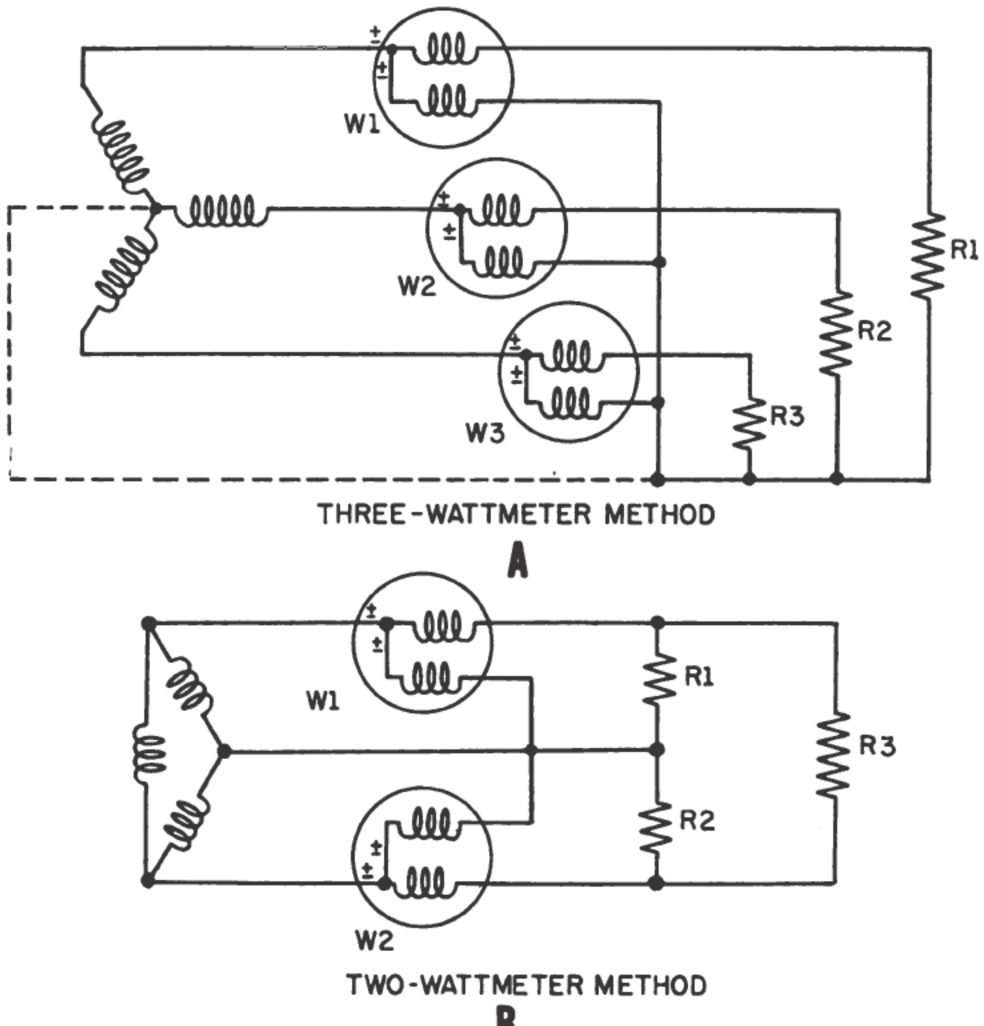


Figure 13–16.—Wattmeter connections for measurement of power in a 3-phase system.

the loads are balanced, the total true power will be equal to the difference in the two wattmeter readings. If the load power factor is 0.5, one meter will indicate the total true power and the other will indicate zero. If the load power factor is above 0.5, the total true power is equal to the sum of the two wattmeter readings.

QUIZ

1. What is the inductive reactance in ohms developed by a 0.159–henry coil if the frequency is (a) 50 cycles per second? (b) 100 cycles per second?
2. In figure 13–1, A, if the resistance of branch (1) is 100 ohms, the inductance of branch (2) is 0.159 henry, the frequency of the source is 100 cycles per second, and the applied voltage is 100 volts, find the following:
   (a) Current in branch (1).
   (b) Inductive reactance of branch (2).
   (c) Current in branch (2).
   (d) Total circuit current.
   (e) True power in branch (1).
   (f) Reactive power in branch (2).
   (g) Total apparent power of the combined parallel circuit.
   (h) Total circuit impedance.
   (i) Total circuit power factor.
   (j) Phase angle between the total circuit current and the applied voltage.
3. A low-loss inductor has a power factor of 1 percent and an inductance of 15.9 henries. When operated at a frequency of 60 cycles per second, find the
   (a) Inductive reactance.
   (b) Effective series resistance.
   (c) Equivalent shunt resistance.
   (d) Phase angle between E and I for the low-loss inductor.
4. In a circuit similar to figure 13–3, A, E is a 250-volt a-c source. The series resistance in branch (1) is 12.5 ohms and the series inductive reactance is 21.65 ohms. The series resistance in branch (2) is 17.68 ohms and the series inductive reactance is 17.68 ohms. Referring to figure 13–3, find the following:
   (a) Impedance, Z_1.
   (b) Impedance, Z_2.
   (c) Current in branch (1).
   (d) Current in branch (2).
   (e) Total current.
   (f) Total circuit power factor.
   (g) Apparent power in branch (1).
   (h) Apparent power in branch (2).
   (i) Total circuit true power.
   (j) Total circuit impedance.
   (k) Equivalent series resistance of parallel circuit.
   (l) Equivalent series reactance of parallel circuit.

5. A circuit similar to figure 13–5, A, has a 50 cycle per second 250-volt supply. The resistor has a value of 125 ohms and the capacitor a value of 25.5 microfarads and negligible losses. Referring to figure 13–5, find the following:
   (a) Capacitive reactance of branch (2).
   (b) Energy current of branch (1).
   (c) Nonenergy current of branch (2).
   (d) Total parallel circuit current.
   (e) True power in branch (1).
   (f) Reactive power in branch (2).
   (g) Apparent power of the total circuit.
   (h) Total circuit power factor.
   (i) Total circuit impedance.
   (j) Equivalent series resistance of parallel circuit.
   (k) Equivalent series capacitive reactance of parallel circuit.
   (l) Equivalent series capacitance of parallel circuit.
6. A 7.96-microfarad capacitor is connected across a 400 cps, 250-volt source. The capacitor has a power factor of 2%. Referring to figure 13–7, find the following:
   (a) Capacitive reactance.
   (b) Equivalent series resistance.
   (c) Total impedance.
   (d) Total current.
   (e) Energy component of current.
   (f) Nonenergy component of current.
   (g) Equivalent shunt resistance.
   (h) True power.
   (i) Reactive power.
   (j) Apparent power.
7. In a circuit similar to figure 13–8, A, $E=120$ volts, $f=400$ cps, $R_1=20$ ohms, $R_2=80$ ohms, and $C=6.64$ microfarads. Referring to figure 13–8, find the following:
   (a) Energy current in branch (1).
   (b) Capacitive reactance in branch (2).
   (c) Impedance of branch (2).
   (d) Current in branch (2).
   (e) Total circuit current.
   (f) Phase angle of capacitive branch.
   (g) Total circuit phase angle.
   (h) Total circuit impedance.
   (i) Capacitive reactance of the equivalent series circuit.
   (j) Capacitance of the equivalent series circuit.
   (k) Total apparent power of the parallel circuit.
   (l) Total true power of the parallel circuit.

8. In a parallel circuit similar to figure 13–9, A, $E=250$ volts, $f=50$ cps, $R_1=25$ ohms, $R_2=17.32$ ohms, $L=0.0318$ henry, and $C=181.8$ microfarads with a zero percent power factor. Referring to figure 13–9, find the following:
   (a) Current in branch (1).
   (b) Inductive reactance of branch (2).
   (c) Impedance of branch (2).
   (d) Current in branch (2).
   (e) Phase angle of branch (2).
   (f) Energy component of current in branch (2).
   (g) Nonenergy component of current in branch (2).
   (h) Capacitive reactance of branch (3).
   (i) Impedance of branch (3).
   (j) Nonenergy component of current in branch (3).
   (k) Total circuit current.
   (l) Total circuit phase angle.
   (m) Total impedance of the parallel circuit.
   (n) Resistance of the equivalent series circuit.
   (o) Capacitive reactance of the equivalent series circuit.
   (p) True power of the parallel circuit.
   (q) Apparent power of the parallel circuit.
9. In a circuit similar to figure 13–10, A, if the load voltage is 250 volts, the load current is 50 amperes, the line resistance is 1.0 ohm, and the load power factor is 60 percent lagging, find the following:
   (a) True power in the load.
   (b) True power lost in the line.
   (c) Efficiency of transmission.
   (d) Magnitude of the 90° leading current to be supplied by a parallel-connected capacitor to improve the load power factor to unity.
   (e) Efficiency of transmission with a unity power factor load.
10. (a) In a circuit similar to figure 13–11, A, the voltage applied to the unity-power-factor load is reduced from 120 volts to 75 volts through the series dropping resistor. If the circuit current is 20 amperes, find the circuit efficiency.
   (b) If the voltage applied to the load is reduced to 75 volts through a series inductor, as in figure 13–11, B, find the circuit power factor.
   (c) If a capacitor is placed in parallel with the series combination of the inductor and load, as in figure 13–11, D, what 90° leading current should the capacitor supply to give unity power factor to the parallel circuit?
   (d) What then is the line current at unity power factor?
11. List the stated seven possible nonuseful losses in a-c circuits.
12. How can the inductance of a hollow core coil operating at a frequency of 60 cycles per second be increased?

13. What symbols are always used to designate the three phases of a 3-phase system?
14. In a wye-connected 3-phase winding, the line voltage is the product of the phase voltage and ____________.
15. In a delta-connected 3-phase winding, the line current is the product of the phase current and ____________.
16. State the expression for the true power P, delivered to a balanced 3-phase wye-connected load in terms of phase current I_{phase}, phase voltage, E_{phase}, and phase power factor, $\cos \theta_{phase}$.
17. A 3-phase wye-connected alternator having a terminal voltage of 460 volts delivers 250 amperes per terminal at a power factor of 0.75, find the following:
    (a) Phase voltage.
    (b) Phase current.
    (c) Apparent power in kva.
    (d) True power in kw.
18. A 3-phase delta-connected alternator has a terminal voltage of 460 volts. The current in each phase is 300 amperes and the power factor is 78 percent, find the
    (a) Line voltage.
    (b) Line current.
    (c) Apparent power in kva.
    (d) True power in kw.

CHAPTER 14

ALTERNATORS AND TRANSFORMERS

ALTERNATORS

A large percentage of the electric power generated on shipboard or produced for industrial applications is by means of alternating-current generators. Also, aircraft a-c generator applications are increasing steadily. As a result, the a-c generator, or alternator, is a most important means of electric power production.

Alternators vary greatly in size, depending upon their power requirements. For example, the alternators used at hydroelectric plants such as Boulder Dam are tremendous in size, generating thousands of kilowatts at voltages of over 13,000 volts.

Regardless of their size, all electric generators, whether d-c or a-c, depend upon the action of a coil cutting through a magnetic field, or a magnetic field cutting through a coil. As long as there is relative motion between a conductor and a magnetic field, a voltage will be generated in the conductor. That part of the alternator which establishes the magnetic field is called the FIELD, and that part in which the voltage is generated is called the ARMATURE. In order to have relative motion between a conductor and a magnetic field, all d-c and a-c generators are made up of two mechanical parts—a ROTOR and a STATOR. As mentioned before, in most d-c generators the armature is the rotor and the field is the stator, or stationary member.

Types

There are two types of alternators—the revolving-armature type and the revolving-field type. The revolving-armature type alternator is similar in construction to the d-c generator, in that the armature rotates through a stationary magnetic field. The revolving-armature alternator is found only in alternators of small power rating and is not generally used. In the d-c gen-

erator the emf generated in the armature windings is converted into a unidirectional voltage (d-c) by means of the commutator. In the revolving-armature type of alternator, the generated a-c voltage is applied unchanged to the load by means of slip rings and brushes.

The revolving-field type of alternator has a stationary armature winding (stator) and a rotating-field winding (rotor). The advantage of having a stationary armature winding is that the armature can be connected directly to the load without having sliding contacts in the load circuit. A rotating armature would require slip rings and brushes to conduct the load current from the armature to the external circuit. Because large alternators supply load currents of the order of thousands of amperes at generated voltages that are frequently over 13,000 volts, sliding contacts would have a short life and arc-over would be a constant hazard. Therefore, high-voltage alternators are usually of the stationary-armature, rotating-field type. The voltage and current supplied to the rotating field are relatively small, and slip rings and brushes for this circuit are adequate. The direct connection to the armature circuit makes possible the use of large cross-section conductors, adequately insulated for high voltage.

Rating

The maximum current that can be supplied by an alternator depends upon (1) the maximum heating loss (I^2R power loss) that can be sustained in the armature and (2) the maximum heating loss that can be sustained in the field. The armature current of an alternator varies with the load. This action is similar to that of d-c generators. In a-c generators, however, lagging power factor loads tend to demagnetize the field of an alternator; and terminal voltage is maintained only by increasing the d-c field current. Therefore, alternators are rated in terms of armature load current and voltage output or kilovolt-ampere (kva) output at a specified frequency and power factor. The specified power factor is usually 80 percent lagging. For example, a single-phase alternator designed to deliver 100 amperes at 1,000 volts is rated at 100 kva. This machine would supply a 100-kw load at unity power factor or an 80-kw load at 80 percent power factor. If, however, the alternator supplied a 100-kva load at 20 percent power factor, the required increase in d-c field current

to maintain the terminal voltage at the desired value would cause excessive temperature rise in the field.

Construction

Alternators used in the Navy are divided into three classes according to the type of prime mover—(1) low-speed engine-driven alternators, (2) high-speed turbine-driven alternators, and (3) aircraft-engine driven high-speed alternators 4,000 to 8,000 rpm.

The stator, or armature, of the revolving-field alternator is built up from steel punchings, or laminations. This construction is somewhat similar to that of the armature core of a d-c generator. The laminations of an alternator stator, however, form a steel ring that is keyed or bolted to the inside circumference of a steel frame. The inner surface of the laminated ring has slots in which the stator winding is placed.

The low-speed engine-driven alternator (fig. 14–1) has a large-diameter revolving field with many poles, and a stationary armature relatively short in axial length. The stator (fig. 14–1, A) contains the armature windings, and the rotor (fig. 14–1, B) consists of salient poles, on which are mounted the d-c field windings.

The high-speed turbine-driven alternator (fig. 14–2) is connected either directly or through gears to a steam turbine. The enclosed metal structure is a part of a forced ventilation system that carries away the heat by circulation of the air through the stator (fig. 14–2, A) and rotor (fig. 14–2, B). The enclosed stator directs the paths of the circulating air-cooling currents and also reduces windage noise.

In both low-speed and high-speed alternators, the field windings are connected either in series or in series groups connected in parallel. The ends of the windings connect to slip rings mounted on the rotor shaft. Regardless of the type of rotor field used, its windings are separately excited by a d-c generator called an "exciter". The exciter armature for the engine-driven alternator (fig. 14–1, B) is shown mounted on the alternator shaft. The excitation voltage is usually 120 or 240 volts.

Single Phase

A single-phase alternator has a stator made up of a number of windings in series, which form a single circuit in which an output

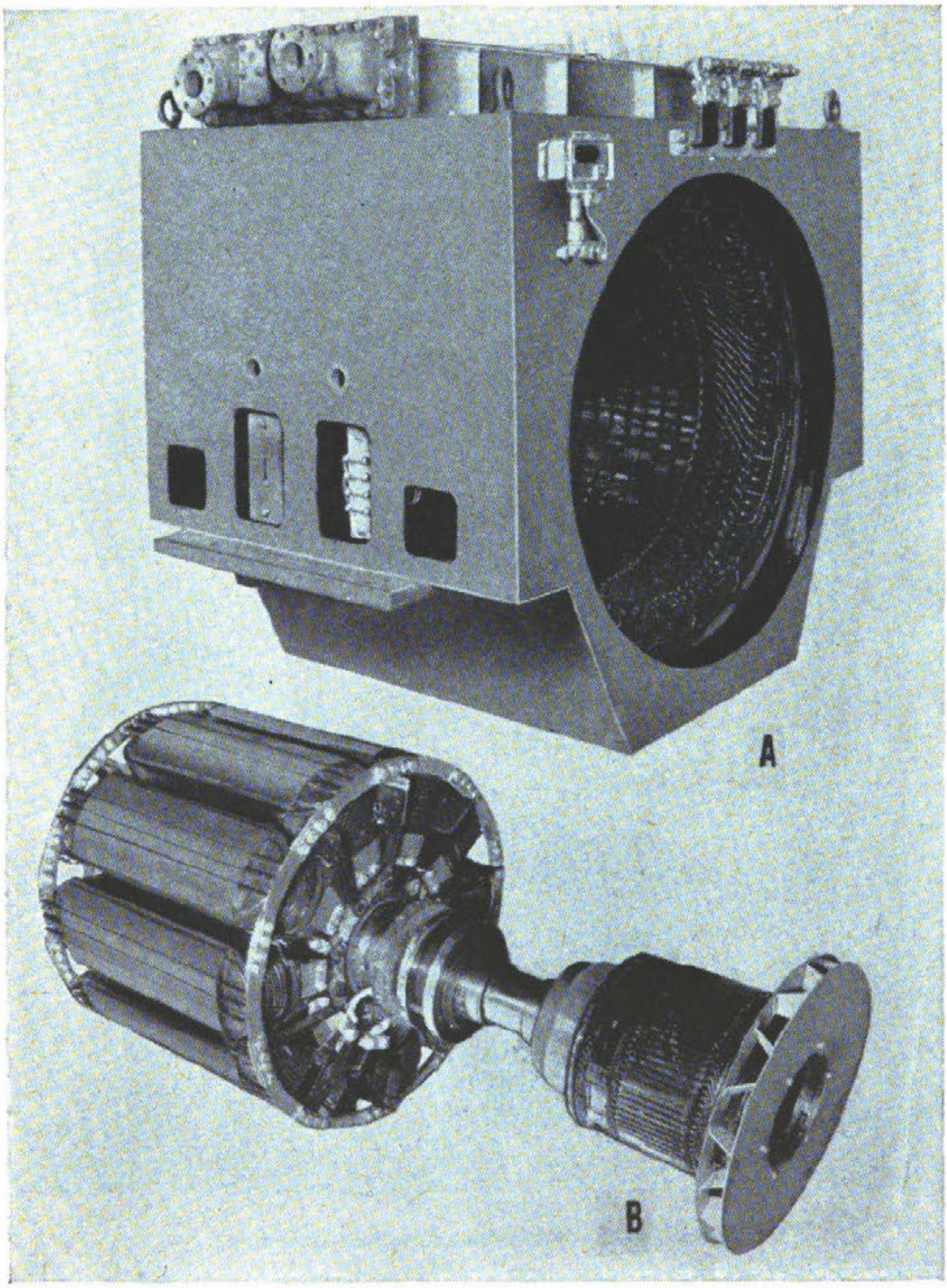


Figure 14–1.—Low-speed engine-driven alternator.

voltage is generated. The principle of the single-phase alternator is described first, and the polyphase alternator is described later.

Figure 14–3 illustrates a schematic diagram of a single-phase alternator having four poles. The stator has four polar groups evenly spaced around the stator frame. The rotor has four poles,

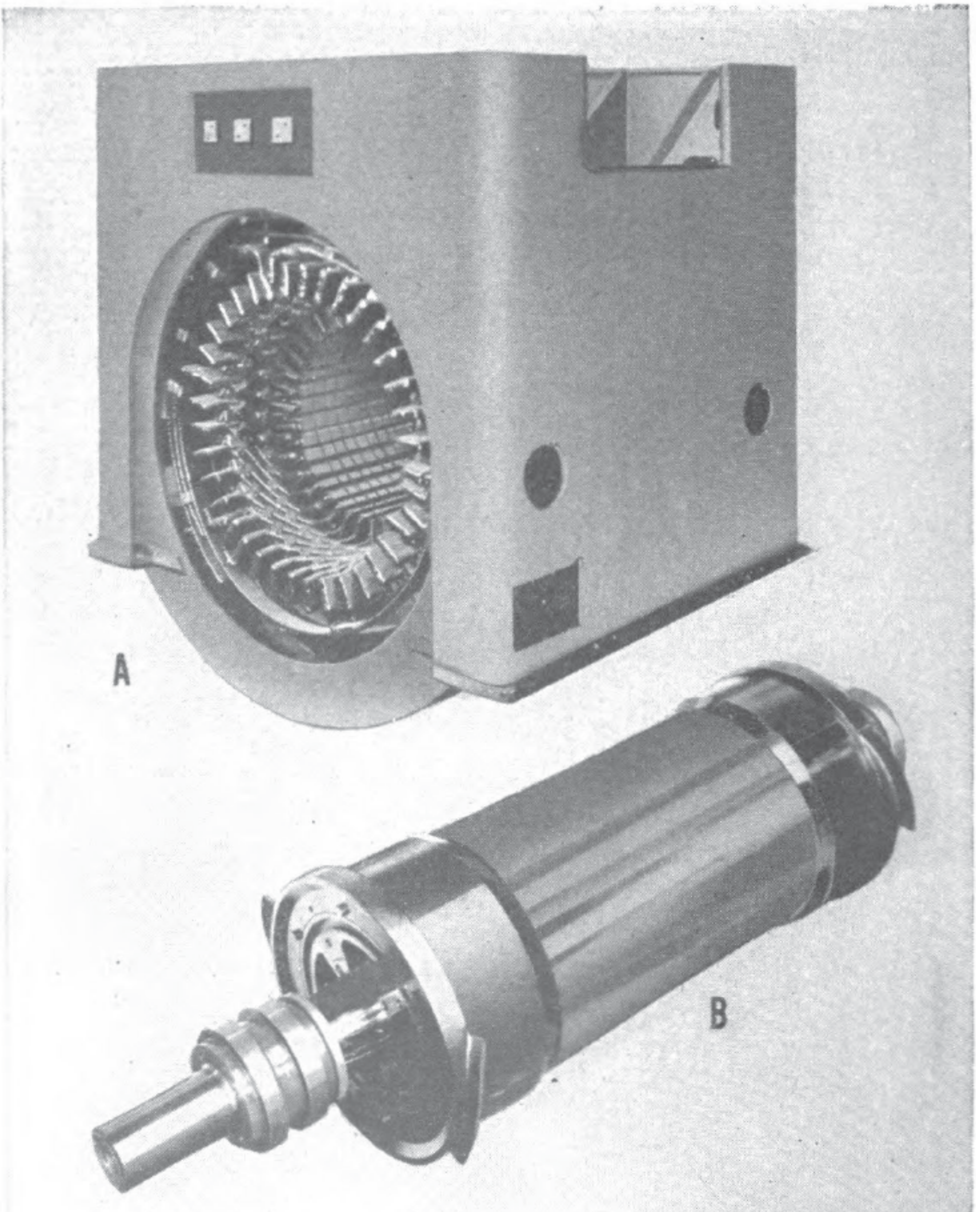


Figure 14–2.—High-speed turbine alternator.

with adjacent poles of opposite polarity. As the rotor revolves, a-c voltages are induced in the stator windings. Since one rotor pole is in the same position relative to a stator winding as any other rotor pole, all stator polar groups are cut by equal amounts of magnetic lines of force at any time. As a result, the voltages induced in all the windings have the same amplitude, or value, at any given instant. The four stator windings are connected to each other

so that the a-c voltages are in phase, or "series aiding". Assume that rotor pole 1, a south pole, induces a voltage in the direction indicated by the arrow in stator winding 1. Since rotor pole 2 is a north pole, it will induce a voltage in the opposite direction in stator coil 2 with respect to that in coil 1.

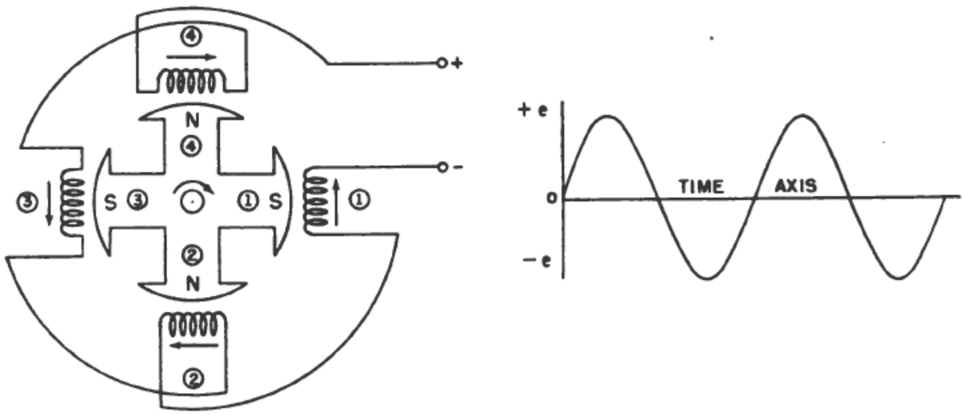


Figure 14–3.—Single-phase alternator.

In order that the two induced voltages be in series addition, the two coils are connected as shown. Applying the same reasoning, the voltage induced in stator coil 3 (clockwise rotation of the field) is in the same direction (counterclockwise) as the voltage induced in coil 1. Similarly, the direction of the voltage induced in winding 4 is opposite to the direction of the voltage induced in coil 1. All four stator coil groups are connected in series so that the voltages induced in each winding add to give a total voltage that is four times the voltage in any one winding.

Two Phase

Multi-phase or polyphase alternators have two or more single-phase windings symmetrically spaced around the stator. In a 2-phase alternator there are two single-phase windings phsyically spaced so that the a-c voltage induced in one is 90° out of phase with the voltage induced in the other. The windings are electrically separate from each other. When one winding is being cut by maximum flux, the other is being cut by no flux. This condition establishes the 90° relation between the two phases.

Figure 14–4 illustrates a schematic diagram of a 2-phase 4-pole alternator. The stator consists of two single-phase windings (phases) completely separated from each other. Each phase is made up of four windings, which are connected in series so that their voltages add. The rotor is identical with that used in the single-phase alternator. In figure 14–4, A, the rotor poles are opposite all of the coils of phase *A*. Therefore, the voltage induced in phase *A* is maximum and the voltage induced in phase *B* is zero. As the rotor continues rotating in a clockwise direction, it moves away from the windings of phase *A* and approaches those of phase *B*. As a result, the voltage in phase *A* decreases from its maximum value and the voltage in phase *B* increases from zero. In figure 14–4, B, the rotor poles are opposite the windings of phase *B*. Now the voltage induced in phase *B* is maximum; whereas the voltage induced in phase *A* has dropped to zero. Notice that in the 4-pole alternator a 45° mechanical rotation of the rotor corresponds electrically to one quarter cycle, or 90 electrical degrees. Figure 14–4, C, illustrates the waveforms of

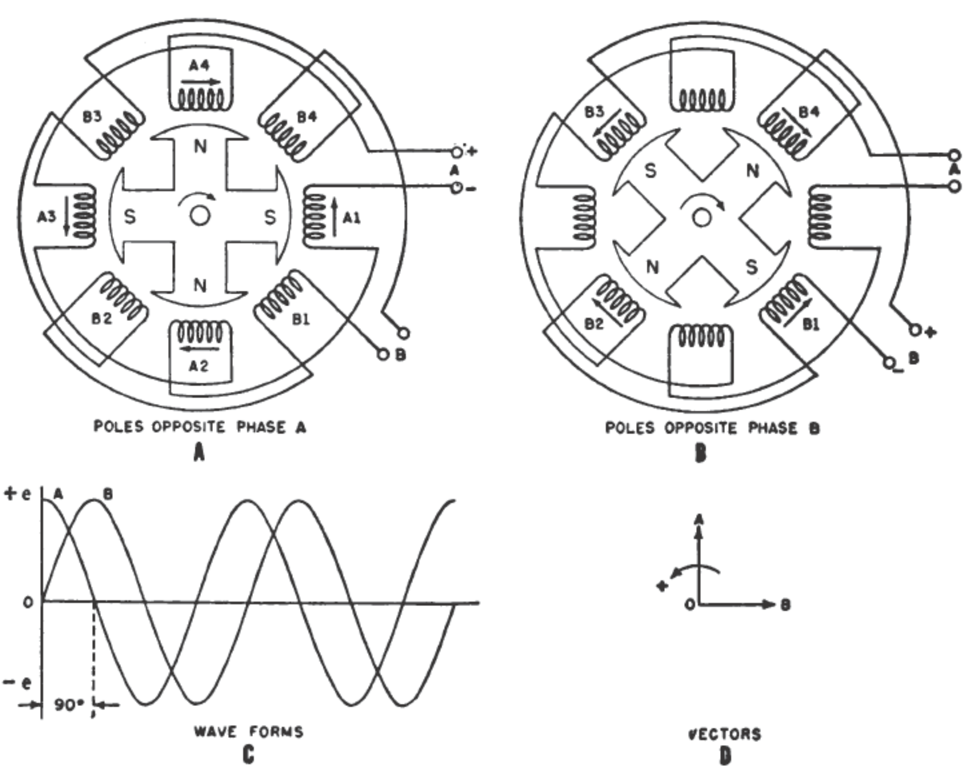


Figure 14–4.—Two-phase 4-pole alternator.

the voltage generated in each of the two phases. Both are sine curves, and A leads B by 90°. Figure 14–4, D, illustrates the vectors representing the 2-phase voltages. Vector A leads vector B by 90°.

The two phases of a 2-phase alternator can be connected to each other as shown in figure 14–5, so that only three leads are

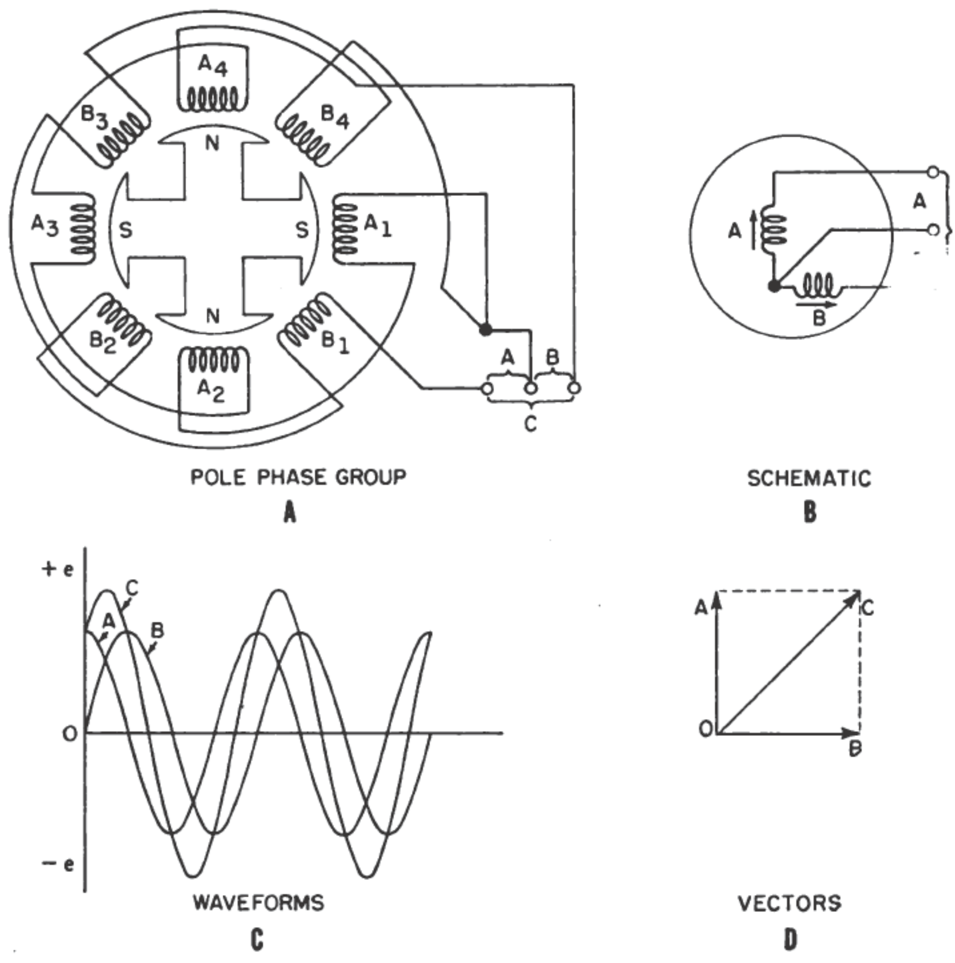


Figure 14–5.—Two-phase, 3-wire alternator.

brought out to the load. The alternator is then called a 2-phase 3-wire alternator. The two phases for a 4-pole alternator are shown in figure 14–5, A. A simplified schematic diagram is shown in figure 14–5, B, in which the rotor is omitted and each phase is indicated as a single coil. The two coils are drawn at right angles to each other to represent the 90° phase displacement

between their respective voltages. The three wires make possible three different load connections—(A) and (B) across phases A and B, respectively, and (C) across both phases in series. The third voltage (C) is displaced 45° from the A and B phase voltages and is their vector sum. If each phase voltage has an effective value of 100 volts, the vector sum of these voltages will be the hypotenuse of a right triangle, the base and altitude of which are each 100 volts. This hypotenuse is equal to

$$\frac{100}{\cos 45^\circ}, \text{ or } \frac{100}{0.707}=141 \text{ volts.}$$

Three Phase

The 3-phase alternator, as the name implies, has three single-phase windings spaced so that the voltage induced in each winding is 120° out of phase with the voltages in the other two windings. A schematic diagram of a 3-phase stator showing all the coils becomes complex, and it is difficult to see what is actually happen-

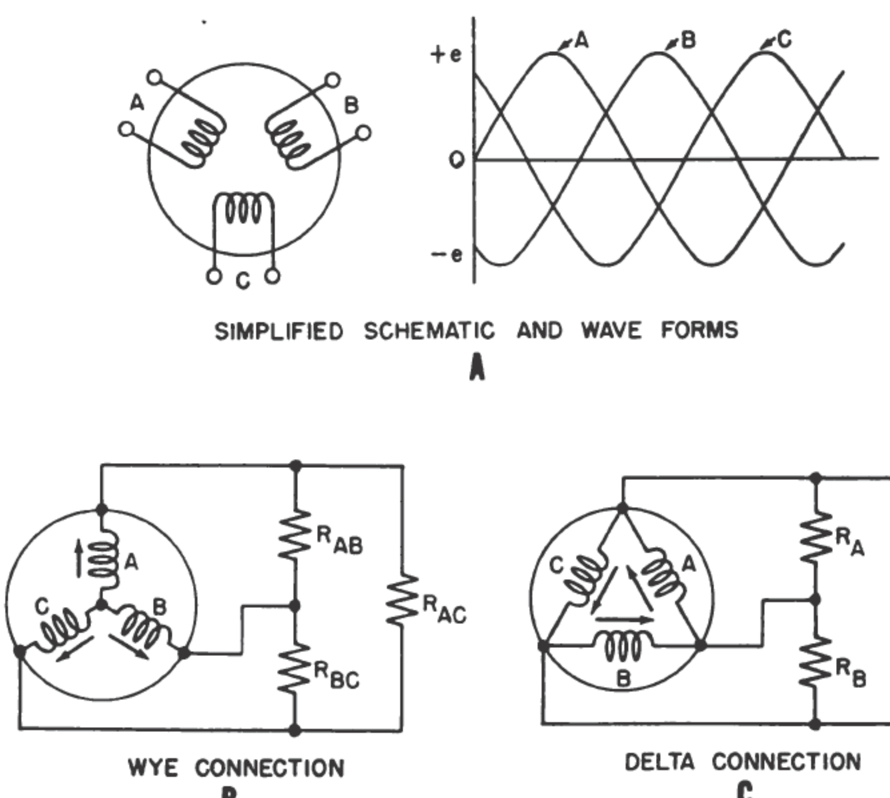


Figure 14–6.—Three-phase alternator.

ing. A simplified schematic diagram, showing all the windings of a single phase lumped together as one winding, is illustrated in figure 14–6, A. The rotor is omitted for simplicity. The waveforms of voltage are shown to the right of the schematic. The three voltages are 120° apart and are similar to the voltages that would be generated by three single-phase alternators whose voltages are out of phase by angles of 120°. The three phases are independent of each other.

Rather than have six leads come out of the 3-phase alternator, one of the leads from each phase may be connected to form a common junction. The stator is then called WYE, or STAR, connected. The common lead may or may not be brought out of the machine. If it is brought out, it is called the NEUTRAL. The simplified schematic (fig. 14–6, B) shows a wye-connected stator with the common lead not brought out. Each load is connected across two phases in series. Thus, R_{ab} is connected across phases A and B in series, R_{ac} is connected across phases A and C in series, and R_{bc} is connected across phases B and C in series. Thus, the voltage across each load is larger than the voltage across a single phase. The total voltage, or line voltage, across any two phases is the vector sum of the individual phase voltages. For balanced conditions, the line voltage is 1.73 times the phase voltage. Since there is only one path for current in a line wire and the phase to which it is connected, the line current is equal to the phase current.

A 3-phase stator can also be connected so that the phases are connected end to end as shown in figure 14–6, C. This arrangement is called a DELTA connection. In the delta connection the line voltages are equal to the phase voltages. The line currents are equal to the vector sum of the phase currents. The line current is equal to 1.73 times the phase current when the loads are balanced.

For equal loads (equal k-w output), the delta connection supplies increased line current at a value of line voltage equal to phase voltage, and the wye connection supplies increased line voltage at a value of line current equal to phase current.

In both wye- and delta-connected alternators the total output power equals $\sqrt{3}\ EI \cos\theta$, where E is the line voltage, I the line current, $\cos\theta$ the phase power factor, and $\sqrt{3}$ a constant for balanced 3-phase loads. For example, a 3-phase 440-volt alter-

nator may supply 655 amperes to each line wire at full load. Its kva rating is

$$\frac{\sqrt{3}\times 440\times 655}{1{,}000}=500\text{ kva.}$$

Frequency

The frequency of the alternator voltage depends upon the speed of rotation of the rotor and the number of poles. The faster the speed, the higher the frequency will be, and the lower the speed, the lower the frequency becomes. The more poles there are on the rotor, the higher the frequency will be for a given speed. When a rotor has rotated through an angle such that two adjacent rotor poles (a north and a south pole) have passed one winding, the voltage induced in that winding will have varied through one complete cycle. For a given frequency, the more pairs of poles there are, the lower will be the speed of rotation. A 2-pole alternator rotates at twice the speed of a 4-pole alternator for the same frequency of generated voltage. The frequency of the alternator in cycles per second is related to the number of poles and the speed as expressed by the equation

$$f=\frac{P}{2}\times\frac{N}{60}=\frac{PN}{120},$$

where P is the number of poles and N the speed in rpm. For example, a 2-pole 3,600-rpm alternator has a frequency of $\frac{2\times 3{,}600}{120}=60$ cycles per second; a 4-pole 1,800-rpm alternator has the same frequency; and a 6-pole 500-rpm alternator has a frequency of $\frac{6\times 500}{120}=25$ cycles per second, and a 12-pole 4,000 rpm alternator has a frequency of $\frac{12\times 4{,}000}{120}=400$ cycles per second.

Generated Voltage

Once a machine has been designed and built, the generated voltage of an alternator is controlled, in practice, by varying the d-c excitation voltage applied to the field winding.

In the design, however, many factors must be taken into account, as the following text illustrates.

As mentioned before, the conductors in an alternator arma-

ture are arranged in one or more groups called phases, according to whether the machine is designed to deliver single-phase, 2-phase, or 3-phase voltages. The effective voltage, E, per phase is

$$E=2.22\Phi Zf10^{-8}K_bK_p,$$

where Φ is the number of magnetic lines of flux per pole, Z the number of conductors in series per phase, f the frequency in cycles per second, K_b a breadth (or belt) factor, and K_p a pitch factor.

Generally, the coils of each phase are distributed uniformly around the stator. The voltages generated in the various coils are not in time phase with each other because of the displacement of the coils. A breadth, or belt, factor K_b takes the displacement into account and reduces the total generated voltage per phase below the voltage that would be generated if all the active conductors were in the form of a concentrated winding. Also, if the two halves of a coil are less than 180 electrical degrees apart, the voltage generated in the coil is less than it would be if the coil pitch were a full 180 electrical degrees. The pitch factor, K_p, accounts for this reduction in voltage.

For example, a certain 3-phase, 60-cycle alternator has 96 conductors in series per phase and a field pole flux of 2.54×10^6 lines per pole. The breadth factor is 0.958, and the pitch factor is 0.966. The effective voltage per phase is

$$E=\frac{2.22\times2.54\times10^6\times96\times60\times0.958\times0.966}{10^8}=300 \text{ volts.}$$

Characteristics

When the load on an alternator is changed, the terminal voltage varies with the load. The amount of variation depends on the design of the alternator and the power factor of the load. With a load having a lagging power factor, the drop in terminal voltage with increased load is greater than for unity power factor. With a load having a leading power factor, the terminal voltage tends to rise. The causes of a change in terminal voltage with load change are (1) armature resistance, (2) armature reactance, and (3) armature reaction.

Armature resistance.—When current flows through an alternator armature winding, there is an IR drop due to the resistance of the winding. This drop increases with load, and the terminal

voltage is reduced. The armature resistance drop is small because the resistance is low.

ARMATURE REACTANCE.—The armature current of an alternator varies approximately as a sine wave. This action is in marked contrast to the alternating current in the armature of a d-c generator where the coil current is unvarying except during the time that it is being commutated. The continuously varying current

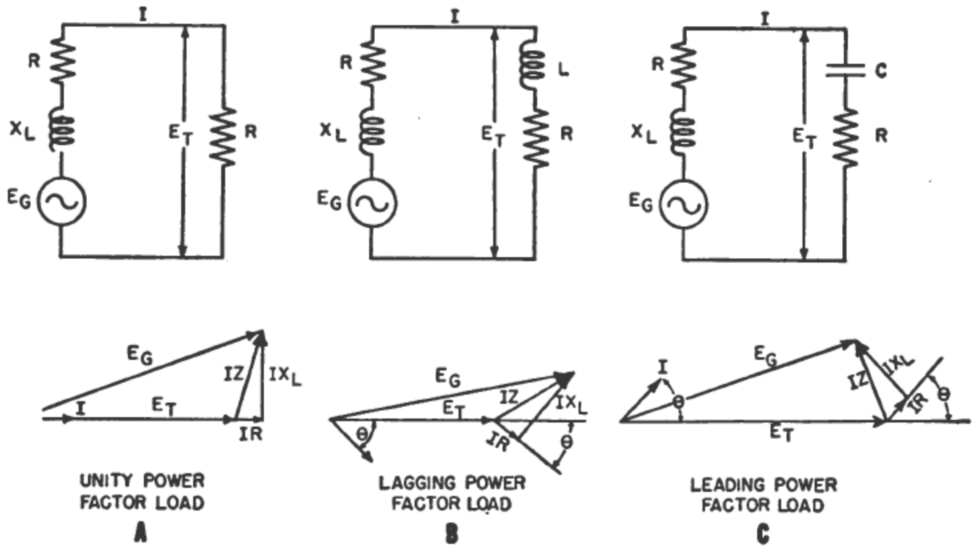


Figure 14–7.—Alternator voltage characteristics.

in the alternator armature is accompanied by an IX_L voltage drop in addition to the IR drop. Armature reactance in an alternator may be from 30 to 50 times the value of armature resistance because of the relatively large inductance of the coils compared with their resistance. As the lagging power factor is reduced, the IX_L voltage drop affects the terminal voltage to a greater extent.

A simplified series equivalent circuit of one phase of an alternator is shown in figure 14–7. The voltage generated in the phase winding is equal to the vector sum of the terminal voltage for the phase and the internal voltage loss in the armature resistance, R, and the armature reactance, X_L, associated with that phase. The voltage vectors for a unity power factor load are shown in figure 14–7, A. The armature IR drop is in phase with the current, I, and the terminal voltage, E_t. Because the armature IX_L drop is 90° out of phase with the current, the terminal volt-

age is approximately equal to the generated voltage, less the IR drop in the armature.

The voltage vectors for a lagging power-factor load are shown in figure 14–7, B. The load current and IR drop lag the terminal voltage by angle θ. In this example, the armature IZ drop is more nearly in phase with the terminal voltage and the generated voltage. Hence, the terminal voltage is approximately equal to the generated voltage, less the armature IZ drop. Because the IZ drop is much greater than the IR drop, the terminal voltage is reduced that much more. The voltage vectors for a leading power factor load are shown in figure 14–7, C. The load current and IR drop lead the terminal voltage by angle θ. This condition results in an increase in terminal voltage above the value of E_g. The total available voltage of the alternator phase is the combined effect of E_g (rotationally induced) and the self-induced voltage (not shown in the vectors). The self-induced voltage, as in any a-c circuit, is caused by the varying field (accompanying the varying armature current) linking the armature conductors. The self-induced voltage always lags the current by 90°; hence, when I leads E_t, the self-induced voltage aids E_g, and E_t increases.

ARMATURE REACTION.—When an alternator supplies no load, the d-c field flux is distributed uniformly across the air gap in a manner similar to that in a d-c generator. When an alternator supplies a reactive load, however, the current flowing through the armature conductors produces an armature magnetomotive force that influences the terminal voltage by changing the magnitude of the field flux across the air gap. When the load is inductive, the armature mmf opposes the d-c field and weakens it, thus lowering the terminal voltage. When a leading current flows in the armature, the d-c field is aided by the armature mmf, and the flux across the air gap is increased, thus increasing the terminal voltage. Thus, the terminal voltage of an alternator may vary through a wider range than that of a d-c generator of comparable rating.

Voltage Regulation

The voltage regulation of an alternator is the change of voltage from full load to no load, expressed in percentage of full-load volts, when the speed and d-c field current are held constant.

$$\text{Percent regulation} = \frac{E_{NL} - E_{fL}}{E_{fL}} \times 100.$$

For example, the no-load voltage of a certain alternator is 250 volts and the full-load voltage is 220 volts. The percent regulation is

$$\frac{250-220}{220} \times 100 = 13.6 \text{ percent.}$$

Voltage Control

As has been stated, the voltage regulation of an alternator is relatively poor, when compared with that of a d-c generator. Under practical operating conditions (aircraft or shipboard) the load varies widely with the starting and stopping of motors and so forth. In order to maintain the terminal voltage constant, the field current must be increased when the load on the alternator is increased or when the power factor is low and lagging; likewise, it must be decreased when the load is decreased or when the power factor is leading. This variation in field current is accomplished by various means. One method is to adjust the resistance of the alternator field rheostat in low-power installations. This may be accomplished manually or by automatic means.

Manual adjustment may be practicable when the load is fairly constant. It is not practicable when the load fluctuates rapidly. However, manual adjustment is also available on automatic systems.

Vibrating-type regulators, widely used in the past, have been replaced to a large extent by rheostatic-type regulators. The latter type has lower maintenance cost, and the contacts are not as likely to stick. These regulators may be used in the main-field circuit; however, a more common practice is to use them in the exciter field circuit. When a wide range of control is necessary, the exciter is generally separately excited. Carbon-pile regulators are a form of rheostatic regulator that has been widely used in the past and is used extensively today in aircraft electrical systems.

In this chapter only the rheostatic-type regulators, which may be of the direct- or indirect-acting type, are considered.

One type of direct-acting voltage regulator (Diactor) manufactured by the General Electric Company is shown in simplified

form in figure 14–8. It regulates automatically the direct current in the exciter field coils and thus regulates the alternating output voltage of the generator. A special type of variable resistor (rheostatic element) is connected in series with the exciter shunt field. The rheostatic element is similar to the unit described in the tilted-plate regulator in chapter 10.

The operation of the system may be explained in the following manner. Under normal operating conditions, the force of the spring and the force of the torque motor are balanced, and the rotor of the torque motor is stationary. The stabilizer damps out slight current variations that would cause the system to "hunt" or to oscillate.

If the a-c output voltage of the generator drops below normal, the voltage supplied to the torque motor is reduced, and the spring will turn the armature clockwise. The resistance of the rheostatic element is reduced, and more current is fed to the exciter shunt field. The increase in current through the exciter field raises the

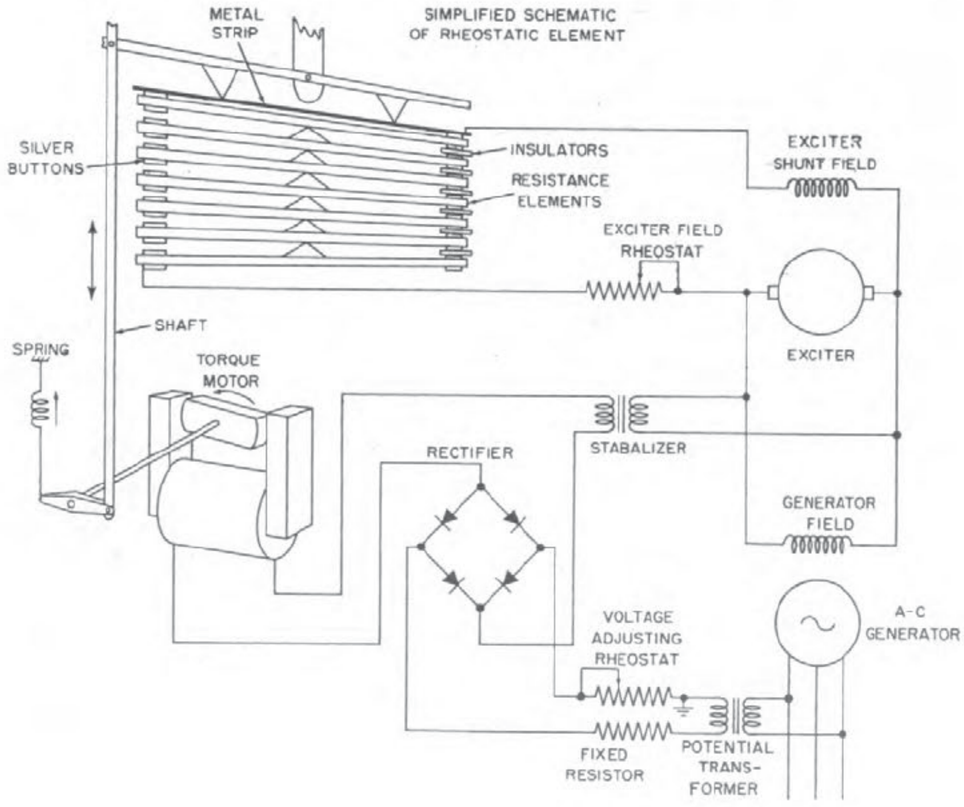


Figure 14–8.—Direct-acting voltage regulator.

d-c output of the exciter. Consequently, the generator field flux is increased, with the result that the a-c generated voltage increases and prevents the terminal voltage from decreasing further.

If the alternating output voltage of the generator increases above normal, the voltage supplied to the torque motor is increased, and the armature will turn counterclockwise against the pull of the spring. The resistance of the rheostatic element is increased, and less current is fed to the exciter shunt field. The decrease in current through the exciter field reduces the d-c output of the exciter. Consequently, the generator field flux is reduced, with the result that the alternating generated voltage decreases and prevents any further increase in the output voltage.

Another type of direct-acting voltage regulator, the SILVERSTAT, manufactured by the Westinghouse Electric Corporation, regulates the current through the exciter shunt field (and thus controls the output alternating voltage) by means of a different type of variable resistor. A multitapped resistor is used in place of the rheostatic element. The taps on the resistor are connected to silver contacts by means of flat single-leaf spring conductors, which are insulated from each other. These contracts are arranged in a row so that as the pressure is increased (due to increased pull of the spring) on one end of the row, the number of contacts closed is increased, thus shorting out more of the resistance. If the a-c output voltage rises, more force is exerted by the torque element and the pull of the spring is reduced. The contacts separate and more resistance is added in the exciter shunt-field circuit. This action reduces the current in the shunt field, thereby checking the rise in alternating voltage. The reverse of this action also occurs when the a-c output voltage falls below a certain value.

The indirect-acting-type regulator is used to regulate the output of a-c generators, the exciters of which are too large to be controlled by direct-acting regulators. In this type of regulator a pilot motor is used to operate the exciter field rheostat. The motor is actuated by voltage-sensitive contacts when the change in output voltage is small. When there is a large change in voltage, quick-response contacts either introduce or short out a block of resistance in the exciter field circuit until the motor-controlled rheostat can again take control.

Detailed explanations of the last two types of regulators are included in the Electrician's Mate (EM) series.

Parallel Operation

Alternators are connected in parallel to (1) increase the plant capacity beyond that of a single unit, (2) serve as additional reserve power for expected demands, or (3) permit shutting down one machine and cutting in a standby machine without interrupting the power supply.

The following conditions must be obtained to parallel two alternators:

1. The terminal voltage of both alternators must be the same. This condition is accomplished by varying the voltage of the incoming alternator by means of the field rheostat.
2. The frequency of both alternators must be the same. This condition is accomplished by adjustment of the speed of the prime mover of the incoming alternator.
3. The voltages of both alternators must be in the proper phase relation with respect to each other. This condition is indicated by means of a synchronizing device (synchroscope) that is connected between the incoming alternator and the bus bars. The synchroscope is described in detail in chapter 16.
4. The voltages of both alternators must have the same phase sequence. This condition is accomplished when the alternator is initially installed by reversing the phase sequence if it is not correct.

TRANSFORMERS

A transformer is a device that has no moving parts and that transfers energy from one circuit to another by electro-magnetic induction. The energy is always transferred without a change in frequency, but usually with changes in voltage and current. A step-up transformer receives electrical energy at one voltage and delivers it at a higher voltage. Conversely, a step-down transformer receives energy at one voltage and delivers it at a lower voltage. Transformers require little care and maintenance because of their simple, rugged, and durable construction. The efficiency of transformers is high, and it is probably because of this fact that transformers are responsible for the more extensive use of alternating current than direct current. The conventional constant-potential transformer is designed to operate with the primary connected across a constant-potential source and to

provide a secondary voltage that is substantially constant from no load to full load.

Various types of small single-phase transformers are used as component parts of shipboard equipment. In many installations transformers are used on switchboards to step-down the voltage for indicating lights. Low-voltage transformers are included in some motor control panels to supply control circuits or to operate overload relays. Other common uses include low-voltage supply for gun-firing circuits, special signal lights, and high-voltage ignition circuits for automatic oil burners.

Instrument transformers include POTENTIAL, or VOLTAGE, TRANSFORMERS and CURRENT TRANSFORMERS. Instrument transformers are commonly used with a-c instruments when high voltages or large currents are involved. These transformers are described in chapter 16.

Electronic circuits and devices employ many types of transformers to provide necessary voltages for proper electron-tube operation, interstage coupling, signal amplification, and so forth. The physical construction of these transformers differs widely.

The POWER-SUPPLY TRANSFORMER used in electronic circuits is a single-phase constant-potential transformer with one or more secondary windings, or a single secondary with several tap connections. These transformers have a low volt-ampere capacity and are less efficient than large constant-potential power transformers. Most power-supply transformers for electronic equipments aboard ship are designed to operate at a frequency of 50 to 60 cycles per second. Aircraft power-supply transformers are designed for a frequency range of from 400 or 1,600 cycles per second. The higher frequencies permit a saving in size and weight of transformers and associated equipment. Power-supply transformers, audio-frequency transformers, radio-frequency transformers, and other types associated with electron-tube circuits are described in the training course, *Basic Electronics,* NavPers 10087.

Construction

The ordinary transformer has two windings insulated electrically from each other. These windings are wound on a common magnetic circuit made of laminated sheet steel. The principal parts are: (1) The CORE, which provides a circuit of low reluctance for the magnetic flux; (2) the PRIMARY WINDING, which receives the energy from the a-c source; (3) the SECONDARY WIND-

ING, which receives the energy by mutual induction from the primary and delivers it to the load, and (4) the ENCLOSURE.

When a transformer is used to step up the voltage, the low-voltage winding is the primary. Conversely, when a transformer is used to step down the voltage, the high-voltage winding is the primary. It is common practice to refer to the windings as the primary and secondary rather than the high-voltage and low-voltage windings.

The principal types of transformer construction are the core-type and the shell-type, as illustrated respectively in figure 14–9, A and B. The cores are built of thin stampings of silicon steel.

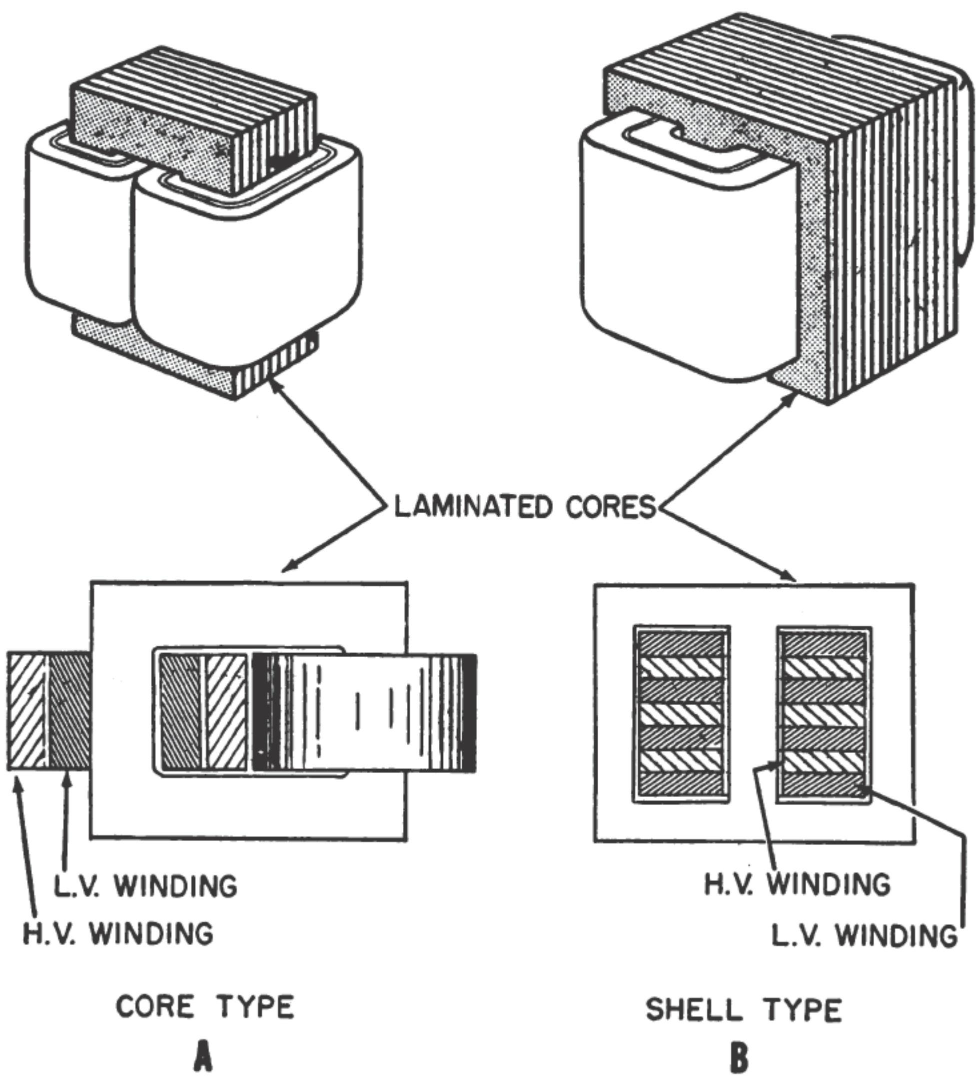


Figure 14–9.—Transformer types.

Eddy currents, generated in the core by the alternating flux as it cuts through the iron, are minimized by using thin laminations and by insulating adjacent laminations with insulating varnish. Hysteresis losses, caused by the friction developed between magnetic particles as they are rotated through each cycle of magnetization, are minimized by using a special grade of heat-treated grain-oriented silicon-steel laminations.

In the core-type transformer, the copper windings surround the laminated iron core. In the shell-type transformer the iron core surrounds the copper windings. Distribution transformers are generally of the core type; whereas some of the largest power transformers are of the shell type.

If the windings of a core-type transformer were placed on separate legs of the core, a relatively large amount of the flux produced by the primary winding would fail to link the secondary winding and a large leakage flux would result. The effect of the leakage flux would be to increase the leakage reactance drop, IX_L, in both windings. To reduce the leakage flux and reactance drop, the windings are subdivided and half of each winding is placed on each leg of the core. The windings may be cylindrical in form and placed one inside the other with the necessary insulation, as shown in figure 14–9, A. The low-voltage winding is placed with a large part of its surface area next to the core, and the high-voltage winding is placed outside the low-voltage winding in order to reduce the insulation requirements of the two windings. If the high-voltage winding were placed next to the core, two layers of high-voltage insulation would be required, one next to the core and the other between the two windings.

In another method, the windings are built up in thin flat sections called pancake coils. These pancake coils are sandwiched together, with the required insulation between them, as shown in figure 14–9, B.

The complete core and coil assembly (fig. 14–10, A) are placed in a steel tank. In commercial transformers the complete assembly is usually immersed in a special mineral oil to provide a means of insulation and cooling. No oil is used in the transformer enclosures on naval vessels. Transformer banks installed in these vessels normally consist of a single-phase air-cooled transformer mounted in drip-proof enclosure for each phase used, as shown in figure 14–10, B. The primary and secondary windings are delta-connected. These transformer banks provide 3-phase 117-volt

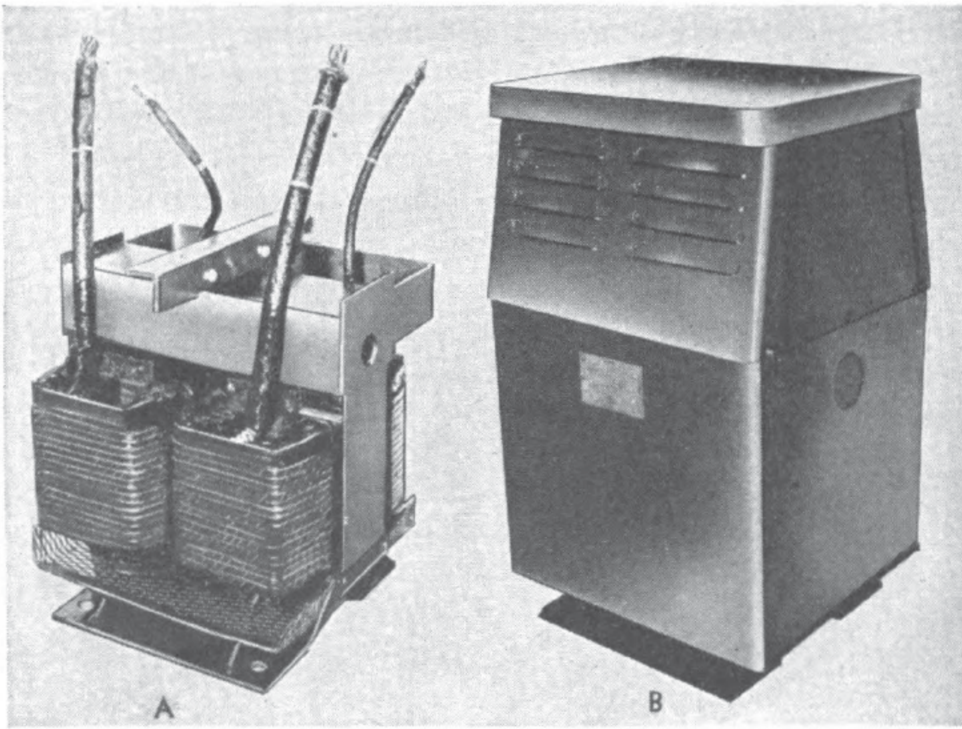


Figure 14—10.—Single-phase transformer construction.

service for lighting circuits and for IC (interior-communications) and fire-control circuits.

Transformers are built in both single-phase and polyphase units. A three-phase transformer consists of separate insulated windings for the different phases, wound on a 3-legged core capable of establishing three magnetic fluxes displaced 120° in time phase. Three-phase transformers are not installed aboard ship because the loss of a composite unit would result in an interruption to service. To ensure continuity of service, three separate single-phase delta-connected transformers are installed to supply low-voltage service. If one transformer is damaged in such a system, it can be removed and the remaining two transformers can be operated in the open-delta to supply the 3-phase service, but at a reduction in the original load capacity of the transformer bank. Because of the saving in space and weight, three-phase transformers are often used in high-power airborne radar sets, and these are sometimes immersed in oil.

Voltage and Current Relations

The operation of the transformer is based on the principle that electrical energy can be transferred efficiently by mutual induction

from one winding to another. When the primary winding is energized from an a-c source, an alternating magnetic flux is established in the transformer core. This flux links the turns of both primary and secondary, thereby inducing voltages in them. Because the same flux cuts both windings, the same voltage is induced in each turn of both windings. Hence, the total induced

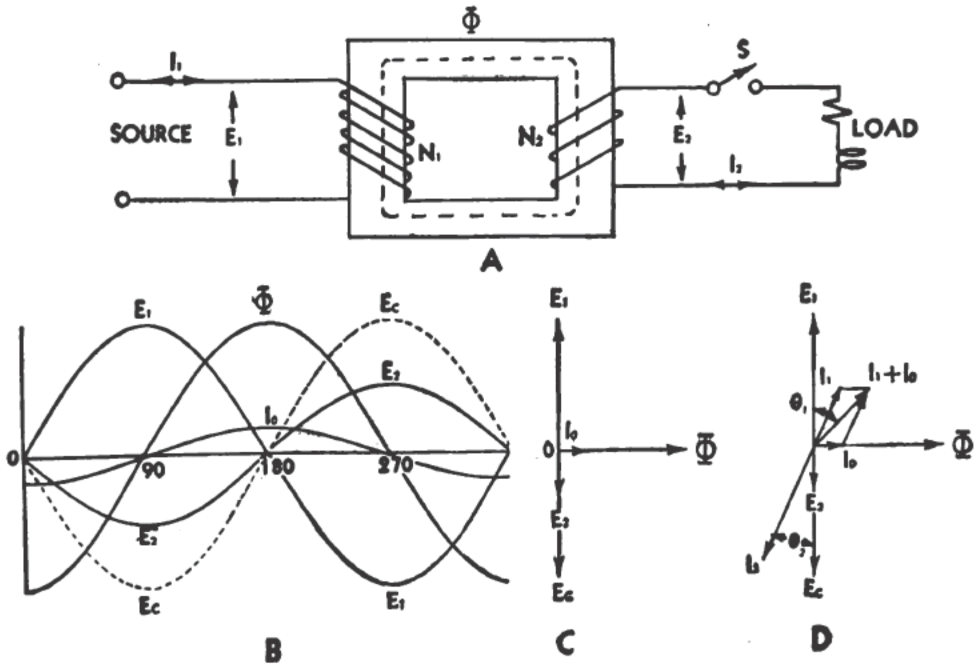


Figure 14–11.—Constant-potential transformer.

voltage in each winding is proportional to the number of turns in that winding. That is,

$$\frac{E_1}{E_2}=\frac{N_1}{N_2},$$

where E_1 and E_2 are the induced voltages in the primary and secondary windings, respectively, and N_1 and N_2 are the number of turns in the primary and secondary windings, respectively. In ordinary transformers the induced primary voltage is almost equal to the applied primary voltage; hence, the applied primary voltage and the secondary induced voltage are approximately proportional to the respective number of turns in the two windings.

A constant-potential single-phase transformer is represented by the schematic diagram in figure 14–11, A. For simplicity, the

primary winding is shown as being on one leg of the core and the secondary winding on the other leg. The equation for the voltage induced in one winding of the transformer is

$$E=\frac{4.44\ BSfN}{10^8},$$

where E is the rms voltage, B the maximum value of the magnetic flux density in lines per square inch in the core, S the cross-sectional area of the core in square inches, f the frequency in cycles per second, and N the number of complete turns in the winding.

For example, if the maximum flux density is 90,000 lines per square inch, the cross-sectional area of the core is 4.18 square inches, the frequency is 60 cycles per second, and the number of turns in the high-voltage winding is 1,100, the voltage rating of this winding is

$$E_1=\frac{4.44\times 90{,}000\times 4.18\times 60\times 1{,}100}{10^8}=1{,}100 \text{ volts.}$$

If the primary-to-secondary turns ratio of this transformer is 10 to 1, the number of turns in the low-voltage winding will be

$$\frac{1{,}100}{10}=110 \text{ turns,}$$

and the voltage induced in the secondary will be

$$E_2=\frac{1{,}100}{10}=110 \text{ volts.}$$

The waveforms of the ideal transformer with no load are shown in figure 14–11, B. When E_1 is applied to the primary winding, N_1, with the switch, S, open, the resulting current, I_0, is small and lags E_1 by almost 90° because the circuit is highly inductive. This no-load current is called the EXCITING, or MAGNETIZING, CURRENT because it supplies the magnetomotive force that produces the transformer core flux, Φ. The flux produced by I_0 cuts the primary winding, N_1, and induces a counter voltage E_c 180° out of phase with E_1 in this winding. The voltage, E_2, induced in the secondary winding is in phase with the induced (counter) voltage, E_c, in the primary winding, and both lag the exciting current and flux, whose variation produce them, by an angle of 90°. These

relations are shown in vector form in figure 14–11, C. The values are only approximate and are not drawn exactly to scale.

When a load is connected to the secondary by closing switch S (fig. 14–11, A), the secondary current I_2, depends upon the magnitude of the secondary voltage, E_2, and the load impedance, Z. For example, if E_2 is equal to 110 volts and the load impedance is 22 ohms, the secondary current will be

$$I_2=\frac{E_2}{Z_2}=\frac{110}{22}=5 \text{ amperes.}$$

If the secondary power factor is 86.6 percent, the phase angle, θ_2, between secondary current and voltage will be the angle whose cosine is 0.866, or 30°.

The secondary load current flowing through the secondary turns comprises a load component of magnetomotive force, which according to Lenz's law is in such a direction as to oppose the flux which is producing it. This opposition tends to reduce the transformer flux a slight amount, like the reduction in speed of a shunt motor with added load. The reduction in flux is accompanied by a reduction in the counter voltage induced in the primary winding of the transformer. Because the internal impedance of the primary winding is low and the primary current is limited principally by the counter emf in the winding (just as the counter voltage in a shunt motor armature limits the load current), the transformer primary current increases when the counter emf in the primary is reduced.

The increase in primary current continues until the primary ampere turns are equal to the secondary ampere turns, neglecting losses. For example, in the transformer being considered, the magnetizing current, I_0, is assumed to be negligible in comparison with the total primary current, I_1+I_0, under load conditions because I_0 is small in relation to I_1 and lags it by an angle of 60°. Hence, the primary and secondary ampere turns are equal and opposite. That is,

$$N_1I_1=N_2I_2.$$

In this example,

$$I_1=\frac{N_2}{N_1}I_2=\frac{110}{1,100}\times 5=0.5 \text{ ampere.}$$

Neglecting losses, the power delivered to the primary is equal to the power supplied by the secondary to the load. If the load

power is $P_2 = E_2 I_2 \cos \theta_2$, or 110×5 ($\cos 30 = 0.866$) $= 476$ watts, the power supplied to the primary is approximately $P_1 = E_1 I_1 \cos \theta$, or 110×0.5 ($\cos 30° = 0.866$) $= 476$ watts.

The load component of primary current, I_1, increases with secondary load and maintains the transformer core flux at nearly its initial value. This action enables the transformer primary to take power from the source in proportion to the load demand, and to maintain the terminal voltage approximately constant in a manner similar to the speed regulation of a shunt motor as load is added to it. The lagging-power-factor load vectors are shown in figure 14–11, D. Note that the load power factor is transferred through the transformer to the primary and that θ_2 is approximately equal to θ_1, the only difference being that θ_1 is slightly larger than θ_2 because of the presence of the exciting current which flows in the primary winding but not in the secondary.

The copper loss of a transformer varies as the square of the load current; whereas the core loss depends on the terminal voltage applied to the primary and on the frequency of operation. The core loss of a constant-potential transformer is constant from no-load to full-load because the frequency is constant and the effective values of the applied voltage, exciting current, and flux density are constant.

If the load supplied by a transformer has unity power factor, the kilowatt (true power) output is the same as the kilovolt ampere (apparent power) output. If the load has a lagging power factor, the kilowatt output is proportionally less than the kilovolt ampere output. For example, a transformer having a full-load rating of 100 kva can supply a 100-kw load at unity power factor, but only an 80-kw load at a lagging power factor of 80 percent.

Navy shipboard transformers are generally rated in terms of the kva load that they can safely carry continuously without exceeding a temperature rise of 80° C. when maintaining rated secondary voltage at rated frequency and when operating with an ambient (surrounding atmosphere) temperature of 40° C. The actual temperature RISE of any part of the transformer is the difference between the total temperature of that part and the ambient temperature.

It is possible to operate transformers on a higher frequency than that for which they are designed, but it is not permissible to

operate them at more than 10 percent below their rated frequency, because of the resulting overheating. The exciting current in the primary varies directly with the applied voltage and, like any impedance containing inductive reactance, the exciting current varies inversely with the frequency. Thus, at reduced frequency, the exciting current becomes excessively large and the accompanying heating may damage the insulation and the windings.

Efficiency

The efficiency of a transformer is the ratio of the output power at the secondary terminals to the input power at the primary terminals. It is also equal to the ratio of the output to the output plus losses. That is,

$$\text{efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{copper loss} + \text{core loss}}.$$

The ordinary power transformer has an efficiency of 97 to 99 percent. The losses are due to the copper losses in both windings and the hysteresis and eddy-current losses in the iron core.

The copper losses vary as the square of the current in the windings and as the winding resistance. In the transformer being considered, if the primary has 1,100 turns of number 23 copper wire, having a length of 1,320 feet, the resistance of the primary winding is 26.9 ohms. If the load current in the primary is 0.5 ampere, the primary copper loss is $(0.5)^2 \times 26.9 = 6.725$ watts. Similarly, if the secondary winding contains 110 turns of number 13 copper wire having a length of approximately 132 feet, the secondary resistance will be 0.269 ohm. The secondary copper loss is $I_2^2 R_2$, or $(5)^2 \times 0.269 = 6.725$ watts, and the total copper loss is $6.725 \times 2 = 13.45$ watts.

The core losses, consisting of the hysteresis and eddy-current losses, caused by the alternating magnetic flux in the core are approximately constant from no load to full load, with rated voltage applied to the primary.

In the transformer of figure 14–11, A, if the core loss is 10.6 watts and the copper loss is 13.4 watts, the efficiency is

$$\frac{\text{output}}{\text{output} + \text{copper loss} + \text{core loss}} = \frac{476}{476 + 13.4 + 10.6} = \frac{476}{500} = 0.952,$$

or 95.2 percent. The kva rating of the transformer is

$$\frac{E_1 I_1}{1{,}000}=\frac{1{,}100\times 0.5}{1{,}000}=0.55 \text{ kva.}$$

The efficiency of this transformer is relatively low because it is a small transformer and the losses are disproportionately large.

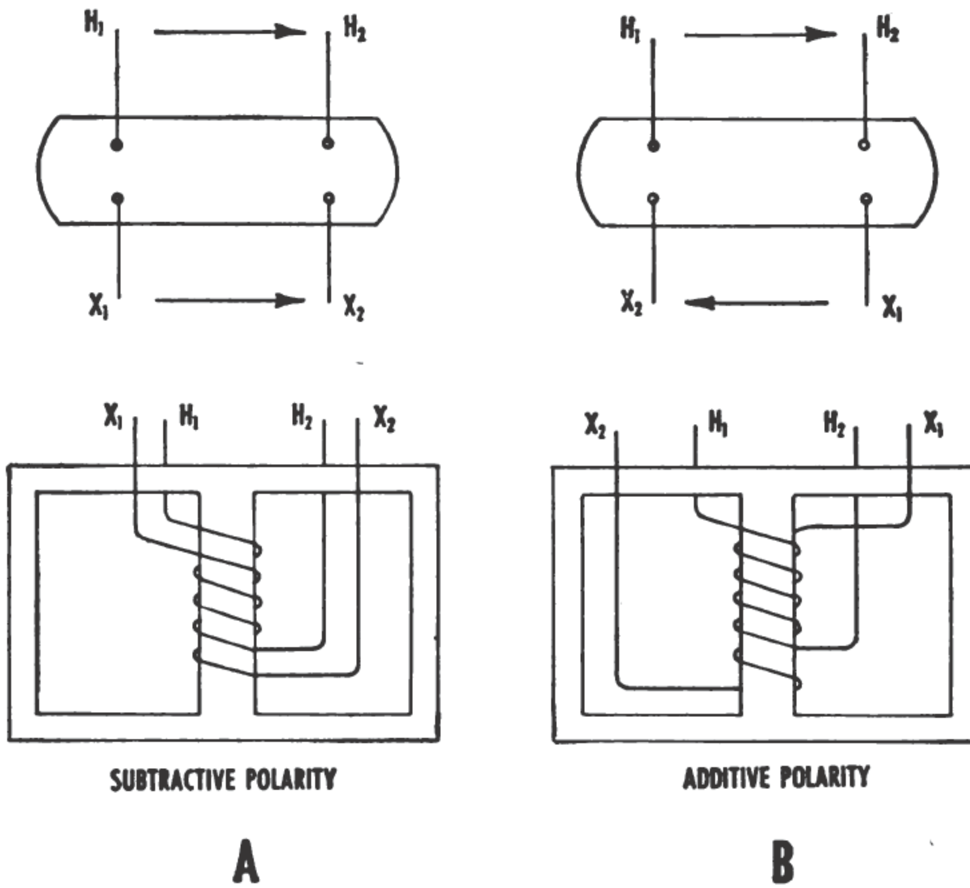


Figure 14–12.—Transformer polarity markings.

Polarity

Standard markings have been adopted for transformer terminals to ensure that the instantaneous polarity of the induced voltages of the windings will be known. This knowledge is necessary in paralleling transformers or in connecting them in wye or delta in 3-phase circuits and in properly metering power, current, and voltage. These markings are called POLARITY MARKINGS. The terminals for the high-voltage, or *H*, winding are marked H_1, H_2, H_3, and

so forth. Facing the high voltage side of the transformer, the extreme right-hand lead is always marked H_1. The lead marked H_2 is brought out next, H_3 next, and so on. The increasing numerical subscript designates an increasing voltage. Thus, the voltage between H_1 and H_3 is higher than the voltage between H_1 and H_2.

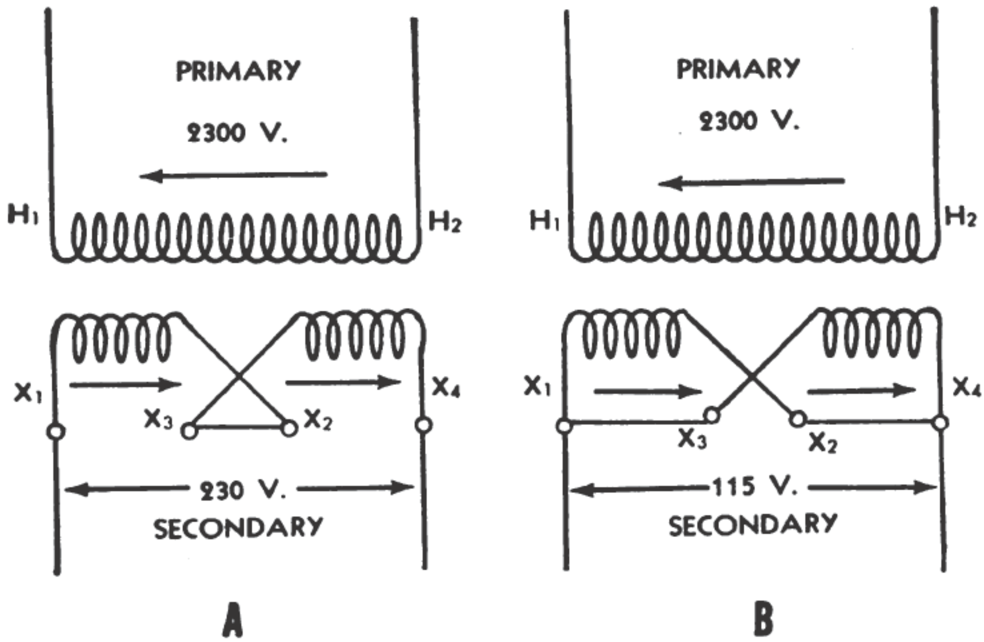


Figure 14–13.—Transformer secondary connections.

The secondary terminals are marked X_1, X_2, X_3, and so forth. There are two types of markings that may be employed on the secondaries. These are shown in figure 14–12. When the H_1 and X_1 leads are brought out on the same side of the transformer (fig. 14–12, A) the polarity is called SUBTRACTIVE. The reason this arrangement is called subtractive is as follows: If the H_1 and X_1 leads are connected together and a reduced voltage is applied across the H_1 and H_2 leads, the resultant voltage which appears across the H_2 and X_2 leads in the series circuit formed by this connection will equal the difference in the voltages of the two windings. The voltage of the low-voltage winding opposes that of the high-voltage winding and subtracts from it, hence the term "subtractive polarity."

When the H_1 and X_1 leads are brought out on opposite corners of the transformer (fig. 14–12, B), the polarity is ADDITIVE. Navy

shipboard transformers have additive polarity. If the H_1 and X_2 leads are connected together and a reduced voltage is applied across the H_1 and H_2 leads, the resultant voltage across the H_2 and X_1 leads in the series circuit formed by this connection will equal the sum of the voltages of the two windings. The voltage of the low-voltage winding aids the voltage of the high-voltage winding and adds to it, hence the term "additive polarity."

Polarity markings do not indicate the internal voltage stress in the windings but are useful only in making external connections between transformers, as previously mentioned.

Single-Phase Connections

Single-phase distribution transformers usually have their windings divided into two or more sections, as shown in figure 14–13. When the two secondary windings are connected in series (fig. 14–13, A), their voltages add. In figure 14–13, B, the two secondary windings are connected in parallel, and their currents add. For example, if each secondary winding is rated at 115 volts and 100 amperes, the series-connection output rating will be 230 volts at 100 amperes, or 23 kva; the parallel-connection output rating will be 115 volts at 200 amperes, or 23 kva.

In the series connection, care must be taken to connect the coils so that their voltages add. The proper arrangement is indicated in the figure. A trace made through the secondary circuits from X_1 to X_4 is in the same direction as that of the arrows representing the maximum positive voltages.

In the parallel connection, care must be taken to connect the coils so that their voltages are in opposition. The correct connection is indicated in the figure. The direction of a trace made through the secondary windings from X_1 to X_2 to X_4 to X_3 and returning to X_1 is the same as that of the arrow in the right-hand winding. This condition indicates that the secondary voltages have their positive maximum values in directions opposite to each other in the closed circuit, which is formed by paralleling the two secondary windings. Thus, no circulating current will flow in these windings on no load. If either winding were reversed, a short-circuit current would flow in the secondary, and this would cause the primary to draw a short-circuit current from the source.

This action would, of course, damage the transformer as well as the source.

Three-Phase Connections

Power may be supplied through 3-phase circuits containing transformers in which the primaries and secondaries are connected in various wye and delta combinations. For example, three single-phase transformers may supply 3-phase power with four possible combinations of their primaries and secondaries. These connec-

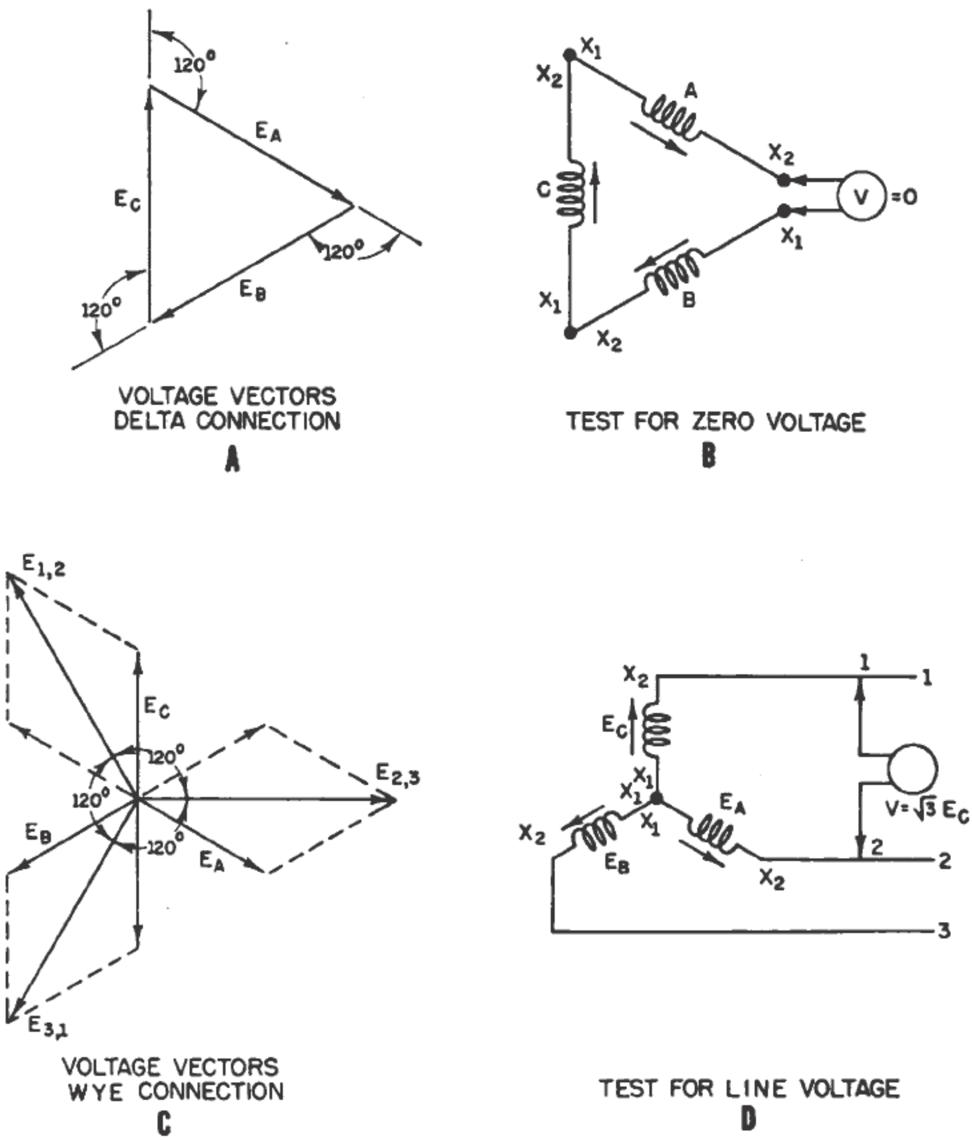


Figure 14–14.—Delta-connected secondaries.

tions are: (1) primaries in delta and secondaries in delta, (2) primaries in wye and secondaries in wye, (3) primaries in wye and secondaries in delta, (4) primaries in delta and secondaries in wye.

Delta and wye connections were described earlier in this chapter under the subject of 3-phase alternators, and the relations between line and phase voltages and currents were described under three-phase circuits in chapter 13. These relations apply to transformers as well as to alternators and to 3-phase motors. Three-phase motors are described in chapter 15.

If the primaries of three single-phase transformers are properly connected (either in wye or delta) to a 3-phase source, the secondaries may be connected in delta, as shown in figure 14–14. A topographic vector diagram of the 3-phase secondary voltages is shown in figure 14–14, A. The vector sum of these three voltages is zero. This may be seen by combining any two vectors, for example, E_A and E_B, and noting that their sum is equal and opposite to the 3rd vector, E_C. A voltmeter inserted within the delta will indicate zero voltage, as shown in figure 14–14, B, when the windings are connected properly.

Assuming all three transformers have the same polarity, the delta connection consists in connecting the X_2 lead of winding A to the X_1 lead of B, the X_2 lead of B to X_1 of C, and the X_2 lead of C to X_1 of A. If any one of the three windings is reversed with respect to the other two windings, the total voltage within the delta will equal twice the value of one phase; and if the delta is closed on itself, the resulting current will be of short-circuit magnitude, with resulting damage to the transformer windings and cores. The delta should never be closed until a test is first made to determine that the voltage within the delta is zero or nearly zero. This may be done with a voltmeter, fuse wire, or test lamp. In the figure, when the voltmeter is inserted between the X_2 lead of A and the X_1 lead of B, the delta circuit is completed through the voltmeter, and the indication should be approximately zero. Then the delta is completed by connecting the X_2 lead of A to the X_1 lead of B.

If the three secondaries of a live transformer bank are properly connected in delta and are supplying a balanced 3-phase load, the line current will be equal to 1.73 times the phase current. If the rated current of a phase (winding) is 100 amperes, the

rated line current will be 173 amperes. If the rated voltage of a phase is 120 volts, the voltage between any two line wires will be 120 volts.

The three secondaries of the live transformer bank may be reconnected in wye in order to increase the output voltage. The voltage vectors are shown in figure 14–14, C. If the phase voltage is 120 volts, the line voltage will be $1.73 \times 120 = 208$ volts. The line voltages are represented by vectors, $E_{1,2}$, $E_{2,3}$, and $E_{3,1}$. A voltmeter test for the line voltage is represented in figure 14–14, D. If the three transformers have the same polarity, the proper connections for a wye-connected secondary bank are indicated in the figure. The X_1 leads are connected together to form a common or neutral connection and the X_2 leads of the three secondaries are brought out to the line leads. If the connections of any one winding are reversed, the voltages between the 3 line wires will become unbalanced, and the loads will not receive their proper magnitude of load current. Also the phase angle between the line currents will be changed, and they will no longer be 120° out of phase with each other. Therefore, it is important to properly connect the transformer secondaries in order to preserve the symmetry of the line voltages and currents.

Transformer installations aboard naval vessels usually consist of 450/117-volt delta-connected banks to supply the general lighting circuits and service for the interior-communications and fire-control circuits. Some ships also include 450/230-volt delta-connected banks to supply the galley equipment. Power is distributed over an ungrounded 3-wire, 3-phase, 450-volt system.

Three single-phase transformers with both primary and secondary windings delta-connected are shown in figure 14–15. The

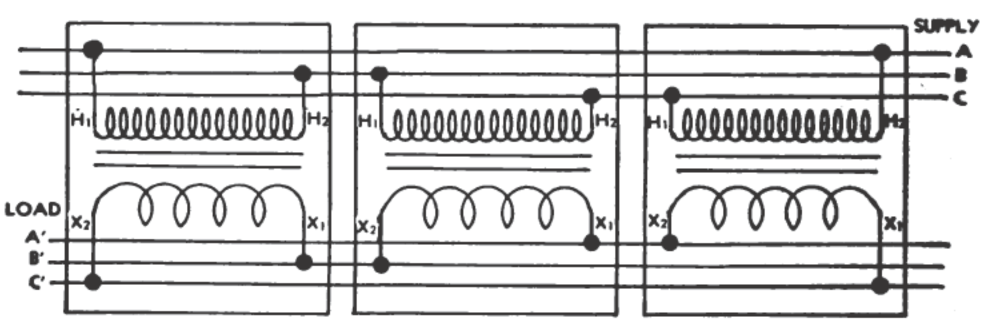


Figure 14–15.—Delta-delta transformer connections.

H_1 lead of one phase is always connected to the H_2 lead of an adjacent phase; the X_1 lead is connected to the X_2 terminal of the corresponding adjacent phase, and so on; and the line connections are made at these junctions. This arrangement is based on the assumption that the three transformers have the same polarity.

An open-delta connection results when any one of the three transformers is removed from the delta-connected transformer bank without disturbing the 3-wire 3-phase connections to the

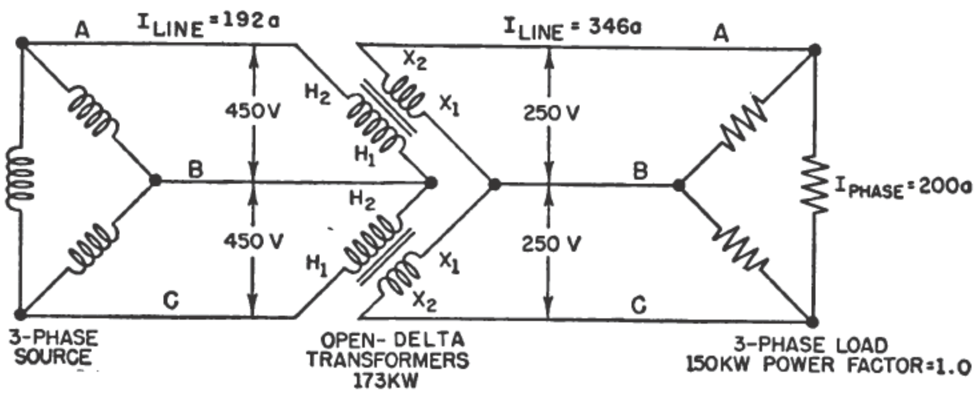


Figure 14–16.—Open-delta transformer connections.

remaining two transformers. These transformers will maintain the correct voltage and phase relations on the secondary to supply a balanced 3-phase load. An open-delta connection is shown in figure 14–16. The 3-phase source supplies the primaries of the two transformers, and the secondaries supply a 3-phase voltage to the load. The LINE CURRENT is equal to the transformer PHASE CURRENT in the open-delta connection. In the closed delta connection, the transformer phase current, $I_{\text{phase}}=\frac{I_{\text{line}}}{\sqrt{3}}$. Thus, when one transformer is removed from a delta-connected bank of three transformers, the remaining two transformers will carry a current equal to $\sqrt{3}\ I_{\text{phase}}$. This value amounts to an overload current on each transformer of 1.73 times the rated current, or an overload of 73.2 percent.

Thus, in an open-delta connection, the line current must be reduced so as not to exceed the rated current of the individual transformers if they are not to be overloaded. The open-delta

connection therefore results in a reduction in system capacity. The full-load capacity in a delta connection at unity power factor is

$$P_{\Delta}=3I_{\text{phase}}E_{\text{phase}}=\sqrt{3}E_{\text{line}}I_{\text{line}}.$$

In an open-delta connection, the line current is limited to the rated phase current of $\frac{I_{\text{line}}}{\sqrt{3}}$, and the full-load capacity of the open-delta, or V-connected, system is

$$P_v=\sqrt{3}E_{\text{line}}\frac{I_{\text{line}}}{\sqrt{3}}=E_{\text{line}}I_{\text{line}}.$$

The ratio of the load that can be carried by two transformers connected in open delta to the load that can be carried by three transformers in closed delta is

$$\frac{P_v}{P_{\Delta}}=\frac{E_{\text{line}}I_{\text{line}}}{\sqrt{3}E_{\text{line}}I_{\text{line}}}=\frac{1}{\sqrt{3}}=0.577, \text{ or } 57.7 \text{ percent}$$

of the closed-delta rating.

For example, a 150-kw 3-phase balanced load operating at unity power factor is supplied at 250 volts. The rating of each of three transformers in closed delta is $\frac{150}{3}=50$ kw, and the phase current is $\frac{50{,}000}{250}=200$ amperes. The line current is $200\sqrt{3}=346$ amperes. If one transformer is removed from the bank, the remaining two transformers would be overloaded $346-200=146$ amperes, or $\frac{146}{200}\times 100=73$ percent. To prevent overload on the remaining two transformers, the line current must be reduced from 346 amperes to 200 amperes and the total load reduced to

$$\frac{\sqrt{3}\times 250\times 200}{1{,}000}=86.6 \text{ kw},$$

or

$$\frac{86.6}{150}\times 100=57.7 \text{ percent}$$

of the original load.

The rating of each transformer in open delta necessary to supply the original 150-kw load is $\frac{E_{\text{phase}} I_{\text{phase}}}{1{,}000}$, or $\frac{250 \times 346}{1{,}000} = 86.6$ kw; and two transformers require a total rating of $2 \times 86.6 = 173.2$ kw, compared with 150 kw for three transformers in closed delta. The required increase in transformer capacity is $173.2 - 150 = 23.2$ kw, or $\frac{23.2}{150} \times 100 = 15.5$ percent, when two transformers are used in open delta to supply the same load as three 50-kw transformers in closed delta.

Three single-phase transformers with both primary and secondary windings wye-connected are shown in figure 14–17. Only 57.7 percent of the line voltage $\left(\frac{E_{\text{line}}}{\sqrt{3}}\right)$ is impressed across each winding, but full-line current flows in each transformer winding. Although the wye-wye connection is never used aboard ship, it is frequently used in Navy shore installations. An objection to the wye-wye connection is that the secondary coil voltages contain frequency components that create serious disturbances in communications circuits in the immediate vicinity. However, these harmonics are eliminated if the transformers have a third, or tertiary, winding that is delta connected.

Three single-phase transformers delta-connected to the primary circuit and wye-connected to the secondary circuit are shown in

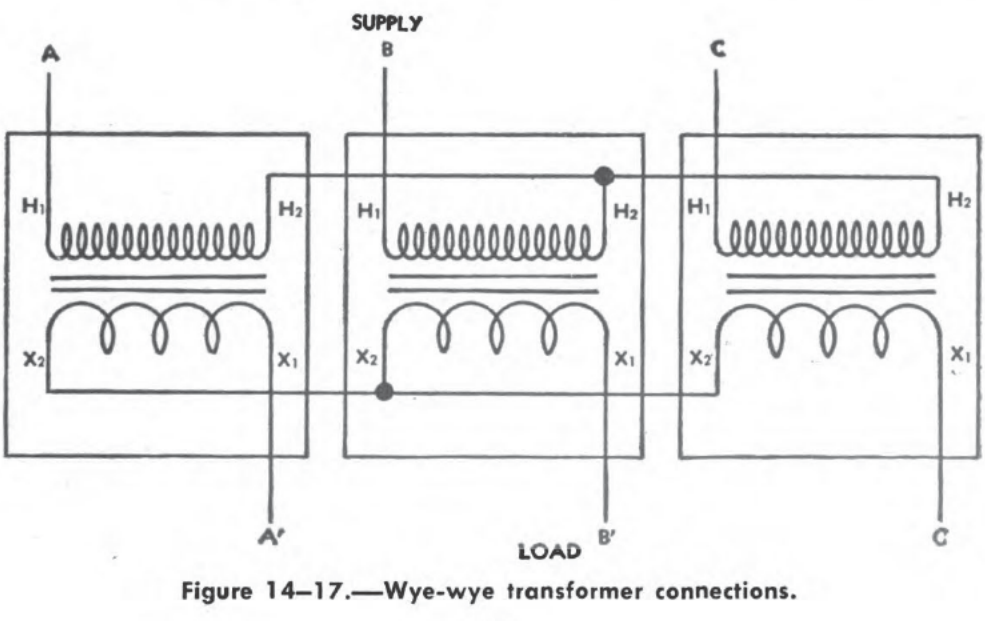


Figure 14–17.—Wye-wye transformer connections.

figure 14–18. This connection provides 4-wire, 3-phase service with 208 volts between line wires $A'B'C'$, and $\frac{208}{\sqrt{3}}$, or 120 volts, between each line wire and neutral N. The wye-connected secondary is desirable in shore installations when a large number of single-phase loads are to be supplied from a 3-phase transformer bank. The neutral, or grounded, wire is brought out from the midpoint of the wye-connection, permitting the single-phase loads to be distributed evenly across the three phases. At the same time, 3-phase motors can be connected directly across the line wires. The single-phase loads have a voltage rating of 120 volts, and the motors are rated at 208 volts. This connection is often used in high-voltage plate-supply transformers in aircraft radar power supplies. The phase voltage is $\frac{1}{1.73}$, or 0.577 of the line voltage.

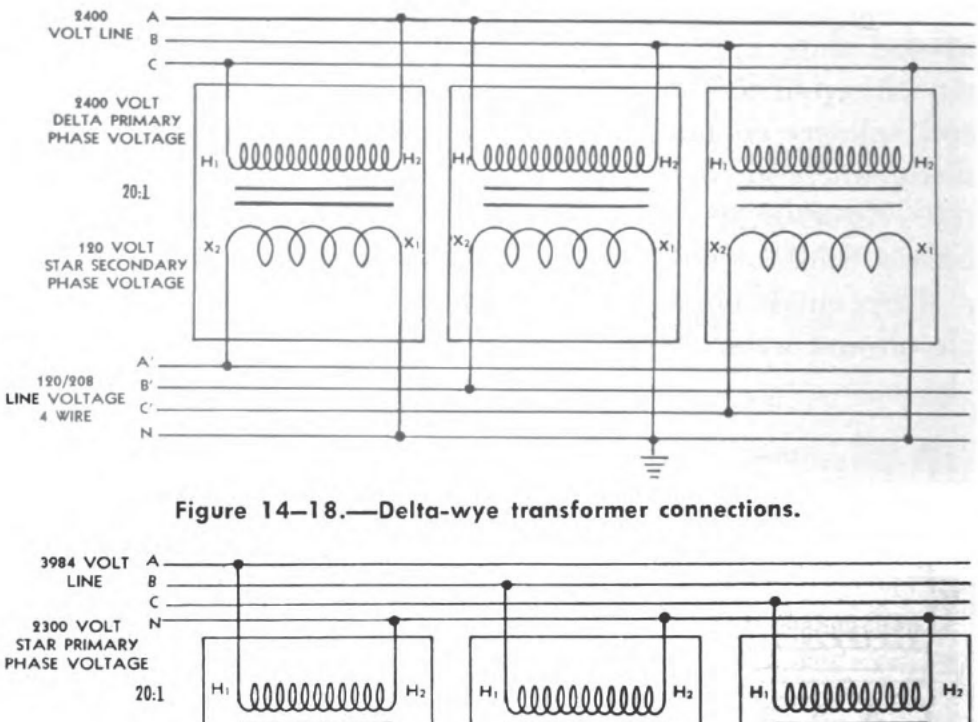


Figure 14–18.—Delta-wye transformer connections.

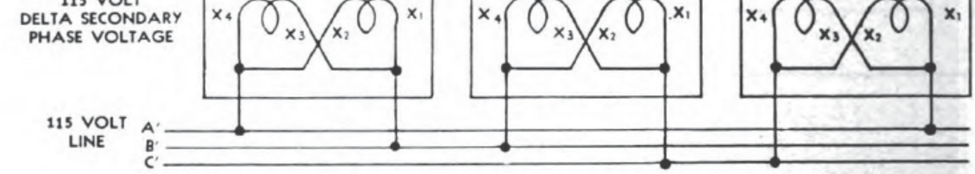


Figure 14–19.—Wye-delta transformer connection.

Three single-phase transformers with wye-connected primaries and delta-connected secondaries are shown in figure 14–19. This arrangement is used for stepping down the voltage from approximately 4,000 volts between line wires on the primary side to either 115 volts or 230 volts, depending upon whether the secondary windings of each transformer are connected in parallel or in series. In the figure, the two secondaries of each transformer are connected in parallel, and the secondary output voltage is 115 volts. There is an economy in transmission with the primaries in wye because the line voltage is 73 percent higher than the phase voltage, and the line current is accordingly less. Thus, the line losses are reduced and the efficiency of transmission is improved.

QUIZ

1. With regard to the field and armature of most a-c generators, what part rotates and what part is stationary?
2. Alternators are rated in terms of what electrical quantities?
3. In figure 14–5, B, what is the magnitude of the voltage between the two outside terminals of the 2-phase, 3-wire alternator, if the phase voltage is 120 volts?
4. In figure 14–6, B, what is the magnitude of the voltage between any two terminals of the 3-phase wye-connected alternator if the phase voltage is 120 volts?
5. What is the frequency in cycles per second of a 4-pole alternator rotating at 12,000 rpm?
6. What is the effective voltage per phase of a 2-phase, 50-cycle alternator that has 100 conductors in series per phase and a field pole flux of 2.27×10^6 lines per pole? The breadth factor is 0.924 and the pitch factor is 0.946.
7. What three factors contribute to terminal voltage change with load change on an alternator?
8. What change in field current is required in an alternator to maintain a constant terminal voltage with (a) increased load when the power factor is lagging and (b) decreased load when the power factor is leading?
9. (a) State the expression for the percent voltage regulation of an alternator in terms of the no-load and full-load voltage.
   (b) If the no-load voltage of an alternator is 490 volts and the full-load voltage is 440 volts, what is the percent voltage regulation?

10. In figure 14–8, how will an increase in load on the alternator affect the (a) torque motor, (b) Diactor resistance, (c) exciter field current, (d) exciter terminal voltage, (e) alternator field current, and (f) alternator generated voltage.
11. Give three reasons for operating a-c generators in parallel.
12. What four conditions must be met when two 3-phase alternators are to be connected in parallel?
13. In a transformer, energy is always transferred without a change in ____________________.
14. What are four principal parts of a transformer?
15. Distinguish between core-type and shell-type transformers.
16. Why are transformer cores laminated?
17. What is the advantage of three separate single-phase transformers connected in delta compared with one 3-phase composite transformer in a common enclosure?
18. An iron-core transformer has 30 turns on the secondary winding and 600 turns on the primary winding. If 120 volts is impressed across the primary winding, what is the approximate voltage across the secondary winding?
19. (a) What is the voltage induced (rating) of the secondary of a transformer if the maximum flux density is 8×10^4 lines per square inch, the cross-sectional area of the core is 5.63 square inches, the frequency is 400 cycles per second, and the number of turns in the secondary is 2,000?
    (b) If the primary-to-secondary-turn ratio is 1 to 133, what voltage should be applied to the primary in order to give rated secondary voltage?
20. A 1.1-kva transformer has a primary voltage of 1,100 volts. If the primary has 1,000 turns and the secondary has 100 turns, find the following:
    (a) Rated primary current.
    (b) Rated secondary current.
    (c) Secondary voltage.
    (d) Primary ampere turns.
21. Where are the losses distributed in a transformer?
22. In what way will a decrease in frequency affect the primary current of a constant-potential transformer?
23. Facing the high-voltage side of a single-phase distribution transformer, (a) how is the right-hand high-voltage lead marked? (b) how is the left-hand high-voltage lead marked? (c) for additive polarity, how is the left-hand low-voltage lead marked? (d) for subtractive polarity, how is the right-hand low-voltage lead marked?

24. (a) In figure 14–13, A, if the rated current of each secondary winding is 6.5 amperes and all other values are as indicated in the figure, what is the kva rating of the transformer?
    (b) In figure 14–13, B, what is the rated output current of the secondary?
25. Three single-phase transformers may supply 3-phase power with what four possible combinations of their primaries and secondaries?
26. In a delta-delta transformer bank, what precaution must be taken before closing the secondary delta?
27. (a) The secondaries of a 3-phase transformer bank are connected in wye to increase the output ________________ of the secondaries.
    (b) The secondaries of a 3-phase transformer bank are connected in delta to increase the output ______________ of the secondaries.
28. A 360-kw, 3-phase balanced load operating at unity power factor is supplied at 120 volts. Find the following values:
    (a) Rating of each transformer in closed delta.
    (b) Secondary phase current in closed delta.
    (c) Secondary line current in closed delta.
    (d) What is the percent overload on the remaining two transformers if one transformer is removed from the line?
    (e) To prevent overload on the open delta, the 3-phase load should not exceed ____ kw.

CHAPTER 15

ALTERNATING-CURRENT MOTORS

INTRODUCTION

Because a large part of the electrical power generated aboard ship, and generated or used at shore stations, is a-c, a large percentage of the motors used by the Navy operate on alternating current.

It was pointed out in the chapter on d-c motors that there are certain advantages in their use. For example, d-c shunt motors are superior to induction motors when variable-speed drive is required. Likewise, d-c series motors are superior to induction motors for electrical traction equipment such as streetcars and electric locomotives.

There are definite advantages, however, in the use of a-c motors other than the fact that a-c power is so widely used by the Navy. In general, a-c motors are less expensive than comparable d-c motors; in many instances brushes and commutators on a-c motors are not used, and therefore sparking at the brushes (which is dangerous in the presence of explosive gas or dust) is avoided; they are reliable; and very little maintenance is needed. Alternating-current motors are well suited for constant-speed applications. However, certain types of a-c motors are manufactured that have, within limits, a variable-speed characteristic. Alternating-current motors are designed to operate on polyphase or single-phase lines and at several voltage ratings.

The subject of alternating-current motors is an extensive one, and in this chapter no attempt is made at an exhaustive treatment of the subject. To prepare the student so that he may more easily understand the operation of 3-phase induction and synchronous motors, the rotating magnetic field is discussed first. The construction, operation, and characteristics of induction and synchronous motors are then treated. The torque developed in the

rotor by the interaction of the rotating magnetic field with the rotor current is also discussed, as well as synchronous speed, slip, and efficiency. Following this, there is a brief discussion of a-c motor starters; and finally, several types of single-phase motors are described. A more detailed treatment will be included as needed in the rating texts.

Rotating Magnetic Field

The operation of an induction or synchronous motor depends on a rotating magnetic field that is established around the air gap (the small gap between the stator and the rotor) of the motor by currents flowing in the stator winding. The stator (fig. 15–1, A) of an induction motor is similar electrically to the stator of a rotating-field alternator (fig. 14–1, A). Both contain the armature windings. A simplified wye-connected stator winding diagram of a 2-pole 3-phase motor is shown in figure 15–1, B. In a 3-phase winding the rotating field is produced by the interaction of three groups of coils—one group in each phase.

The 3-phase currents differ in time phase, as shown in figure 15–1, C, and a magnetic field is established which moves around the stator windings at a uniform speed in accordance with the current variations in the stationary windings. The speed of the revolving field depends on the number of poles, as established by the pole-phase groups (two poles are shown in figure 15–1, D), and the frequency of the line supply. This speed is called SYNCHRONOUS SPEED and is independent of load variations on the motor.

The rotating magnetic field established by the 3-phase stator winding can be visualized by considering the direction of the currents in the three phases at several successive instants, as shown in figure 15–1, D. These instants are marked off also at 30° intervals on the current waves (fig. 15–1, C).

At instant 1, the current in phase *a* is zero and the currents in phase *b* and *c* are each 0.866 of their maximum values and opposite in phase. The value of 0.866 results from the fact that there is a phase difference between *a, b,* and *c* of 120°. The sine of 120° is 0.866, and the sine of 240° is -0.866. The current in phase *b* is negative and flows from the source toward the *b*-phase motor terminal. This current flows inward (away from the observer) through the upper half and outward (toward the observer) through the lower half of the *b*-phase stator coils to the midpoint,

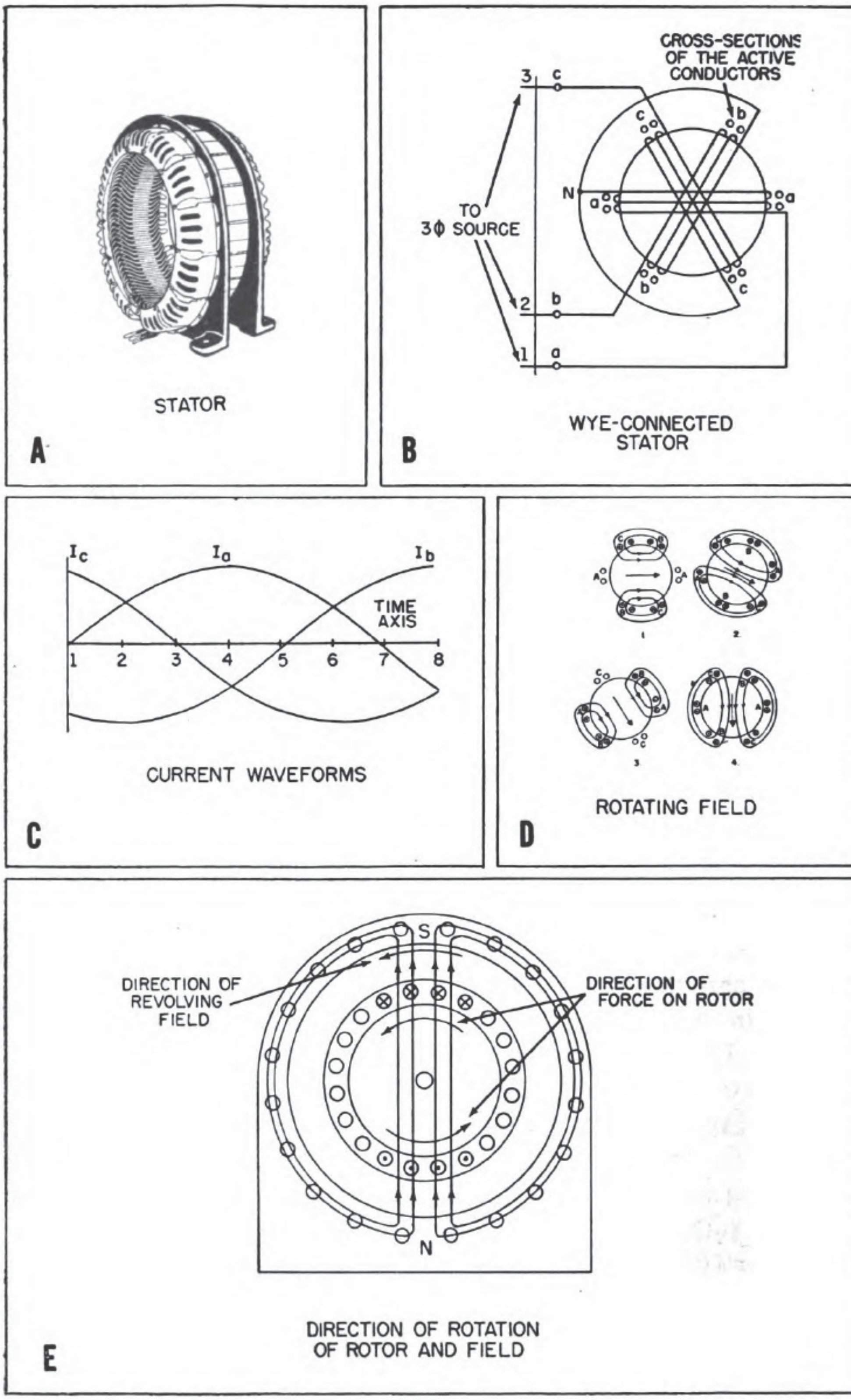


Figure 15–1.—Rotating magnetic field.

N (fig. 15–1, B) of the wye connection. The current in phase c is positive and flows away from the midpoint, N, inward through the upper half and outward through the lower half of the c-phase stator coils, to the c-phase motor terminal, and returns to the source. Because the instantaneous direction (instant 1) of the current in phases c and b is the same (fig. 15–1, D) and flows out at the bottom and in at the top, the magnetic field is horizontal and to the right.

At instant 2, the current in phase b has increased to its maximum negative value and the currents in phases a and c are 0.5 of their maximum positive values. The currents through phases b and c are in the same direction as for instant 1. The current in phase a is positive at this instant and flows away from the a-phase motor terminal toward the source. Thus, the magnetic field has rotated to the right 30° downward from the horizontal.

Further analysis of figure 15–1, C and D, at instants 3 and 4 shows that the magnetic field continues to rotate around the stator traversing a degree of space for each degree of time shown on the sine-curve time axis.

The direction of rotation of the magnetic revolving field may be changed by interchanging any two line leads to the three motor terminals. For example, in figure 15–1, B, if line 1 connects to phase a, line 2 to phase b, and line 3 to phase c, and the line currents reach their positive maximum values in the sequence 1, 2, 3, the phase sequence is a, b, c, and the rotation is arbitrarily clockwise. If lines 1 and 2 are interchanged, the phase sequence becomes b, a, c, and the revolving field turns counterclockwise.

As has been stated, the speed of the revolving field varies directly as the frequency of the applied voltage and inversely as the number of poles. Thus,

$$N=\frac{120f}{P},$$

where N is the number of revolutions that the field makes per minute, f the frequency of the applied voltage in cycles per second, and P the number of poles produced by the 3-phase winding.

A motor having a 2-pole 3-phase stator winding connected to a 60-cycle source has a synchronous speed (magnetic revolving field speed) of 3,600 rpm. A 2-pole 25-cycle motor has a synchronous speed of 1,500 rpm. Increasing the number of poles lowers the speed. Thus, a 4-pole 25-cycle motor has a synchronous speed of

750 rpm. A 12-pole 60-cycle motor has a synchronous speed of 600 rpm. Increasing the frequency of the line supply increases the speed with which the field revolves. Thus, if the frequency is increased from 50 to 60 cycles, and the motor has 4 poles, the speed of the field is increased from 1,500 rpm to 1,800 rpm.

The speed of the rotating field is always independent of load changes on the motor, provided the line frequency is maintained constant. The magnetic revolving field always runs at the same speed, pole for pole, as the alternator supplying it. If a 2-pole 60-cycle alternator supplies a 2-pole motor, the motor has a synchronous speed of 3,600 rpm, which is the same as the speed of the alternator. If a 4-pole 60-cycle alternator runs at 1,800 rpm and supplies a 4-pole 60-cycle motor, the motor has a synchronous speed of 1,800 rpm. If this same alternator supplies an 8-pole 60-cycle motor, the motor has a synchronous speed of 900 rpm.

POLYPHASE INDUCTION MOTORS

Almost all shipboard a-c motors above fractional-horsepower size are polyphase induction motors, and therefore some knowledge of their operating principles is essential.

The driving torque of both d-c and a-c motors is derived from the reaction of current-carrying conductors in a magnetic field. In the d-c motor the magnetic field is stationary and the armature, with the current-carrying conductors, rotates. The current is supplied to the armature through a commutator and brushes.

In induction motors, the rotor currents are supplied by electromagnetic induction. The stator windings contain two or more out-of-time-phase currents, which produce corresponding mmf's. These mmf's combine to establish a rotating magnetic field across the air gap. This magnetic field rotates continuously at constant speed regardless of the load on the motor. The stator winding corresponds to the armature winding of a d-c motor and to the primary winding of a transformer. The rotor is not connected electrically to the power supply. The induction motor derives its name from the fact that mutual induction (transformer action) takes place between the stator and the rotor under operating conditions. The magnetic revolving field around the stator cuts across the rotor conductors, inducing a voltage in the conductors. This induced voltage causes rotor current to flow. Hence, motor torque is developed by the interaction of the rotor current and the magnetic revolving field.

In figure 15–1, E, a 2-pole field is assumed to be rotating in a counterclockwise direction at synchronous speed. At the instant pictured the south pole field cuts across the upper rotor conductors from right to left, and the lines of force extend upward. Applying the left-hand rule for generator action to determine the direction of the voltage induced in the rotor conductors, the thumb is pointed in the direction of motion of the conductors with respect to the field. Since the field sweeps across the conductors from right to left, their relative motion wtih respect to the field is to the right. Hence the thumb points to the right. The index finger points upward and the second finger points into the page, indicating that the rotationally induced voltage in the upper rotor conductors is away from the observer.

Applying the left-hand rule to the lower rotor conductors and the north-pole field, the thumb points to the left, the index finger points upward, and the second finger points toward the observer, indicating that the direction of the rotationally induced voltage is out of the page. The rotor bars are connected to end rings that complete their circuits, and the rotationally induced voltages act in series addition to cause rotor currents to flow in the rotor conductors in the directions indicated. For simplification, the rotor currents are assumed to be in phase with the rotor voltages.

Motor action is analyzed by applying the right-hand rule for motors to the rotor conductors in figure 15–1, E, to determine the direction of the force acting on the rotor conductors. For the upper rotor conductors, the index finger points upward, the second finger points into the page and the thumb points to the left, indicating that the force on the rotor tends to turn the rotor counterclockwise. This direction is the same as that of the rotating field. For the lower rotor conductors the index finger points upward, the second finger points toward the observer, and the thumb points toward the right, indicating that the force tends to turn the rotor counterclockwise—the same direction as that of the field.

The STATOR of a polyphase induction motor consists of a laminated steel ring (fig. 15–1, A), with slots on the inside circumference. The ring is supported by a fabricated steel frame. The motor stator winding is similar to the alternator stator winding and is generally of the two-layer distributed preformed type. Stator phase windings are symmetrically placed on the stator and may be either wye or delta connected.

There are two types of ROTORS—the CAGE ROTOR and the FORM-WOUND ROTOR. Both types have a laminated cylindrical core with parallel slots in the outside circumference to hold the windings in place. The cage rotor has an uninsulated bar winding; whereas the form-wound rotor has a two-layer distributed winding with preformed coils like those on a d-c motor armature.

Cage Rotors

A cage rotor is shown in figure 15–2, A. The rotor bars are of copper, aluminum, or a suitable alloy placed in the slots of the rotor core. These bars are connected together at each end by rings of similar material. The conductor bars carry relatively large currents at low voltage. Hence, it is not necessary to insulate these bars from the core because the currents follow the path of least resistance and are confined to the cage winding.

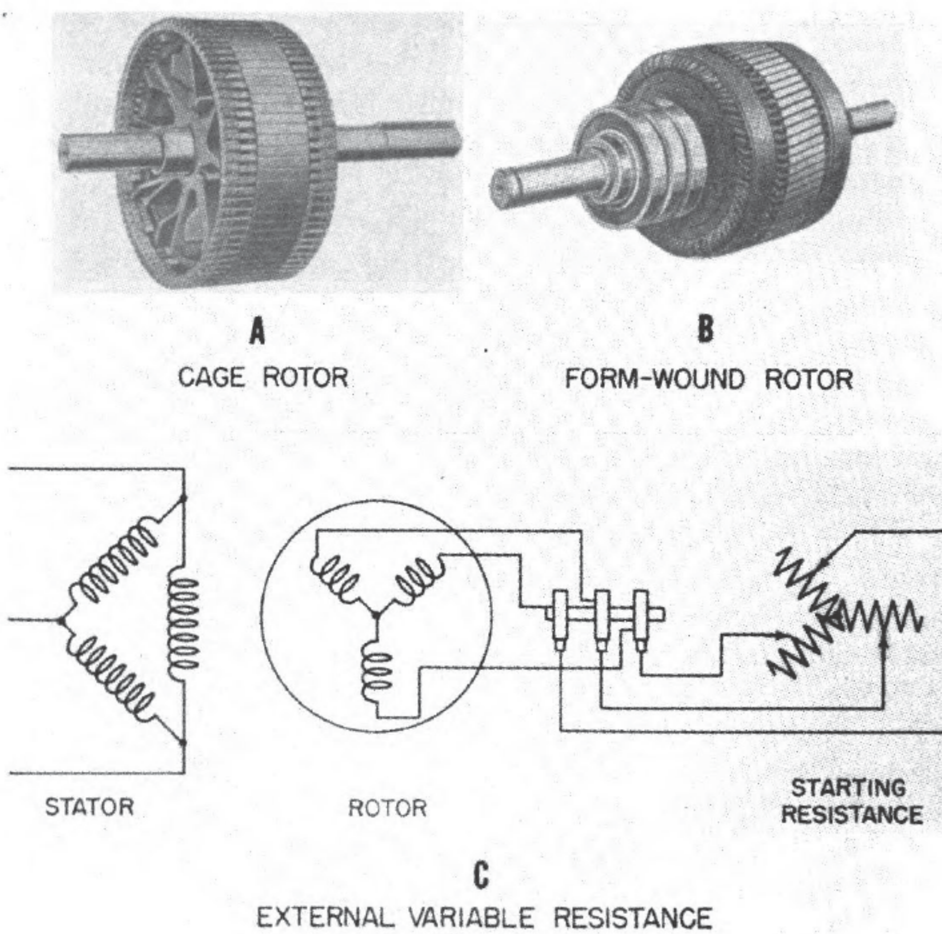


Figure 15–2.—Induction motor rotors.

Form-Wound Rotor

A form-wound rotor (fig. 15–2, B) has a winding similar to 3-phase stator windings. Rotor windings are usually wye connected with the free ends of the winding connected to three slip rings mounted on the rotor shaft. An external variable wye-connected resistance (fig. 15–2, C) is connected to the rotor circuit through the slip rings. The variable resistance provides a means of increasing the rotor-circuit resistance during the starting period to produce a high starting torque. As the motor accelerates, the rheostat is cut out. When the motor reaches full speed, the slip rings are short-circuited and the operation is similar to that of the cage motor.

Torque

As previously described, the revolving field produced by the stator windings cuts the rotor conductors and induces voltages in the conductors. Rotor currents flow because the rotor end-rings provide continuous metallic circuits. The resulting torque tends to turn the rotor in the direction of the rotating field. This torque is proportional to the product of the rotor current, the field strength, and the rotor power factor.

The simplified cross section of a 2-pole cage-rotor motor is shown in figure 15–3. The magnetic field is rotating in a clockwise direction. Applying the left-hand rule for generator action, note that in figure 15–3, A, the induced currents flow outward in the upper half and inward in the lower half of the rotor conductors. Applying the right-hand rule for motor action, note that the force acting on the rotor conductors is to the right on the upper group and to the left on the lower group.

As previously stated, a d-c motor receives its armature current by means of conduction through the commutator and brushes; whereas an induction motor receives its rotor current by means of induction. In this respect the induction motor is like a transformer with a rotating secondary. The primary is the stator which produces the revolving field; the secondary is the rotor. At start, the frequency of the rotor current is that of the primary stator winding. The reactance of the rotor is relatively large compared with its resistance, and the power factor is low and lagging by almost 90°. The rotor current therefore lags the rotor voltage by approximately 90°, as shown in figure 15–3, B.

Because almost half of the conductors under the south pole carry current outward and the remainder of the conductors carry current inward, the net torque on the rotor as a result of the interaction between the rotor and the rotating field is small.

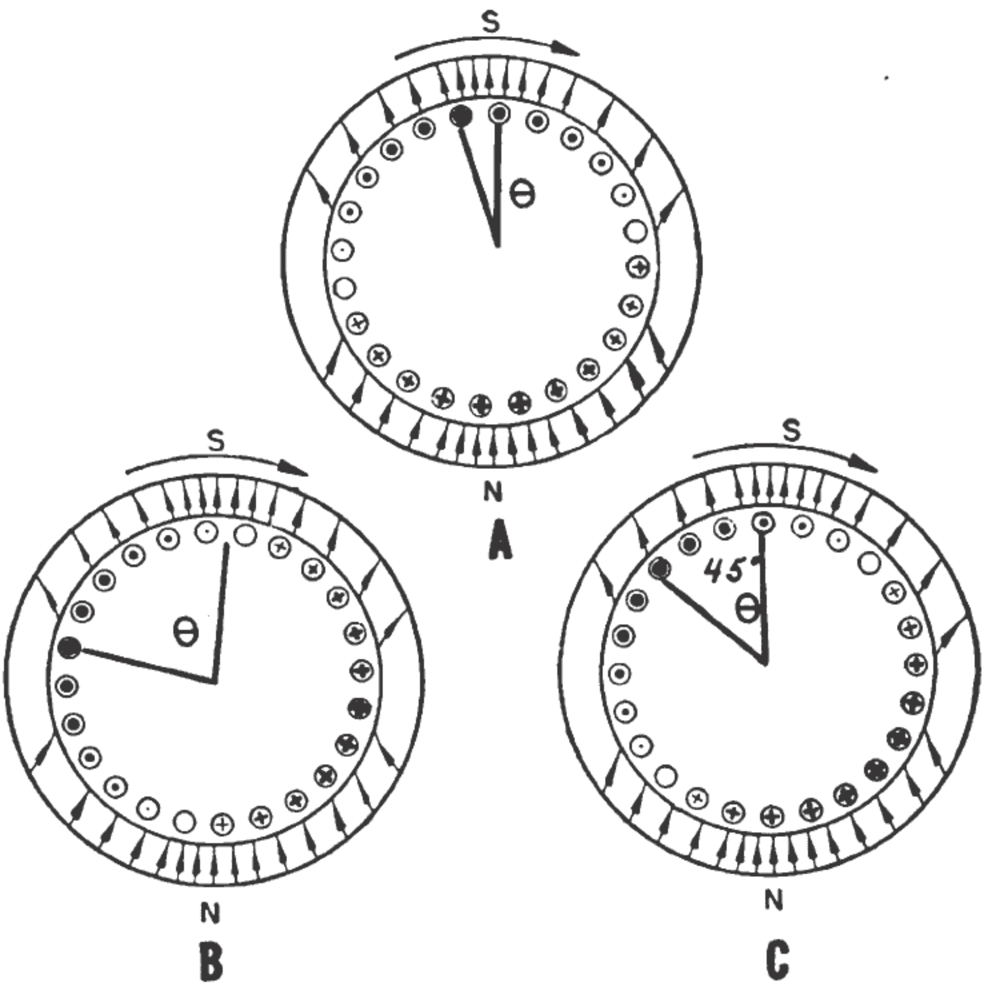


Figure 15–3.—Development of torque.

As the rotor comes up to speed in the same direction as the revolving field, the rate at which the revolving field cuts the rotor conductors is reduced and the rotor voltage and frequency of rotor currents are correspondingly reduced. Hence, at almost synchronous speed the voltage induced in the rotor is very small. The rotor reactance, X_L, also approaches zero, as may be seen from the relationship

$$X_L = 2\pi f_o L S,$$

where f_0 is the frequency of the stator current, L the rotor inductance, and S the ratio of the difference in speed (between the stator field and the rotor) to the synchronous speed—that is, the slip. Slip is expressed mathematically as

$$S=\frac{N_s-N_r}{N_s},$$

where N_s is the number of revolutions per minute of the stator field, and N_r the number of revolutions per minute of the rotor. The frequency of the induced rotor currents is f_0S.

In figure 15–3, A, the rotor current is nearly in phase with the rotor voltage, and the direction of flow under the south pole is the same in all conductors. The torque would be ideally high except for the very small rotor current, which is produced by the low rotor voltage. The rotor voltage is proportional to the difference in speed between the rotor and the rotating field—that is, proportional to the slip.

The frequency of the rotor current varies directly with the slip. Thus, when the slip and the frequency of rotor current are almost zero, the rotor reactance and angle of lag are very small. When the rotor is starting, the difference in speed between the rotor and the rotating field is maximum; hence, the rotor reactance is maximum because the frequency of rotor current is maximum and approaches that of the line supplying the stator primary.

Normal operation is between these two extremes of rotor slip—that is, when the rotor is not turning at all or when it is turning almost at synchronous speed. The motor speed under normal load conditions is rarely more than 10 percent below synchronous speed. At the extreme of 100-percent slip, the rotor reactance is so high that the torque is low because of low power factor. At the other extreme of zero rotor slip, the torque is low because of low rotor current. The equation for torque, T, is

$$T=K\Phi I_r \cos \theta_r,$$

where K is a constant, Φ the strength of the magnetic revolving field, I_r the rotor current, and $\cos \theta_r$ the power factor of the rotor current.

In the previous formula the product of rotor current and rotor power factor for a given strength of magnetic revolving field is a

maximum value when the phase angle between rotor current and rotor induced voltage is 45° lagging, as indicated in figure 15–3, C. In this case the reactance of the rotor equals the resistance of the rotor circuit, and the rotor power factor is 70.7 percent. This condition of operation is called the PULL-OUT POINT. Beyond this point, the motor speed falls off rapidly with added load, and the motor stalls.

Variations in applied stator voltage affect the motor torque. This applied voltage establishes mmf's which create the rotating field. The rotating field, in turn, establishes the rotor current. Because rotor torque varies as the product of these factors, the torque of an induction motor varies as the square of the voltage applied to the stator primary winding.

The rotating field also sweeps across the stator winding which produces it. This action induces a counter emf in the winding that is similar to the counter emf generated in the armature of a d-c shunt motor. The counter emf opposes the applied voltage and limits the stator currents. If the applied voltage is increased to magnetic saturation, the counter emf is limited, and the primary current becomes dangerously high.

Synchronous Speed and Slip

The speed, N, of the rotating field is called the SYNCHRONOUS SPEED of the motor. As previously stated, the torque on the rotor tends to turn the rotor in the same direction as the revolving field. If the motor is not driving a load, it will accelerate to nearly the same speed as the revolving field. During the starting period, the increase in rotor speed is accompanied by a decrease in induced rotor voltage because the relative motion between the rotating field and rotor conductors is less. If it were possible for the rotor to attain synchronous speed there would be no relative motion between the rotor and the rotating field. There would then be no induced emf in the rotor, no rotor current, and thus no torque.

It is obvious that an induction motor cannot run at exactly synchronous speed. Instead, the rotor always runs just enough below synchronous speed at no load to establish sufficient rotor current to produce a torque equal to the resisting torque that is caused by the rotor losses.

The frequency of the alternating voltage induced in the rotor

depends on the speed of the revolving field with respect to the rotor. One cycle of alternating voltage is induced in the rotor when the stator field sweeps completely around the rotor once. The rotor frequency, f_r, is directly proportional to the percent slip—

$$f_r=\frac{Sf_s}{100},$$

where S is the percent slip and f_s is the frequency of the supply. For example, if the frequency of the supply is 60 cycles per second and the slip is 5 percent, the rotor frequency will be $\frac{5\times 60}{100}=3$ cycles per second. The frequency and magnitude of the induced rotor voltage decrease as the rotor speed increases. Both the rotor voltage and frequency would become zero if the rotor could attain synchronous speed.

Losses and Efficiency

The losses of an induction motor include (1) stator copper loss, $I_s^2R_s$, and rotor copper loss, $I_r^2R_r$; (2) stator and rotor core loss; and (3) friction and windage loss. For all practical purposes, the core, bearing friction, and windage losses are considered to be constant for all loads of an induction motor having a small slip. The power output may be measured on a mechanical brake or calculated from a knowledge of the input and the losses. The efficiency is equal to the ratio of the output power to input power; and at full load, it varies from about 85 percent for small motors to more than 90 percent for large motors.

Characteristics of the Cage-Rotor Motor

As stated previously, the cage-rotor induction motor is comparable to a transformer with a rotating secondary. At no load, the magnetic revolving field produced by the primary stator winding cuts the turns of the stator winding. This action generates a counter emf in the stator winding, which limits the line current to a small value. This no-load value is called EXCITING CURRENT. Its function is to maintain the revolving field. Because the circuit is highly inductive, the power factor of the motor with no load is very poor. It may be 30 percent lagging. Because there is no

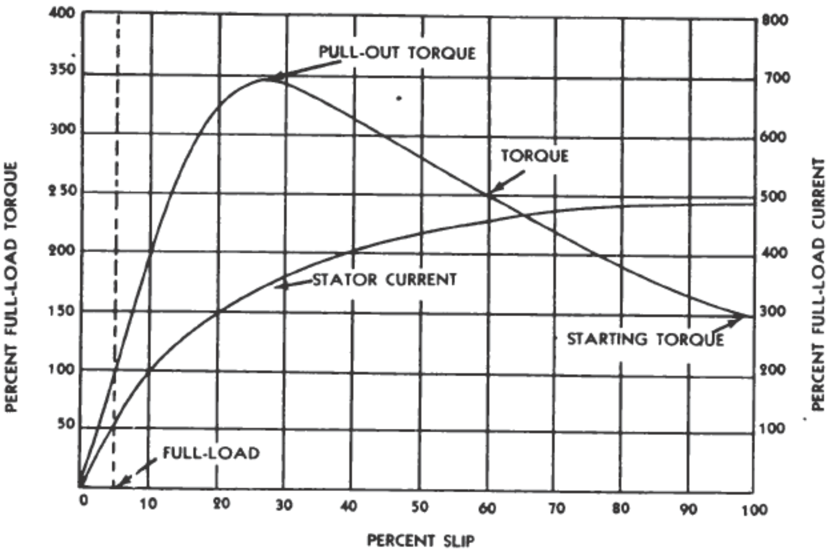
drag on the rotor, it runs at almost synchronous speed and the rotor current is quite small. Hence, the reaction of the rotor mmf on the primary revolving field is small.

When load is added to the motor, the rotor slows down slightly; but the rotating field continues at synchronous speed. Therefore, the rotor current and slip increase. The motor torque increases more than the decrease in speed and the power output increases. The increased rotor mmf opposes the primary field flux and lowers it slightly. The primary counter emf therefore decreases slightly and primary current increases. The load component of primary current maintains the rotating field and prevents its further weakening because of the rotor-current opposition. Because of the relatively low internal impedance of the motor windings, a small reduction in speed and counter emf in the primary may be accompanied by large increases in motor current, torque, and power output. In this respect the cage-rotor motor is like a d-c shunt motor. The cage-rotor motor has essentially constant-speed variable-torque characteristics.

If the induction motor is stalled by overload, the resulting increased rotor current lowers the primary counter emf and causes excessive primary current. This excessive current may damage the motor winding. When the rotor of an induction motor is locked, the voltage applied to the primary stator winding should never exceed 50 percent of rated voltage.

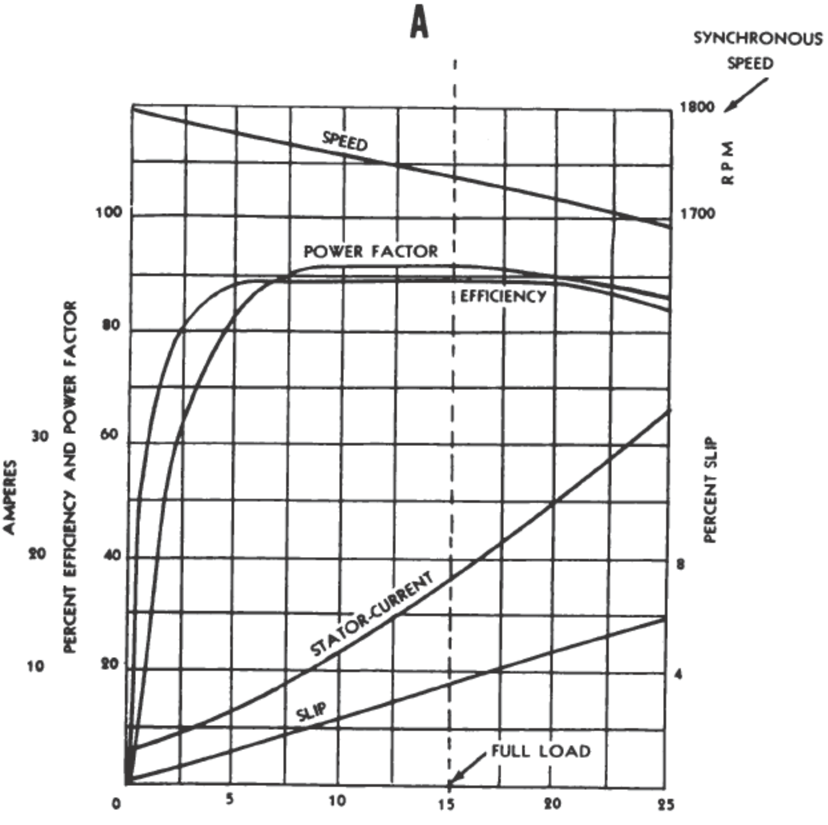
When the motor is operating at full load, the load component of stator current is more nearly in phase with the voltage across each stator phase because of the mechanical output (true power component) of the motor. The power factor is considerably improved over the no-load condition.

The torque and current curves for a 3-phase induction motor with cage rotor are shown in figure 15–4, A. Rotor reactance increases with slip and increasingly affects the rotor current and power factor as the motor load is increased. The pull-out point on the torque curve occurs at about 25-percent slip. Maximum torque at this condition is about 3.5 times the normal full-load value, and, as previously mentioned, corresponds to a rotor power factor of 70.7 percent. Thus, for the pull-out condition, the rotor resistance equals the rotor reactance and the rotor power factor angle equals 45°. Any additional load on the motor beyond this



TORQUE AND CURRENT CURVES

A



PERFORMANCE CURVES FOR 4-POLE 3-PHASE 450-VOLT 15-HP CAGE-ROTOR MOTOR

B

Figure 15–4.—Characteristic curves of a cage-rotor motor.

point causes the rotor to pull out of its normal speed range and to stall quickly. At standstill, stator current is nearly 5 times normal; hence, constant-potential motor circuits like the one supplying this motor are equipped with time-delay automatic-overload protective devices. Sustained overload causes a circuit breaker to open and to protect both the motor and the circuit from damage.

The performance curves of a 4-pole 3-phase 450-volt 15-horsepower cage-rotor motor are shown in figure 15–4, B. The full-load slip is only about 3.5 percent. At standstill, the rotor reactance of this type of motor is nearly 5 times as great as the rotor resistance. At full load, however, the rotor reactance is much less than the rotor resistance. When the motor is running, the rotor current is determined principally by the rotor resistance. The torque increases up to the pull-out point as the slip increases. Beyond this point the torque decreases and the motor stalls. Because the change in speed from no load to full load is relatively small, the motor torque and the horsepower output are considered to be directly proportional.

The cage-rotor induction motor has a fixed rotor circuit. The resistance and inductance of the windings are determined when the motor is designed and cannot be changed after it is built. The standard cage-rotor motor is a general-purpose motor. It is used to drive loads that require a variable torque at approximately constant speed with high full-load efficiency—such as blowers, centrifugal pumps, motor-generator sets, and various machine tools. As stated previously, this motor has operating characteristics similar to those of a d-c shunt motor.

If the load requires special operating characteristics, such as high-starting torque, the cage rotor is designed to have high resistance. The starting current of a motor with a high-resistance rotor is less than that of a motor with a low-resistance rotor. The high-resistance rotor motor, like the cumulative compounded d-c motor, has wider speed variations than the low-resistance rotor motor. The high rotor resistance also increases the rotor copper losses, resulting in a lower efficiency than that of the low resistance type. These motors are used to drive cranes and elevators when high-starting torque and moderate-starting current are required and when it is desired to slow down the motor without drawing excessive currents.

Characteristics of Wound-Rotor Motor

The wound-rotor, or slip-ring, induction motor is used when it is necessary to vary the rotor resistance in order to limit the starting current or to vary the motor speed. As previously explained, the starting torque may be made equal to the pull-out torque by increasing the rotor resistance to the point where it equals the rotor reactance at standstill. Maximum torque at start can be obtained with a wound-rotor motor with about 1.15 times full-load current; whereas a cage-rotor motor may require 5 times full-load current to produce maximum torque at start. Because the rotor-circuit copper losses are largely external to the rotor winding, the wound-rotor motor is desirable for an application which requires frequent starts.

The advantages of the wound-rotor induction motor over the cage-rotor induction motor are: (1) high-starting torque with moderate starting current, (2) smooth acceleration under heavy loads, (3) no excessive heating during starting, (4) good running characteristics, and (5) adjustable speed. The chief disadvantage of the wound-rotor motor is that the initial and maintenance costs are greater than those of the cage-rotor motor.

SYNCHRONOUS MOTORS

The synchronous motor differs from the induction motor in several ways. The synchronous motor requires a separate source of d-c for the field. It also requires special starting components. These include a salient-pole field with starting grid winding. The rotor of the conventional type synchronous motor is essentially the same as that of the salient-pole alternator. The stator windings of induction and synchronous motors are essentially the same. The stator of a synchronous motor is illustrated in figure 15–5, A.

As previously stated, the d-c generator operates satisfactorily as a d-c motor, and there is practically no difference in construction and rating between the two. Similarly, an alternator becomes a synchronous motor if electric power is supplied to its terminals from an external source. Synchronous-motor rotating fields are generally of the salient-pole type, as shown in figure 15–5, B; whereas alternators are either of the salient-pole type (fig. 14–1, B), or the turbo type (fig. 14–2, B).

Assume, for example, that two alternators of the salient-pole type are operating in parallel and feeding the same bus. If the prime mover is disconnected from one of the generators, it will continue to run at the same speed, as a synchronous motor, drawing its power from the other alternator.

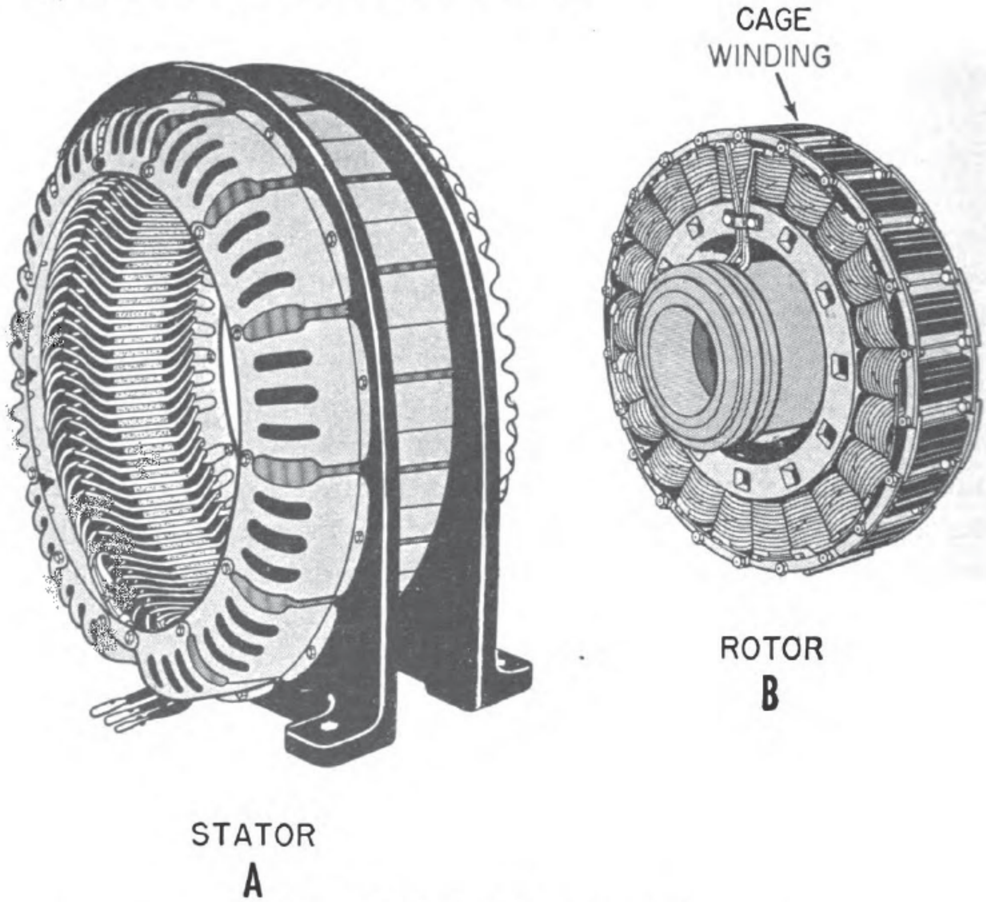


Figure 15–5.—Synchronous motor.

With the exception of certain modifications to make its operation more efficient and also to make it self-starting, the synchronous motor is very similar to the salient-pole rotating-field a-c generator. The rotor fields of both are separately excited from a d-c source, and both run at synchronous speeds under varying load conditions. An 8-pole generator rotor revolving at 900 rpm generates 60 cycles per second; likewise, an 8-pole synchronous motor supplied with 60-cycle current will rotate at 900 rpm.

Principle of Operation

A polyphase current is supplied to the stator winding of a synchronous motor and produces a rotating magnetic field the

same as in an induction motor. A direct current is supplied to the rotor winding, thus producing a fixed polarity at each pole. If it could be assumed that the rotor had no inertia and that no load of any kind were applied, then the rotor would revolve in step with the revolving field as soon as power was applied to both of the windings. This, however, is not the case. The rotor has inertia, and in addition there is a load.

The reason a synchronous motor has to be brought up to synchronous speed by special means, may be understood from a consideration of figure 15–6.

If the stator and rotor windings are energized, then as the poles of the rotating magnetic field approach rotor poles of opposite polarity (fig. 15–6, A), the attracting force tends to turn the rotor in the direction opposite to that of the rotating field. As the rotor starts in this direction, the rotating-field poles are leaving the rotor poles (fig. 15–6, B), and this tends to pull the rotor poles in the same direction as the rotating field. Thus, the rotating field tends to pull the rotor poles first in one direction and then in the other, with the result that the starting torque is zero.

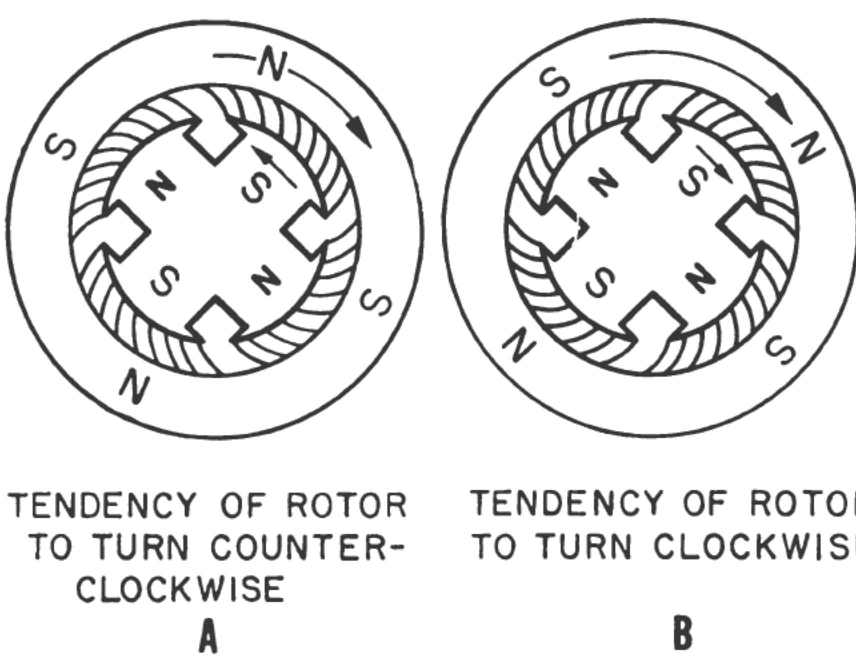


Figure 15–6.—Operating principle of synchronous motor.

Starting

As has been explained, some type of starter must be used with the synchronous motor to bring the rotor up to synchronous speed. Although a small induction motor may be used to bring the rotor up to speed, this is not generally done. Sometimes, if direct current is available, a d-c motor coupled to the rotor shaft may be used to bring the rotor up to synchronous speed. After synchronous speed is attained, the d-c motor is converted to operate as a generator to supply the necessary direct current to the rotor of the synchronous motor.

In general, however, another method is used to start the synchronous motor. A cage-rotor winding is placed on the rotor of the synchronous motor to make the machine self-starting as an induction motor (fig. 15–5, B). At start, the d-c rotor field is deenergized and a reduced polyphase voltage is applied to the stator windings. Thus, the motor starts as an induction motor and comes up to a speed which is slightly less than synchronous speed. The rotor is then excited from the d-c supply (generally a d-c generator mounted on the shaft) and the field rheostat adjusted for minimum line current.

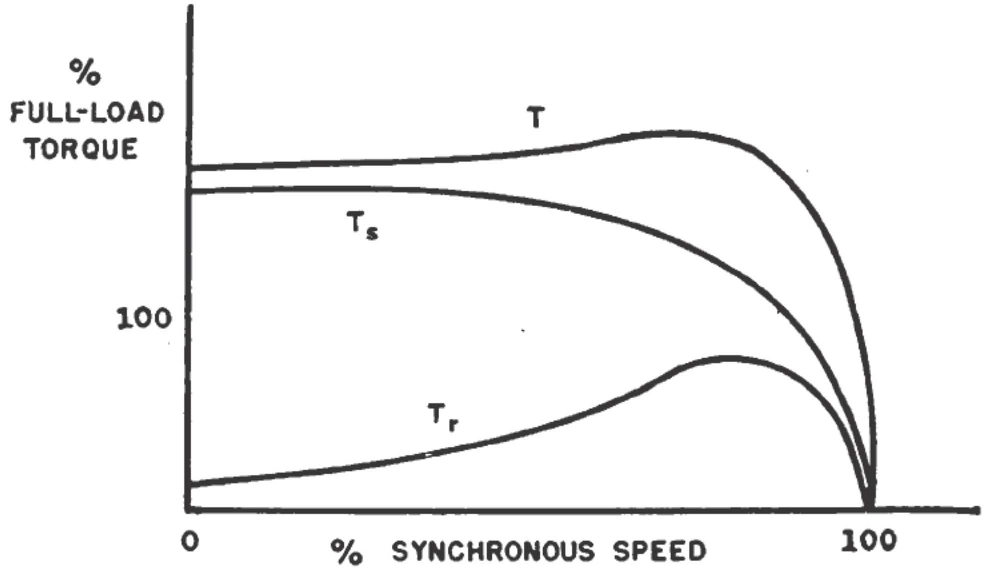


Figure 15–7.—Starting torque of a synchronous motor.

If the armature has the correct polarity at the instant synchronization is reached, the stator current will decrease when the

excitation voltage is applied. If the armature has the incorrect polarity, the stator current will increase when the excitation voltage is applied. This is a transient condition, and if the excitation voltage is increased further the motor will slip a pole and then come into step with the revolving field of the stator.

If the rotor d-c field winding of the synchronous machine is open when the stator is energized, a high a-c voltage will be induced in it because the rotating field sweeps through the large number of turns at synchronous speed.

It is therefore necessary to connect a resistor of low resistance across the rotor d-c field winding during the starting period. During the starting period, the d-c field winding is disconnected from the source and the resistor is connected across the field terminals. This permits alternating current to flow in the d-c field winding. Because the impedance of this winding is high compared with the inserted external resistance, the internal voltage drop limits the terminal voltage to a safe value.

Starting Torque

Both the alternating currents induced in the rotor field winding and the cage-rotor winding during starting are effective in producing the starting torque. The torques produced by the rotor d-c field winding and the cage-rotor winding at different speeds are shown by the curves T_r and T_s, respectively, of figure 15–7. Curve T is the sum of T_r and T_s and indicates the total torque at different speeds during the starting period. Note that T_r is very effective in producing torque as the rotor approaches synchronous speed, but that both windings contribute no torque at synchronous speed because the induced voltage is zero and no d-c excitation is yet applied to the d-c winding.

Effect of Varying Load and Field Strength

The power factor of an induction motor depends on the load and varies with it. The power factor of a synchronous motor carrying a definite load may be unity or less than unity, either lagging or leading, depending on the d-c field strength.

The induced (counter) emf in armature coil C, shown in figure 15–8, A, is maximum when its sides are opposite the pole

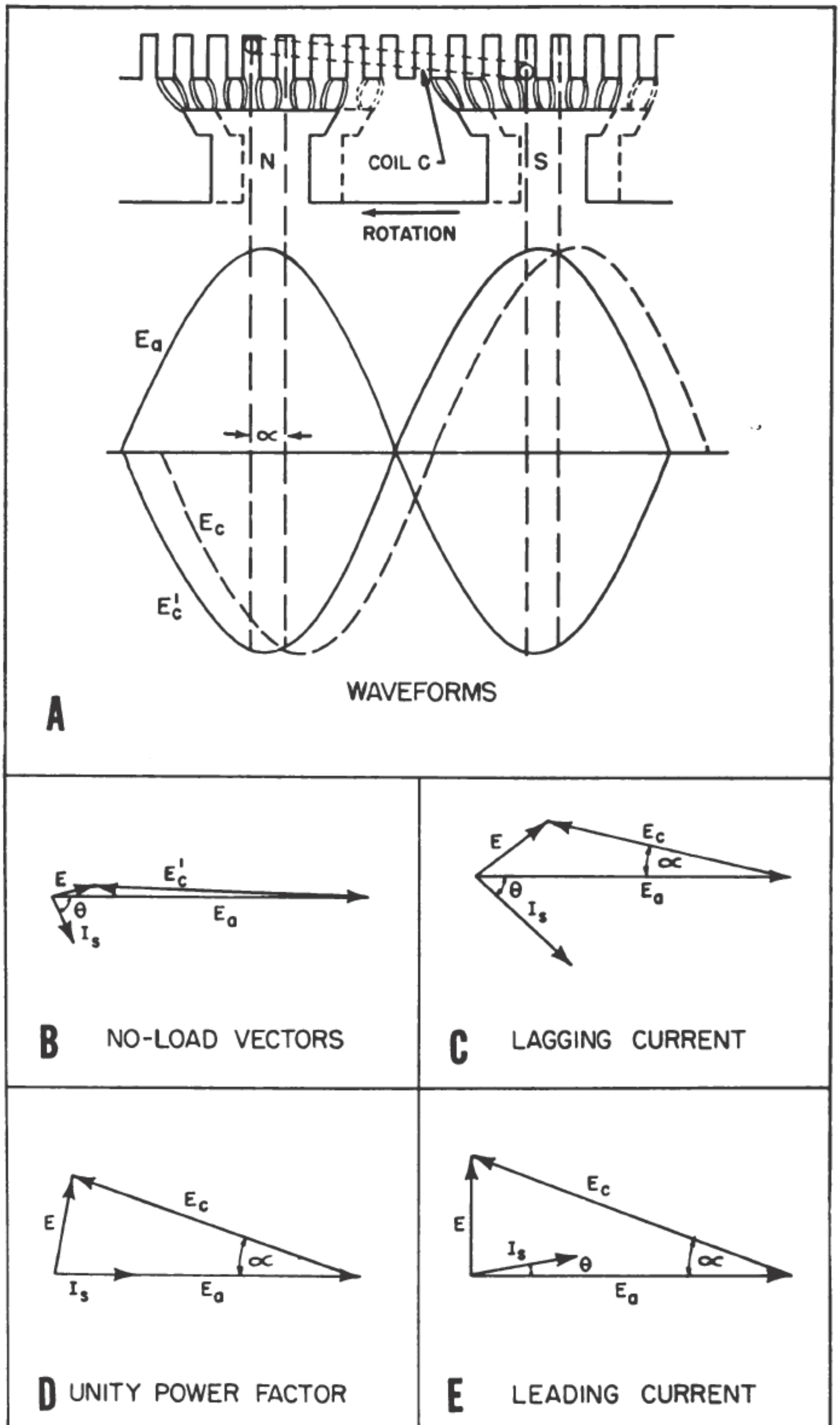


Figure 15–8.—Effect of varying the load and field strength of a synchronous motor.

centers and minimum when its sides are midway between the pole tips; and it varies as indicated by the solid-line curve, E'_c. When the motor carries no load, the counter voltage E'_c is practically 180° out of phase with the applied voltage E_a (fig. 15–8, A and B). If the field is adjusted so that E_c almost equals E_a, the stator current I_s is small and corresponds to the exciting current in a transformer.

When load is applied to the motor it causes the poles to be pulled α degrees behind their no-load position, as indicated by the broken curve in figure 15–8, A, and the counter emf occurs α degrees later. This is indicated by curve E_c in figure 15–8, A, and by vector E_c in figure 15–8, C. The resultant voltage, E, causes the stator current I_s, to lag behind E_a by angle θ.

In a d-c motor the armature current is determined from the equation

$$I_a=\frac{E_a-E_c}{R_a}.$$

Similarly, in a synchronous motor the stator current is determined as

$$I_s=\frac{\text{vector sum of } E_a \text{ and } E_c}{Z_s},$$

where I_s is the stator current, E_a the applied voltage, E_c the counter voltage, and Z_s the stator impedance. This vector sum is indicated by E in figure 15–8, C. The stator reactance is large compared to its resistance, and therefore the current I_s lags the resultant voltage E by nearly 90°. Hence, I_s, lags, E_a, by angle θ. In this condition the synchronous motor operates with a lagging power factor. If the load is increased, the rotor poles are pulled further behind the stator poles, which causes E_c to lag further, and angle α to increase. This action causes the resultant voltage, E, and stator current, I_s, to increase.

Because the speed is constant, if the field excitation is decreased, the counter voltage, E_c, decreases; and the resultant voltage, E, becomes greater. The stator current becomes greater and lags the applied voltage, E_a, a greater amount. On the other hand, if the field excitation is increased until the current, I_s, is in phase with the applied voltage, E_a, the power factor of the motor becomes unity for a given load (fig. 15–8, D). For a definite load

at unity power factor, E and I_s are both at their minimum. If the field excitation is further increased, I_s increases and leads E_a (fig. 15–8, E). Thus, for a definite load, the power factor is governed by the field excitation—that is, a weak field produces lagging current and a strong field produces a leading current. Normal field excitation for a given load occurs when I_s is in phase with E_a.

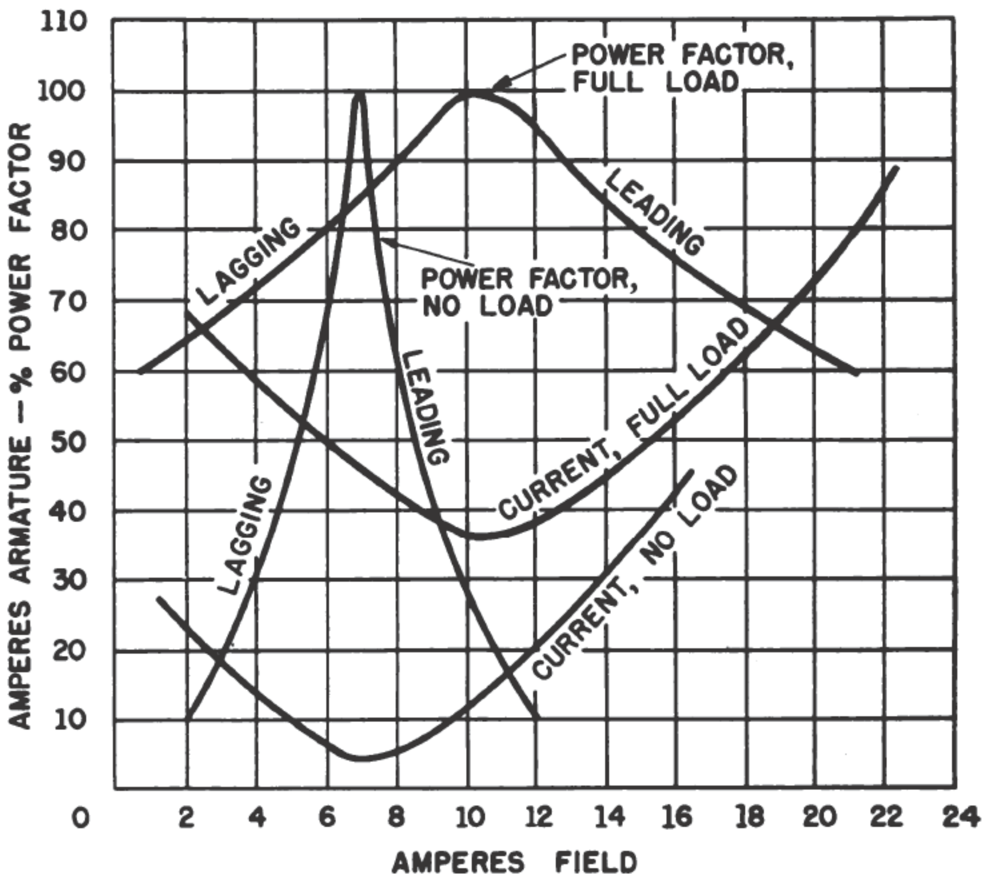


Figure 15–9.—V curves for a synchronous motor of 15 kva.

The so-called synchronous motor **V** curves, which indicate the variations of current for a constant load and varied field excitation, are shown in figure 15–9. The corresponding variations of power factor are shown also.

Although synchronous motors are used as propulsion motors in practically all Navy vessels that have a-c electric propulsion systems, they have very few other uses in the Navy. However, a

synchronous motor is frequently used in shore-based systems to change the power factor of the system to which it is paralleled by adjusting its field excitation. If operated without load, its power factor may be adjusted to a value as low as 10 percent leading. When operated under this condition the motor is generally referred to as a synchronous condenser because it takes a leading current in the same manner as capacitors. In this case the synchronous condenser takes only enough true power from the line to supply its losses. At the same time, it supplies a high leading reactive power, which cancels the lagging reactive power taken by the parallel inductive loads, and the system power factor is thereby improved.

ALTERNATING-CURRENT MOTOR STARTERS

As in the case of d-c motors, some type of starter (controller) may also be employed on a-c motors to limit the initial inrush of current.

ACROSS-THE-LINE starters are the most common form aboard ship because of their simplicity and because ships are generally equipped with sufficient generating capacity to handle the high starting currents of the motors. This type of starter throws the stator winding of the motor directly across the main supply line. This may be feasible if the motor is not too large (5 horsepower or less) and if the generating capacity of the alternator can take care of the added load.

PRIMARY RESISTOR starters insert a resistor in the PRIMARY circuit (the stator circuit) of the motor for starting, or for starting and speed control. This starter is used when it is necessary to limit the starting current of a large a-c motor so as not to put too great a load on the system. If the resistor is used only during starting, its rating is based on intermittent operation, in which case it is relatively small and operates at a higher temperature. If the resistor is used also for speed control of small motors, such as for ventilating fans, its rating is based on continuous operation. In this case the resistor is relatively large and operates at a lower temperature.

SECONDARY RESISTOR starters insert a resistor in the secondary circuit (the rotor circuit of the form-wound type of induction

motor) for starting and speed control. This starter may be used to limit starting currents, but is usually found where speed control of a large a-c motor is required, in which case the resistors are rated for continuous duty. Examples are some elevators and hoists equipped with direct a-c electric drives.

COMPENSATOR, OR AUTOTRANSFORMER, starters start the motor at reduced voltage through an autotransformer, and subsequently connect the motor to full voltage after acceleration. The compensator may be either of two types.

1. The OPEN-TRANSITION type, during the transition period of shifting the motor from the autotransformer to direct connection with the supply lines, disconnects the motor from all power for a short period of time during which, if the motor is of the synchronous type, it may coast and slip out of phase with the power supply. When the motor is then connected directly to the power lines, a high transition current may result.
2. The CLOSED-TRANSITION type keeps the motor connected to the power supply at all times during the transition period, thus not permitting the motor to decelerate. Accordingly, no high transition current is developed.

The AUTOTRANSFORMER starter is the most common form of the reduced-voltage type used for limiting the starting current of a motor. The open-transition form has the disadvantage of allowing a high transition current to develop, which can cause circuit breakers to open. The closed-transition form is preferable because no high transition current is developed.

REACTOR STARTERS insert a reactor in the primary circuit of an a-c motor during starting, and subsequently short-circuit the reactor to apply full voltage to the motor. This type of starter is not very widely used at present, but is becoming more common for starting large motors because it does not have the high transition current problem of the open-transition compensator and is smaller than the closed-transition compensator.

A simplified schematic diagram of a compensator, or autotransformer, starter is shown in figure 15–10. Assume that the line voltage is 100 volts and that when the taps on the autotransformer are positioned as shown (in the starting position) 40 volts will be applied to the 3-phase motor stator. With the reduction

in motor voltage, there is a corresponding reduction in starting current drawn from the line. At the same time, the motor current supplied by the secondary low-voltage windings is proportionately increased by the transformer action.

After the proper time interval, during which acceleration occurs, full-line voltage is applied to the motor.

A resistance starter could be used in place of autotransformer

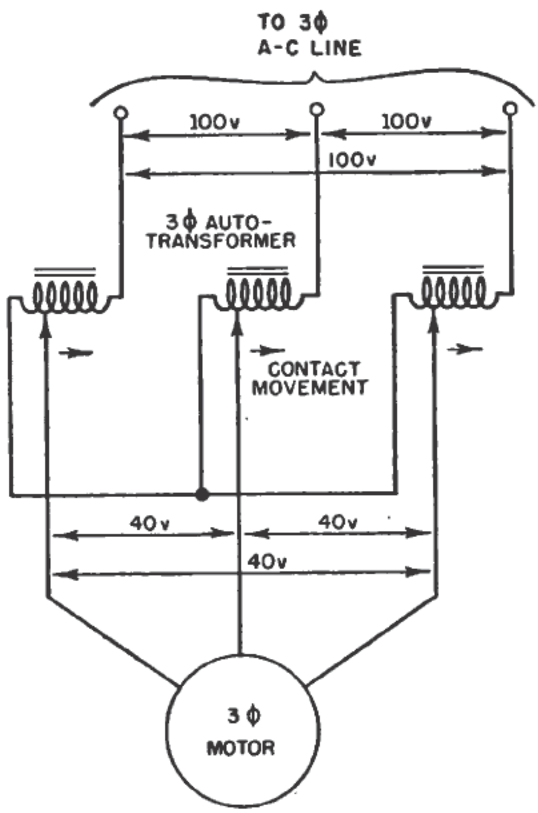


Figure 15–10.—Simplified schematic diagram of autotransformer starter.

to lower the voltage applied to the stator. The power factor would be improved, but the line current would be greater. The autotransformer has the advantage in permitting the motor to draw a relatively large starting current from the secondary with a relatively low line current.

SINGLE-PHASE MOTORS

Single-phase motors, as their name implies, operate on only one phase. These motors are used extensively in fractional horse-

power sizes in the Navy and in commercial and domestic applications. The advantages of using single-phase motors in small sizes are that they are less expensive to manufacture than other types, and they eliminate the need for 3-phase a-c lines. Single-phase motors are used in interior communications equipment, fans, refrigerators, portable drills, grinders, and so forth.

A single-phase induction motor with only one stator winding and a cage rotor is like a 3-phase induction motor with a cage rotor except that the single-phase motor has no magnetic revolving field at start and hence no starting torque. However, if the rotor is brought up to speed by external means, the induced currents in the rotor will cooperate with the stator currents to produce a revolving field, which causes the rotor to continue to run in the direction in which it was started.

Several methods are used to provide the single-phase induction motor with starting torque. These methods identify the motor as split phase, capacitor, shaded pole, repulsion, and so forth.

Another class of single-phase motors is the a-c series (universal) type. These motors have electrical characteristics like d-c series motors. Only the more commonly used types of single-phase motors are described. These include the (1) split-phase motor, (2) capacitor motor, (3) shaded-pole motor, (4) repulsion-start motor, and (5) a-c series motor.

Split-Phase Motor

The split-phase motor (fig. 15–11, A), has a stator composed of slotted laminations that contain an auxiliary (starting) winding and a running (main) winding. The axes of these two windings are displaced by an angle of 90 electrical degrees. The starting winding has fewer turns and smaller wire than the running winding, hence has higher resistance and less reactance. The main winding occupies the lower half of the slots and the starting winding occupies the upper half. The two windings are connected in parallel across the single-phase line supplying the motor. The motor derives its name from the action of the stator during the starting period. The single-phase stator is split into two windings (phases), which are displaced in space by 90°, and which contain currents displaced in time phase by an angle of approximately 15° (fig. 15–11, B). The current, I_s, in the starting winding lags the line voltage by about 30° and is less than the

current in the main winding because of the higher impedance of the starting winding. The current, I_m, in the main winding lags the applied voltage by about 45°. The total current, I_{line}, during the starting period is the vector sum of I_s and I_m.

At start, these two windings produce a magnetic revolving field that rotates around the stator air gap at synchronous speed. As the rotating field moves around the air gap, it cuts across the rotor conductors and induces a voltage in them, which is maximum at the point at maximum field intensity and therefore is in phase with the stator field. The rotor current lags the rotor voltage at start by an angle that approaches 90° because of the high rotor reactance. The interaction of the rotor currents and the stator field cause the rotor to accelerate in the

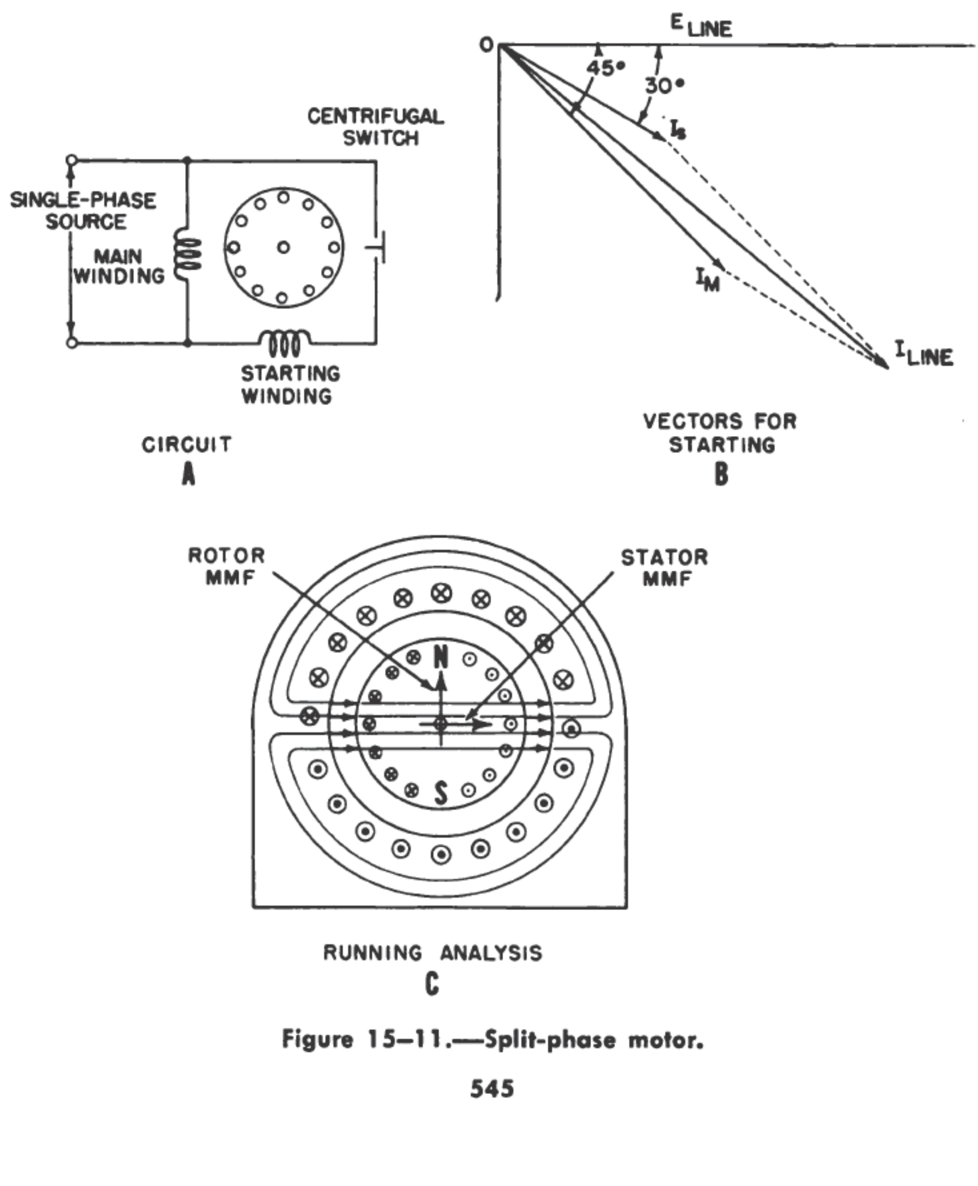


Figure 15–11.—Split-phase motor.

direction in which the stator field is rotating. During acceleration, the rotor voltage, current, and reactance are reduced and the rotor currents come closer to an in-phase relation with the stator field.

When the rotor has come up to about 75 percent of synchronous speed, a centrifugally operated switch disconnects the starting winding from the line supply, and the motor continues to run on the main winding alone. Thereafter, the rotating field is maintained by the interaction of the rotor magnetomotive force and the stator magnetomotive force. These two mmf's are pictured as the vertical and horizontal vectors respectively in the schematic diagram of figure 15–11, C.

The stator field is assumed to be rotating at synchronous speed in a clockwise direction, and the stator currents correspond to the instant that the field is horizontal and extending from left to right across the air gap. The left-hand rule for magnetic polarity of the stator indicates that the stator currents will produce a *N* pole on the left side of the stator and a *S* pole on the right side. The motor indicated in the figure is wound for two poles.

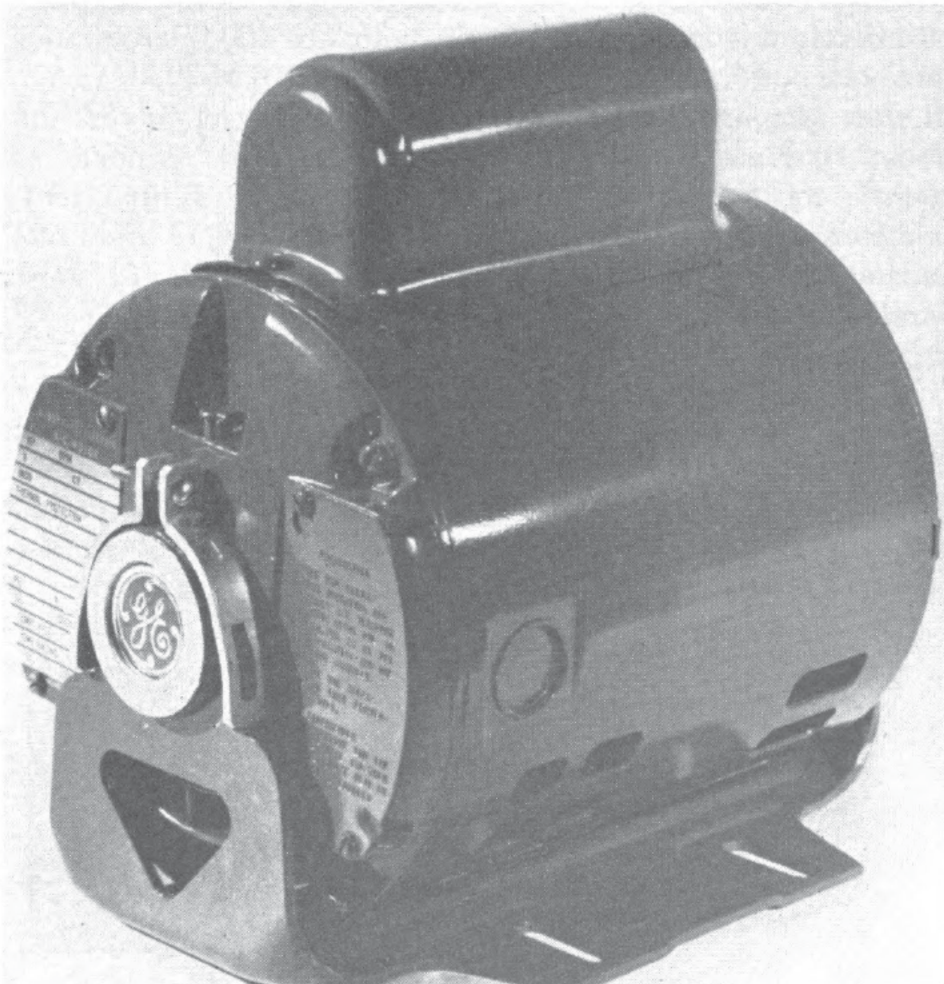
Applying the left-hand rule for induced voltage in the rotor (the thumb points in the direction of motion of the conductor with respect to the field), the direction of induced voltage is back on the left side of the rotor and forward on the right side. The rotor voltage causes a rotor current to flow, which lags the rotor voltage by an angle whose tangent is the ratio of rotor reactance to rotor resistance. This is a relatively small angle because the slip is small. Applying the left-hand rule for magnetic polarity to the rotor winding, the vertical vector pointing upward represents the direction and magnitude of the rotor mmf. This direction indicates the tendency to establish a *N* pole on the upper side of the rotor and a *S* pole on the lower side, as indicated in the figure. Thus, the rotor and stator mmf's are displaced in space by 90° and in time by an angle that is considerably less than 90°, but sufficient to maintain the magnetic revolving field and the rotor speed.

This motor has the constant-speed variable-torque characteristics of the shunt motor. Many of these motors are designed to operate on either 110 volts or 220 volts. For the lower voltage the stator coils are divided into two equal groups and these are connected in parallel. For the higher voltage the groups are

connected in series. The starting torque is 150 to 200 percent of the full-load torque and the starting current is 6 to 8 times the full-load current. Fractional-horsepower split-phase motors are used in a variety of equipments such as washers, oil burners, and ventilating fans. The direction of rotation of the split-phase motor can be reversed by interchanging the starting winding leads.

Capacitor Motor

The capacitor motor is a modified form of split-phase motor, having a capacitor in series with the starting winding. An external view is shown in figure 15–12, with the capacitor located on top of the motor. The capacitor produces a greater phase dis-



Courtesy General Electric Company

Figure 15–12.—Capacitor motor.

placement of currents in the starting and running windings than is produced in the split-phase motor. The starting winding is made of many more turns of larger wire and is connected in series with the capacitor. The starting winding current is displaced approximately 90° from the running winding current. Since the axes of the two windings are also displaced by an angle of 90°, these conditions produce a higher starting torque than that of the split-phase motor. The starting torque of the capacitor motor may be as much as 350 percent of the full-load torque.

If the starting winding is cut out after the motor has increased in speed, the motor is called a CAPACITOR-START MOTOR. If the starting winding and capacitor are designed to be left in the circuit continuously, the motor is called a CAPACITOR MOTOR. Electrolytic capacitors for capacitor-start motors vary in size from about 80 microfarads for 1/8-horsepower motors to 400 microfarads for one-horsepower motors. Capacitor motors of both types are made in sizes ranging from small fractional horsepower motors up to about 10 horsepower. They are used to drive grinders, drill presses, refrigerator compressors, and other loads that require relatively high starting torque. The direction of rotation of the capacitor motor may be reversed by interchanging the starting winding leads.

Shaded-Pole Motor

The shaded-pole motor employs a salient-pole stator and a cage rotor. The projecting poles on the stator resemble those of d-c machines except that the entire magnetic circuit is laminated and a portion of each pole is split to accommodate a short-circuited copper strap called a SHADING COIL (fig. 15–13). This

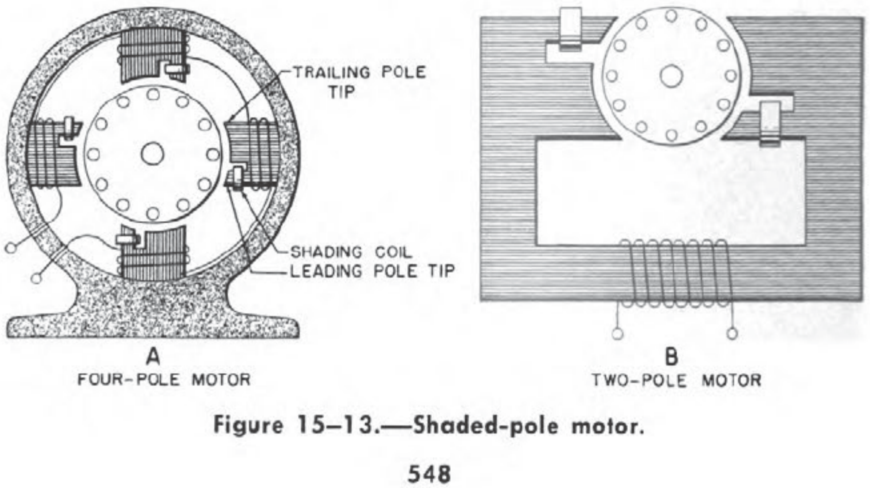


Figure 15–13.—Shaded-pole motor.

motor is generally manufactured in very small sizes, up to $\frac{1}{20}$ horsepower. A 4-pole motor of this type is illustrated in figure 15–13, A. The shading coils are placed around the leading pole tip and the main pole winding is concentrated and wound around the entire pole. The 4 coils comprising the main winding are connected in series across the motor terminals. An inexpensive type of 2-pole motor employing shading coils is illustrated in figure 15–13, B.

During that part of the cycle when the main pole flux is increasing, the shading coil is cut by the flux, and the resulting induced emf and current in the shading coil tend to prevent the flux from rising readily through it. Thus, the greater portion of the flux rises in that portion of the pole that is not in the vicinity of the shading coil. When the flux reaches its maximum value, the rate of change of flux is zero, and the voltage and current in the shading coil also are zero. At this time the flux is distributed more uniformly over the entire pole face. Then as the main flux decreases toward zero, the induced voltage and current in the shading coil reverse their polarity, and the resulting magnetomotive force tends to prevent the flux from collapsing through the iron in the region of the shading coil. The result is that the main flux first rises in the unshaded portion of the pole and later in the shaded portion. This action is equivalent to a sweeping movement of the field across the pole face in the direction of the shaded pole. The cage rotor conductors are cut by this moving field and the force exerted on them causes the rotor to turn in the direction of the sweeping field.

Most shaded-pole motors have only one edge of the pole split, and therefore the direction of rotation is not reversible. However, some shaded-pole motors used in fire control and interior communications circuits have both leading and trailing pole tips split to accommodate shading coils. The leading pole tip shading coils form one series group, and the trailing pole tip shading coils form another series group. Only the shading coils in one group are simultaneously active, while those in the other group are on open circuit.

The shaded-pole motor is similar in operating characteristics to the split-phase motor. It has the advantages of simple construction and low cost. It has no sliding electrical contacts and is reliable in operation. However, it has low starting torque, low

efficiency, and high noise level. It is used to operate small fans. The shading coil and split pole are used in clock motors to make them self-starting.

Repulsion-Start Motor

The repulsion-start motor has a form-wound rotor with commutator and brushes. The stator is laminated and contains a distributed single-phase winding. In its simplest form, the stator

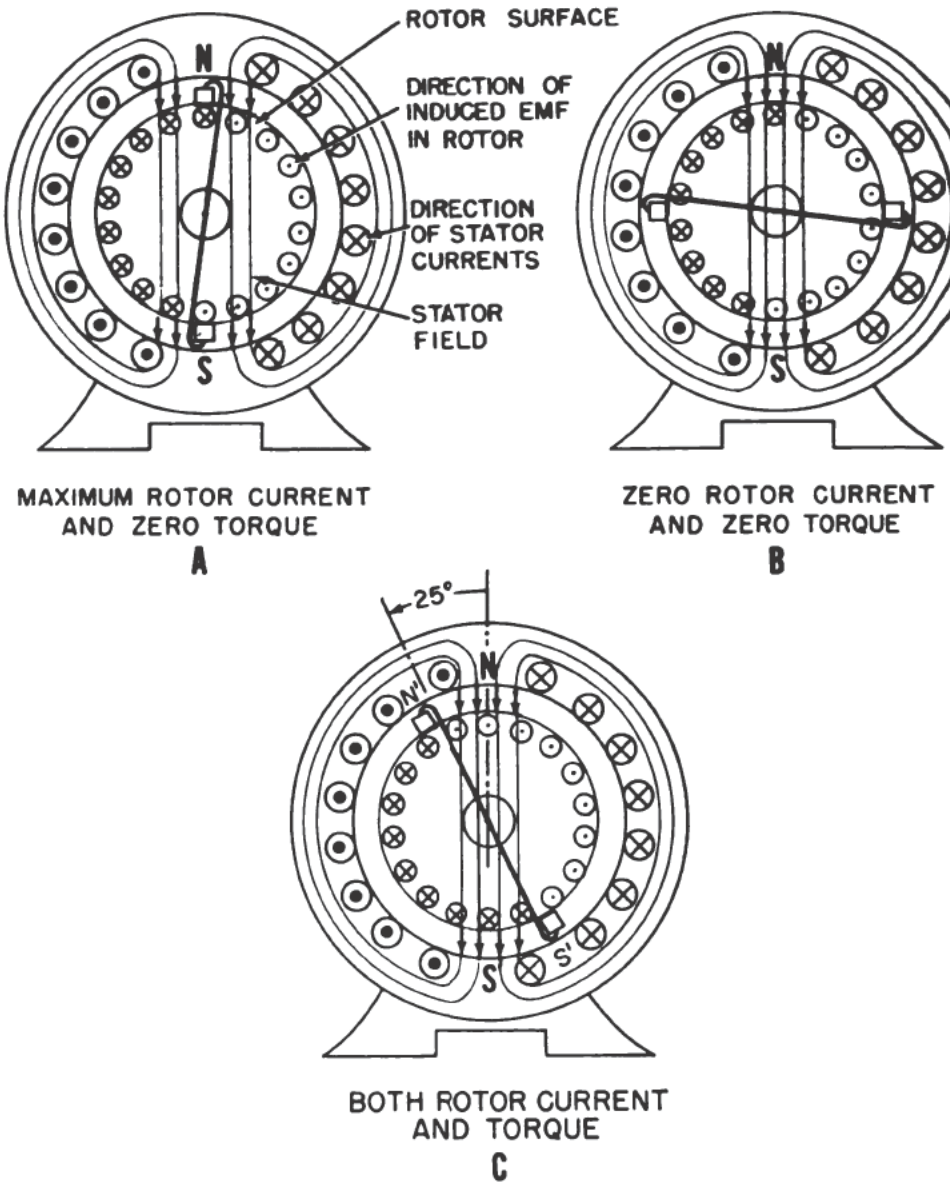


Figure 15–14.—Repulsion-start induction motor.

resembles that of the single-phase motor. In addition, the motor has a centrifugal device which removes the brushes from the commutator and places a short-circuiting ring around the commutator. This action occurs at about 75 percent of synchronous speed. Thereafter, the motor operates with the characteristics of the single-phase induction motor.

The starting torque of the repulsion-start induction motor is developed through the interaction of the rotor currents and the single-phase stator field. Unlike the split-phase motor, the stator field does not rotate at start, but alternates instead. The rotor currents are induced through transformer action. For example, in the 2-pole motor of figure 15–14, A, the stator currents are shown for the instant when a north pole is established on the upper side of the stator, and a south pole on the lower side. The induced voltage in the rotor causes the rotor currents to flow in such a direction as to oppose the stator field. These currents flow in opposite directions under the left and right portions of the north pole and in similar manner under the south pole. Thus, the net force to turn the rotor is zero when the brushes are located in the positions shown.

In figure 15–14, B, the brushes are moved 90° from their original positions, and again there is no rotor turning effort because in this case the rotor current is zero. There can be no rotor current in this position because the transformer induced voltages are equal and opposite to each other in the two halves (upper and lower) of the rotor winding.

In figure 15–14, C, the brush axis is displaced from the stator polar axis by an angle of about 25°, and in this position maximum torque is developed.

The direction of the induced currents in the rotor under the north pole of the stator is toward the observer, and under the south pole, away from the observer. Applying the right-hand rule for motors, the force acting on the conductors under the north pole is toward the left, and under the south pole, toward the right, thus tending to turn the rotor in a counterclockwise direction. When the stator polarity reverses, the direction of the rotor current also reverses, thereby maintaining the same direction of rotation. The function of the commutator and brushes is to divide the rotor currents along an axis that is displaced from the axis of the stator field in a counterclockwise direction. The motor

derives its name from the repulsion of like poles between the rotor and stator. Thus, the rotor currents establish the rotor poles N'–S', which are repelled by the stator poles N–S.

The starting torque is 250 to 450 percent of the full-load torque, and the starting current is 375 percent of the full-load current. This motor is made in fractional horsepower sizes and in larger sizes up to 15 horsepower, but has been replaced in large part by the cheaper and more rugged capacitor motor. The repulsion-start motor has higher pull-out torque (torque at which the motor stalls) than the capacitor-start motor, but the capacitor start motor can bring up to full speed loads that the repulsion motor can start but cannot accelerate.

A-c Series Motor

The a-c series motor will operate on either a-c or d-c circuits. It will be recalled that the direction of rotation of a d-c series motor is independent of the polarity of the applied voltage, provided the field and armature connections remain unchanged. Hence, if a d-c series motor is connected to an a-c source, a torque will be developed which tends to rotate the armature in one direction. However, a d-c series motor does not operate satisfactorily from an a-c supply for the following reasons:

1. The alternating flux sets up large eddy-current and hysteresis losses in the unlaminated portions of the magnetic circuit and causes excessive heating and reduced efficiency.
2. The self-induction of the field and armature windings causes a low power factor.
3. The alternating field flux establishes large currents in the coils that are short-circuited by the brushes; this action causes excessive sparking at the commutator.

To design a series motor for satisfactory operation on alternating current the following changes are made:

1. The eddy-current losses are reduced by laminating the field poles, yoke, and armature.
2. Hysteresis losses are minimized by using high-permeability transformer-type silicon-steel laminations.
3. The reactance of the field windings is kept satisfactorily low by using shallow pole pieces, few turns of wire, low frequency (usually 25 cycles for large motors), low flux density, and low reluctance (a short air gap).

4. The reactance of the armature is reduced by using a compensating winding embedded in the pole pieces. If the compensating winding is connected in series with the armature (fig. 15–15, A), the armature is CONDUCTIVELY compensated.

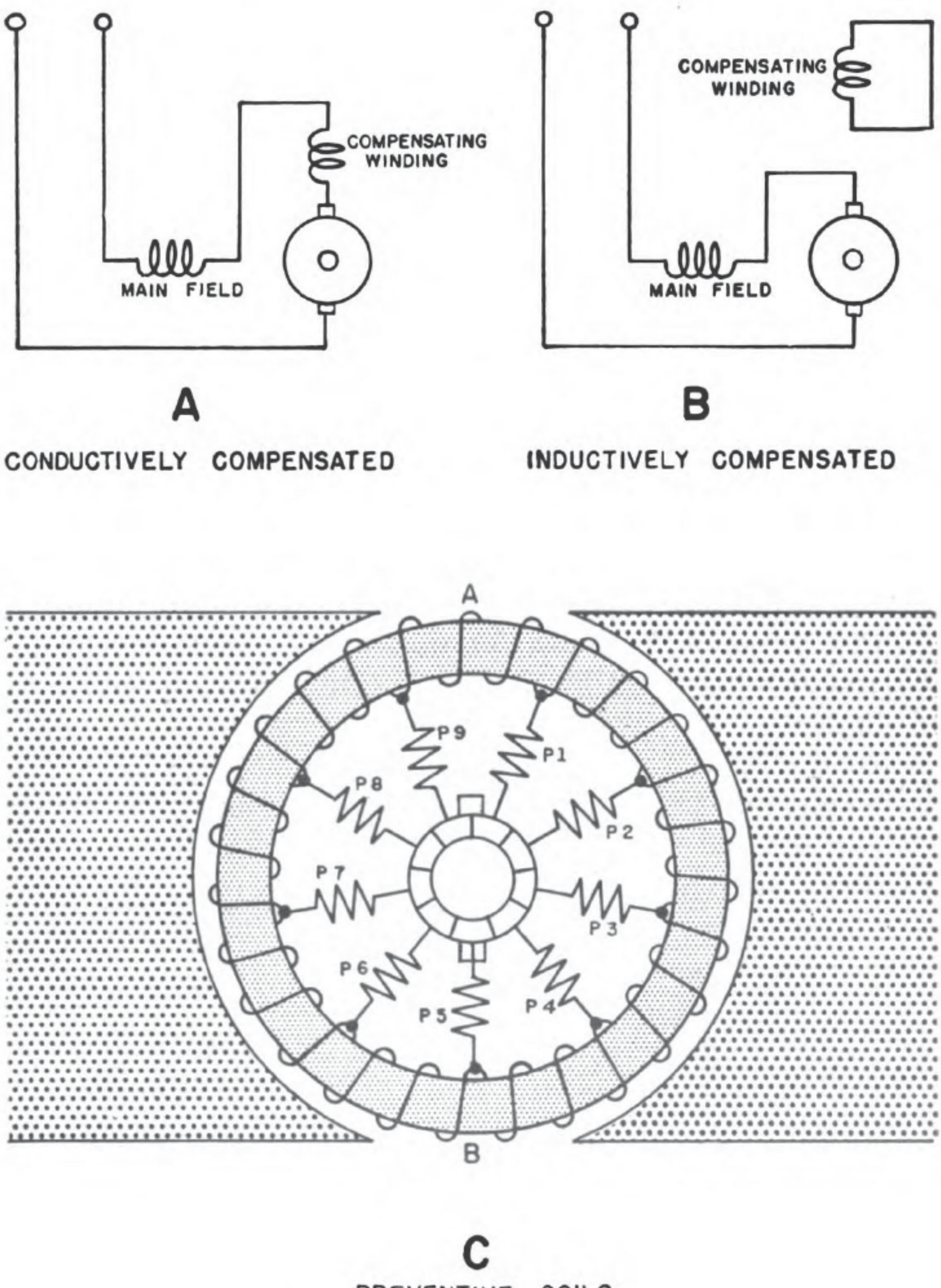


Figure 15–15.—A-c series motor.

If the compensating winding is short-circuited on itself (fig. 15–15, B), the armature is INDUCTIVELY compensated. If the motor is designed for operation on both d-c and a-c circuits, the compensating winding is connected in series with the armature. The axis of the compensating winding is displaced from the main field axis by an angle of 90°. This arrangement is similar to the compensating winding used in some d-c motors and generators to overcome armature reaction. The compensating winding establishes a counter magnetomotive force, which neutralizes the effect of the armature magnetomotive force, thereby preventing distortion of the main field flux and reducing the armature reactance. The inductively compensated armature acts like the primary of a transformer, the secondary of which is the shorted compensating winding. The shorted secondary receives an induced voltage by the action of the alternating armature flux, and the resulting current flowing through the turns of the compensating winding establish the opposing magnetomotive force that neutralizes the armature reactance.

5. Sparking at the commutator is reduced by the use of PREVENTIVE LEADS P_1, P_2, P_3, and so forth (fig. 15–15, C). A ring armature is shown for simplicity. When coils at A and B are shorted by the brushes, the induced current is limited by the relatively high resistance of the leads. Preventive leads are used in some of the older type a-c series motors on electric locomotives. Sparking at the brushes is also reduced by using armature coils having only a single turn and multipolar fields. High torque is obtained by having a large number of armature conductors and a large-diameter armature. Thus, the commutator has a large number of commutator bars that are very thin, so that the armature voltage is limited to about 250 volts.

The operating characteristics of the a-c series motor are similar to those of the d-c series motor. The speed will increase to a dangerously high value if the load is removed from the motor.

Fractional horsepower a-c series motors are called UNIVERSAL MOTORS. They do not have compensating windings or preventive leads. They are used extensively in the Navy to operate fans and portable tools, such as drills, grinders, and saws.

QUIZ

1. Upon what two factors does the speed of the revolving field in an induction or synchronous motor depend?
2. How is the direction of rotation of a magnetic revolving field changed in a 3-phase winding, as in figure 15–1?
3. What is the speed (in rpm) of a magnetic revolving field if the applied voltage has a frequency of 60 cps and there are 8 poles produced by the 3-phase winding?
4. In a cage-rotor induction motor, how is the current supplied to the rotor?
5. To what extent is the speed of the revolving field affected by the load on the motor?
6. How is torque developed in an induction motor?
7. Why is it unnecessary to insulate the bars in a cage rotor?
8. What is the purpose of the variable wye-connected resistance used with the form-wound rotor?
9. How is the torque of an induction motor affected by an increase in (1) rotor current, (2) stator field strength, and (3) rotor power factor?
10. Why is the power factor of a cage rotor low and lagging at start?
11. Why does the inductive reactance of a cage rotor approach zero at no-load speed?
12. At 100-percent slip, what factors limit the torque of a low-resistance-rotor induction motor to a relatively low value?
13. At 0-percent slip, what factor would cause the torque to be zero?
14. For a constant applied stator voltage, at what phase angle between rotor current and rotor voltage is the torque of an induction motor a maximum?
15. If the stator voltage of an induction motor is increased until the iron is saturated magnetically, what is the effect on the stator counter emf and current?
16. If the rotor of an induction motor could reach synchronous speed, what would be the relative values of the induced voltage and frequency?
17. What are the three types of losses found in induction motors?
18. What characteristics of the cage-rotor induction motor are essentially the same as those of the d-c shunt motor?
19. What rotor power-factor angle corresponds to the pull-out condition, as shown in figure 15–4, A?
20. When the cage-rotor motor is running, what is the principal opposition to the rotor current?

21. If a cage-rotor motor is designed to start under a heavy load, will it have a high or a low rotor resistance?
22. The starting torque of an induction motor can be made equal to the pull-out torque by making what two values equal at standstill?
23. Why is a source of d-c potential needed in a synchronous motor?
24. Most synchronous motors and a-c generators are similar in what two respects?
25. What is the function of the cage winding on a synchronous motor?
26. Why is a resistor of low resistance connected across the d-c field winding of a synchronous motor during the starting period?
27. For a given load on a synchronous motor, what governs the power factor?
28. For a given load on a synchronous motor, what is the relative magnitude of the stator current when the field excitation is normal compared to the magnitude of the stator current when the field is over or under normal excitation?
29. What is the name commonly applied to a synchronous motor that is used to correct a lagging power factor in a power circuit by connecting it in parallel with the circuit?
30. What are the two reasons why across-the-line starters are the most common form used aboard ship?
31. In what circuit of the induction motor is (a) the primary resistor starter inserted? (b) the secondary resistor starter?
32. What is the function of the autotransformer starter with respect to changing the voltage applied to the induction motor during and after acceleration?
33. What are two advantages of using single-phase motors in the small sizes rather than 3-phase motors?
34. In the split-phase motor, why does the rotor current lag the rotor voltage at start by approximately 90°?
35. After the split-phase motor comes up to speed, how is the rotating field maintained?
36. How may the direction of rotation of a split-phase motor be reversed?
37. Why is the starting torque greater in the capacitor motor than in the split-phase motor?
38. In the shaded-pole motor, what causes the magnetic field to sweep across the pole face from the unshaded to the shaded portion?
39. What are three disadvantages of shaded-pole motors?
40. Why is the repulsion-start induction motor so called?

CHAPTER 16

A-C INSTRUMENTS

INTRODUCTION

The D'Arsonval-type meter (ch. 9) is basically a d-c instrument. However, it may be used to measure a-c current or voltage if additional equipment is used to convert the meter current to d-c. The proper type of rectifier may be employed for this purpose; thermocouples (ch. 9) may also be employed.

The moving iron-vane meter may be used for d-c or a-c. However, its widest application is in the measurement of a-c values. Likewise, the electrodynamometer-type instrument may be used for both a-c and d-c.

There are other types of instruments, however, that are used only for the measurement of a-c values. These include induction-type instruments, frequency meters, power-factor meters, and synchroscopes. Inductance and capacitance bridges also employ alternating current.

The technician will find that an elementary knowledge of some of the more common types of a-c indicating instruments is of great value in understanding and evaluating a-c circuit operation.

In treating the following instruments, circuit operation is emphasized, and toward this end simplified circuits are employed. In practice, more complicated circuits may be encountered. However, the basic principles are the same, and an understanding of the operation of a specific equipment may be acquired by a careful study of the instruction book provided with the equipment.

RECTIFIER TYPE A-C METERS

If current flows in the proper direction through the moving coil of the D'Arsonval meter, a force acts on the coil that causes the pointer to move up scale. If current is sent through the coil in the reverse direction, the force reverses and the pointer tends

to move off scale to the left of the zero mark. If a current that rapidly reverses in direction (a-c current) is sent through the moving coil, the force acting on the coil rapidly reverses first in one direction and then in the other, and the pointer vibrates about the zero position, giving no useful indication.

However, the D'Arsonval movement may be used to measure alternating current or voltage if the current that passes through the meter is first rectified—that is, changed from alternating current to direct current. Figure 16–1 indicates how copper-oxide rectifiers may be used with the D'Arsonval movement to permit a-c voltage or current to be measured. The subject of rectifiers is treated in detail in chapter 3 of *Basic Electronics,* NavPers 10087, and therefore will not be treated in detail in this chapter.

Essentially, the copper-oxide (metallic) rectifier tends to permit current to flow through it in only one direction. A copper-oxide rectifier symbol is shown in figure 16–1, A. The arrow head of the symbol represents the copper and the heavy horizontal line just above the arrow tip represents the copper oxide. Electrons flow from the copper to the copper oxide but not in appreciable numbers from the oxide to the copper. Therefore, when alternating current is applied to the circuit, electrons flow through the meter when point *A* (and therefore the arrow head of the symbol) is negative and point *B* is positive. During the portion of the cycle when *A* is positive and *B* is negative only a negligible number of electrons flow through the meter. Therefore, unidirectional current flows through the meter, and the D'Arsonval movement will respond to this type of current.

Both halves of the a-c cycle are utilized in the full-wave bridge-type rectifier shown in figure 16–1, B. When point *A* is negative, electrons flow from *A* to *C* to *D,* downward through the meter from *D* to *F,* and from *F* to *E* to *B.* When point *B* is negative, electrons flow from *B* to *E* to *D,* downward through the meter from *D* to *F,* and from *F* to *C* to *A.* During both halves of the a-c cycle, current flows through the meter from *D* to *F.* Therefore, the force acting on the moving coil does not reverse but consists of a series of pulses that moves the needle up scale against the restoring force of the coil spring.

The SINGLE-WAVE arrangement shown in figure 16–1, A, is not efficient. Also, because the rectifier offers a high resistance on the reverse half-cycle (when the copper is positive and the oxide

is negative), practically the entire line voltage is applied across the rectifier. If the breakdown voltage (the voltage that will puncture the oxide film) is exceeded during this half cycle a reverse current will flow through the rectifier and through the meter.

The use of the bridge-connected FULL-WAVE rectifier, as shown in figure 16–1, B, provides a more continuous unidirectional force on the moving coil of the meter. This type of rectifier is used to measure both current and voltage. When the input current or voltage is of a sine waveform, the moving coil is deflected an

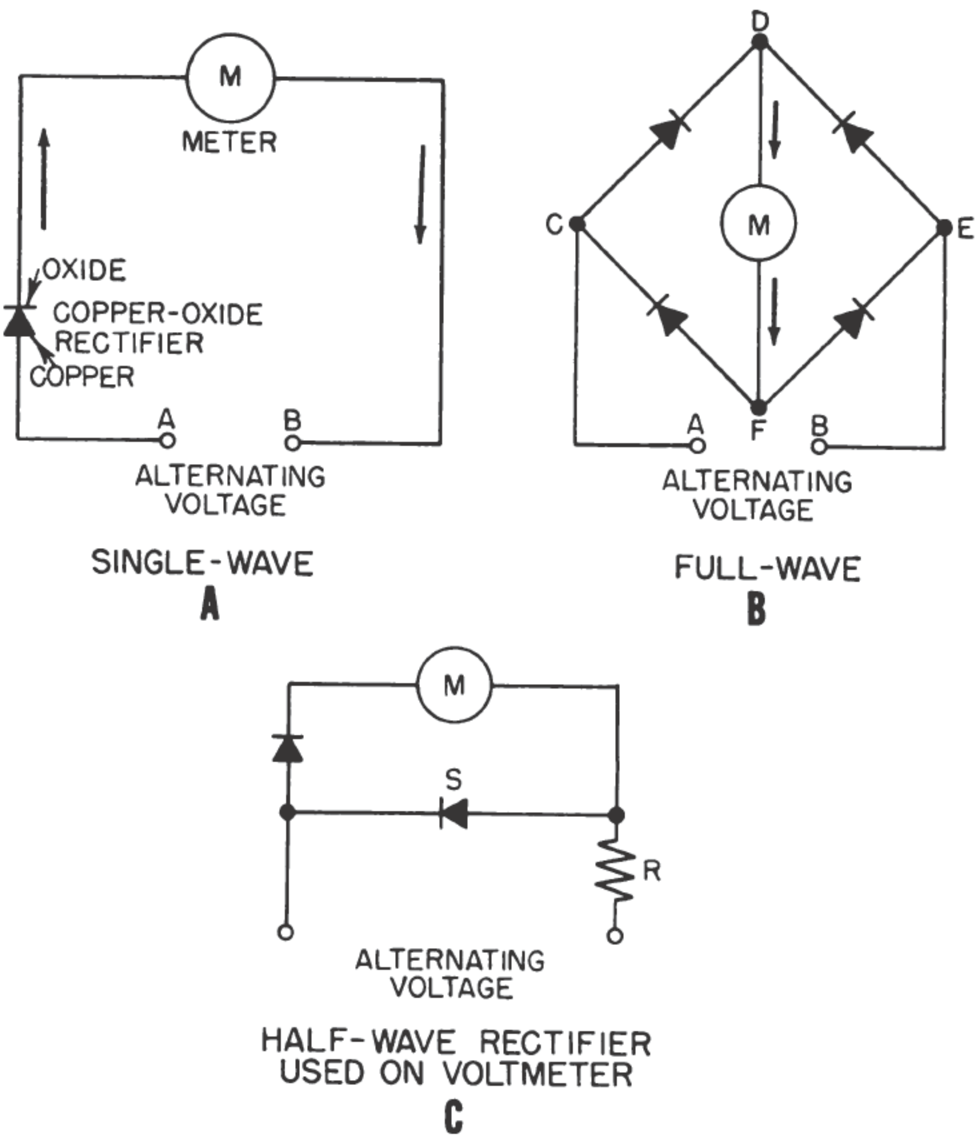


Figure 16–1.—Meter rectifiers.

amount that is proportional to the AVERAGE of all the instantaneous values of the pulsating d-c from the rectifier. However, because the EFFECTIVE, or root-mean-square (rms), values are more useful, a-c meters are generally calibrated to read rms values (rather than average values) that are 1.11 times the average of the instantaneous values. The full-wave rectifier type of instrument does not change appreciably the shape of the a-c wave passing through it.

A rectifier of a size convenient for mounting in the average meter case usually has a current rating of less than 15 milliamperes. It is possible to use shunts with rectifier-type meters to adapt them to different ranges of current measurement. The rectifier part of the instrument may be subjected to current overload for limited periods without undesirable effects, but excessive inverse voltage may cause breakdown.

Series resistors used in half-wave rectifier-type instruments to extend the voltage range will lower the ratio of forward to leakage (reverse) current and impair the efficiency of the rectifier unless certain changes are made. Figure 16–1, C, indicates a method of connecting the resistor in a circuit using two rectifiers. The shunt rectifier, *S*, provides the high ratio of forward to leakage current and tends to keep the needle from vibrating.

There are several advantages as well as certain limitations in the use of rectifier-type instruments for a-c measurements. Among the advantages, the sensitivity is considerably improved by using the D'Arsonval movement instead of the iron-vane movement. The increased sensitivity is especially important in electronic and communication service work. Also on most instruments, the scale is linear. Among the disadvantages are a variation in rectifier resistance with temperature, and a variation in rectifier characteristics with age. These two factors may cause incorrect readings.

As previously stated, most a-c voltmeters and ammeters are calibrated to indicate the rms values of sine waveforms. If the input waveform differs appreciably from the sine waveform the indication will not be accurate.

Rectifier-type meters are suitable for use on circuits in which the frequencies are of the order of power frequencies (60 cps) and audio frequencies (20 cps to 20,000 cps). At higher frequencies they have objectionable error caused by the associated circuit capacitive reactance shunting effect.

WATTMETERS

As has been stated in chapter 9, power is measured by means of a wattmeter. Because power depends on both current and voltage, a wattmeter must employ two elements—one current-operated and one voltage-operated. The electrodynamometer movement, described in chapter 9 and shown in figure 9–9, is well suited for use in a wattmeter. This type of instrument will operate on a-c or d-c. The movable coil forms the voltage element, and the stationary coils form the current element.

The force acting on the movable coil (potential coil) is proportional to the product of the (1) current in the moving coil, (2) magnetic field produced by the stationary coils (current coils), and (3) cosine of the phase angle between them. Because there is no iron in the movable coil and it contains very few turns of wire, the current in the moving coil is in phase with and proportional to the line voltage. The in-phase relationship is further enhanced by inserting noninductive resistance in series with the moving coil in sufficient quantity to make the entire movable coil circuit resistive in nature. This resistance also limits the movable coil current to a relatively low value.

The stationary coils produce a field that is proportional to the line current. Because there is no iron in these coils, the magnitude of the field varies directly with the line current and is in phase with the line current. The force acting on the moving coil of the wattmeter is therefore proportional to the product of line voltage, line current, and the cosine of the phase angle between them. The cosine of this angle is the line power factor. Therefore, the force acting on the moving coil of the wattmeter is proportional to the true power in the line. This force moves the movable coil and indicating pointer up scale against the restoring force of a phosphor bronze coil spring. When a balance is obtained between these two forces the pointer indicates the true power because the angle of displacement of the spring is proportional to the actuating force.

Therefore, the power, as indicated by the movement of the pointer, depends on the magnitude of the potential across the load, the current through the load, and the power factor of the load. In effect, the wattmeter multiplies the voltage, current, and power factor to indicate the true power.

When the power factor is unity (phase angle $=0°$), the currents in both the stationary coils and the movable coil in a wattmeter reverse in unison. The result, as far as the movable element is concerned, is a deflecting force that is always in the same direction. Therefore, the deflecting force and the meter indication are maximum for a given load voltage and current.

When the power factor is less than unity, the currents in the stationary coils and the movable coil do not reverse at exactly the same time, and the result is a reverse component of driving force that reduces the net force on the moving coil during the time the two currents are in opposite directions. Because of its inertia, the moving element (movable coil and pointer) does not follow the individual reversals, but does follow the resultant of the forces acting on it. The magnitude of the wattmeter indication is therefore reduced as the phase angle is increased from 0 to 90°. For a phase angle of 90°, the two opposing forces are equal and the wattmeter indicates zero power. Thus, a wattmeter indicates the true average power of an a-c circuit.

The stationary coils of a wattmeter are connected in series with the load, and the movable coil (with its series resistor) is connected across the load. As will be shown later, current and potential transformers may be employed with wattmeters. Wattmeter scales are cramped only slightly at each end; otherwise, they are practically linear throughout.

Wattmeters are rated in volts, amperes, and watts. Because wattmeters do not indicate volts or amperes directly, but only as a product (including the power factor) the magnitude of the line voltages and current should not exceed the rated values for the meter in order not to overload either the movable (potential) coil circuit or the stationary (current) coil circuit when the meter is connected in the line. For example, in figure 16–2, if the movable coil circuit is rated at 125 volts and the stationary coil circuit is rated at 5 amperes, the full-scale deflection is 5×125, or 625 watts. If the line voltage is 120 volts, the line current 5 amperes, and the load power factor 50 percent ($\theta = 60°$), the wattmeter indication will be $120 \times 5 \times 0.5 = 300$ watts. Although both the movable coil and the stationary coil circuits are operating at their approximate maximum values, the pointer does not indicate this fact because it is at its half-scale reading.

The wattmeter movement is mounted in a laminated steel box to shield it from stray magnetic fields. The moving coil is air-damped by an aluminum vane moving in a stationary closed box. As the pointer moves, the vane forces air from one side of the box to the other, around the edges of the vane.

The polarity markings aid in making the proper connections of the meter to the line (fig. 16–2). The movable coil terminal, which has the polarity mark (±), is connected to the same side

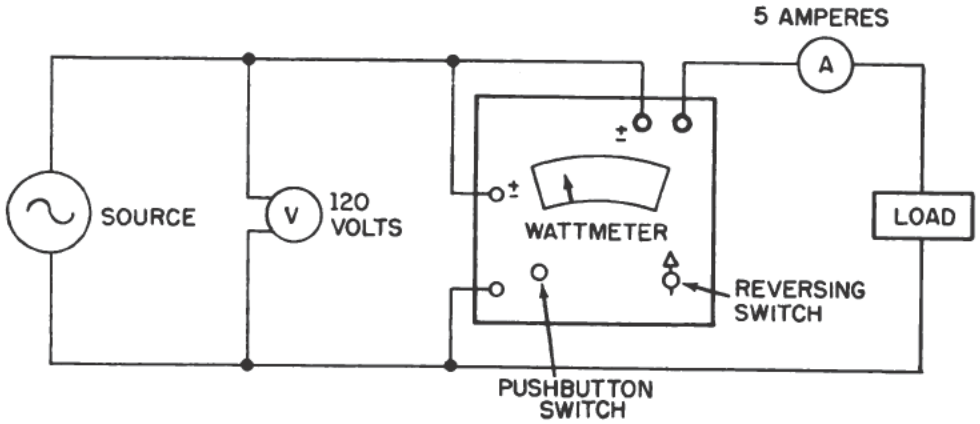


Figure 16–2.—A-c indicating wattmeter connections.

of the line in which the stationary coils are connected. This arrangement reduces electrostatic error. The movable coil is connected in series with a high noninductive resistance. Most of the line voltage is developed across this resistance, leaving only a small voltage across the movable coil itself. The difference in potential between the movable coil and the stationary coils is limited to a low value and the meter calibration is made for this condition.

The high resistance in the movable coil circuit is usually composed of manganin zero-temperature resistance wire wound noninductively on spools or on card frames. Vents in the meter case provide air cooling. The pushbutton switch is in the movable coil circuit and may be locked in the operated position by turning the button. The reversing switch is also in series with the movable coil circuit and, when operated, reverses the connections of the movable coil with respect to the current-limiting resistance. This action reverses the direction of the current in the movable coil with respect to that of the stationary coils and reverses the force acting on the movable coil. This current reversal is sometimes

necessary when measuring the total power in a three-phase circuit when the load is balanced and the power factor is less than 50 percent. This condition is described in the chapter on three-phase power.

WATT-HOUR METERS

A WATTMETER indicates the instantaneous rate of power consumption of a device or a circuit. A WATT-HOUR METER integrates (sums up) all the instantaneous rates so that the total energy utilized in a given time interval can be determined. The watt-hour meter in effect multiplies the power in watts by the time in hours. When large amounts of electrical energy are used, a more convenient unit is the kilowatt-hour, and the instrument used for this purpose is the KILOWATT-HOUR METER. This type of instrument measures the electric energy used in homes, factories, and certain Navy installations.

Thompson Watt-Hour Meter

A simplified diagram of a Thompson watt-hour meter is shown in figure 16–3. This is one of the early types of d-c watt-hour meters. As may be seen in the figure, the instrument is essentially

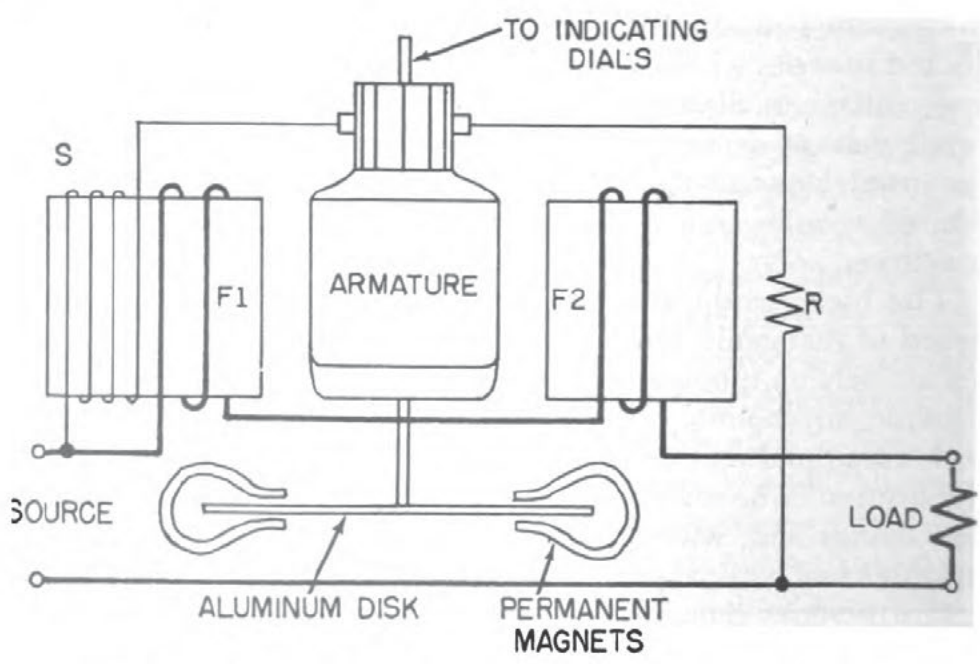


Figure 16–3.—Simplified diagram of Thompson watt-hour meter.

a series motor. The series combination of coil S, the armature, and resistor R is connected across the load. The field coils, $F1$ and $F2$, are wound with a few turns of heavy wire through which the load current flows. This establishes the field in which the armature rotates.

The operation of this type of d-c watt-hour meter may be explained as follows:

1. The current that flows through the load also flows through $F1$ and $F2$, producing a field that is proportional to the load current.
2. The voltage that is applied to the load is also applied to the armature, coil S, and resistor R. The current through the armature is proportional to this voltage.
3. The generated torque that causes the armature to revolve is proportional to the product of the current through the load and the voltage across the load—in other words, proportional to the watts consumed by the load.
4. Some form of resistance (or opposing torque) must be provided that will vary directly with the speed of the armature in order that the motor speed will be directly proportional to the power consumed in the load. The opposing torque is obtained by connecting an aluminum disk to the lower end of the armature shaft so that it will rotate in the two magnetic fields established by the permanent magnets. Eddy currents are induced in the disk as it rotates through the fields. These currents produce a retarding force on the disk that is proportional to the speed of the disk. When the line power increases, the watt-hour meter speeds up until the disk load torque increases to the same value as the increased driving torque. At this condition of equilibrium, the speed is constant and proportional to the increased load power.
5. For any given load, the amount of energy indicated on the dials is proportional to the product of the speed of the disk and the time during which the power is being integrated.

The function of coil S is to produce just enough field flux to interact with the armature flux to overcome static friction so that when a small load is connected, the meter will immediately rotate at a speed proportional to this load. The full-load adjustment is made by moving the position of the drag magnets on the disk. Moving the magnets toward the center of the disk will

lessen the drag and allow the meter to speed up for a given electrical load in the line.

Single-Phase Induction Watt-Hour Meter

As in the case of series motors, the Thompson watt-hour meter may be used with alternating or direct current because both the armature and the field flux reverse at the same time, and the armature continues to rotate in the same direction. The induction watt-hour meter, however, has certain advantages over the Thompson watt-hour meter and is commonly used to measure a-c power.

The SINGLE-PHASE INDUCTION WATT-HOUR METER includes a simple induction-drive motor consisting of an aluminum disk, moving magnetic field, drag magnets, current and potential coils, integrating dials, and associated gears. A simplified sketch of an induction watt-hour meter is shown in figure 16–4, A. The potential coil connected across the load, is composed of many turns of relatively small wire. It is wound on one leg of the laminated magnetic circuit. Because of its many turns, the potential coil has high impedance and high inductance, and therefore, the current through it lags the applied voltage by nearly 90°. The two current coils connected in series with the load, are composed of a few turns of heavy wire. They are wound on two legs of

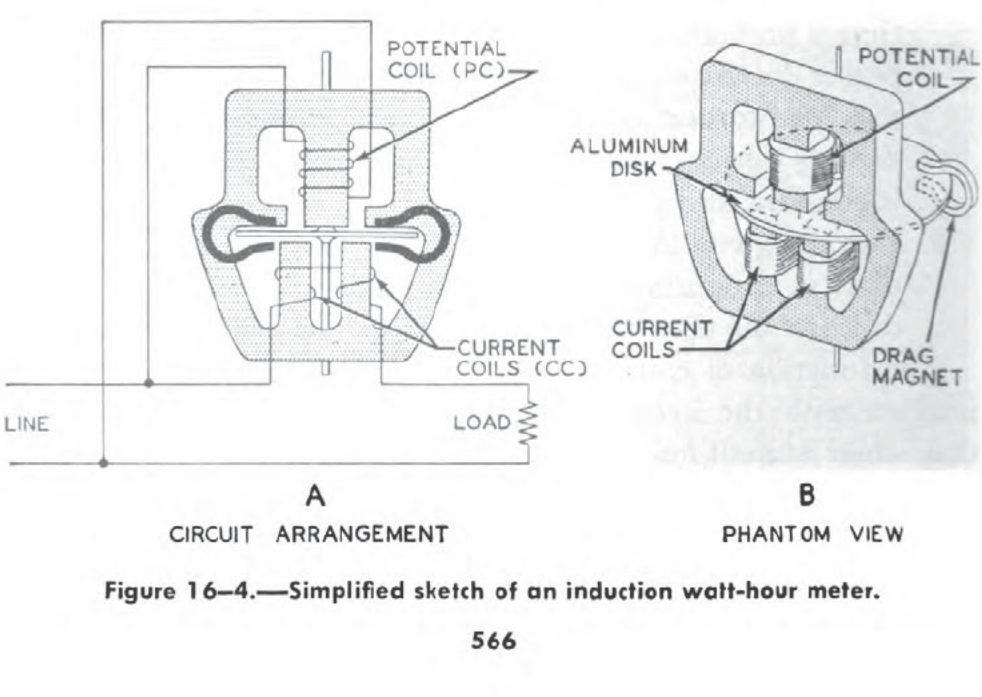


Figure 16–4.—Simplified sketch of an induction watt-hour meter.

the laminated magnetic circuit. Because of the few turns, the current coils have low inductance and low impedance.

The arrangement of the potential coil, the aluminum disk, the current coils, and one of the drag magnets is shown in the phantom view (fig. 16–4, B).

Figure 16–5 shows the potential, current, and flux curves associated with the induction watt-hour meter. Also shown are the instantaneous direction of eddy currents in the aluminum disk and the instantaneous directions of the flux through the disk at four instants during the cycle. To understand the operation of the induction watt-hour meter it is necessary to understand how the force, F, is generated that causes the aluminum disk to rotate.

The force, F, that causes the aluminum disk to move in a clockwise direction, looking down on the disk from above, is explained as follows:

1. At instant 1, assuming a load of unity power factor, the current, i_{pc}, and the flux, Φ_{pc}, of the potential coil are both maximum. The line current, i_{line} (shown as i_{cc} in the figure), and the flux, Φ_{cc}, of the current coils are zero, but they are both CHANGING at their maximum rate. As a result of the rapid flux change through the disk, eddy currents are established in the disk in two distinct paths above the current coils. The two currents join under the potential coil and flow toward the edge of the disk (toward the observer). The right-hand rule for motor action, applied to this area of current flow, indicates a force acting on the disk that tends to turn the disk clockwise.
2. At instant 2, i_{line} and Φ_{cc}, the current and flux respectively of the current coils, are at a maximum. The current, i_{pc}, and the flux, Φ_{pc}, of the potential coil are zero, but they are both CHANGING at their maximum rate. As a result of the rapid flux change through the disk, eddy currents are established in the disk in the region below the potential coil. The current path is shown as a circle around the area covered by the potential coil. On the right-hand side, the current is moving toward the observer and directly through the flux established by the right-hand current coil. On the left-hand side, the current is moving away from the observer and directly through the flux established by the left-hand current coil. As may be seen by the application of the right-

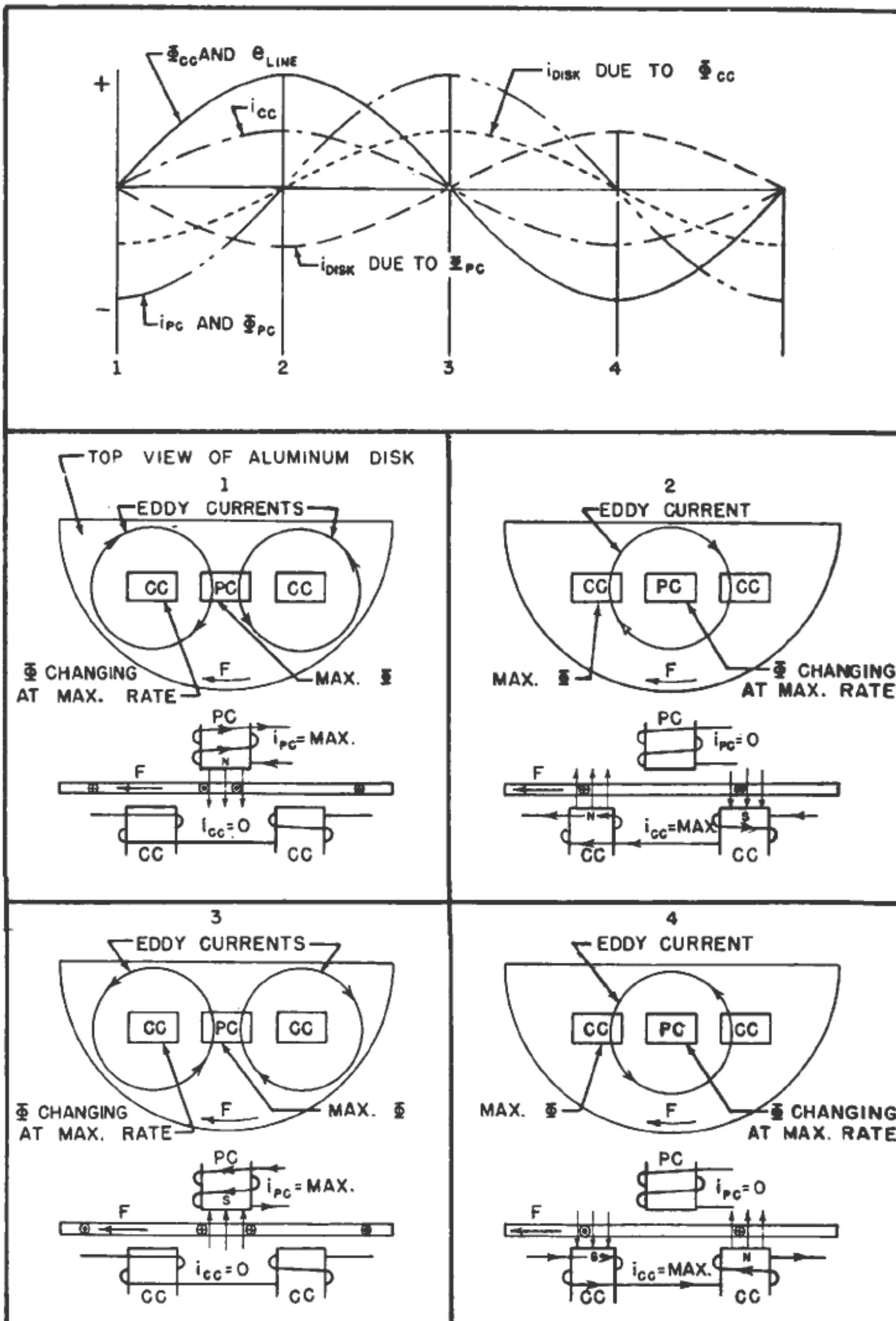


Figure 16–5.—Operation of induction watt-hour meter.

hand motor rule, the force, F, acting on both areas of the disk tends to turn the disk clockwise.

3. Similar reasoning may be applied at instants 3 and 4. In each instant, the direction of the eddy currents and the flux through which the currents flow are such that the resultant forces tend to turn the disk clockwise.

From the foregoing discussion it is seen that there are established in the disk currents that flow about each polar region in circles. The induced currents due to the potential coil are 90° out of phase with those due to the current coils when the load power factor is unity. Part of the current induced by the current coils passes under the pole of the potential coil and therefore through a magnetic field in phase with it. For power factors less than unity, the phase difference between these currents and the magnetic fields through which they pass is the same as the difference in phase between the load current and load voltage. Hence, the driving torque on the disk is proportional to the true power supplied through the line to the load. A braking system similar to that of the d-c Thompson watt-hour meter causes the speed of the disk to be proportional to the average torque and therefore the number of revolutions is proportional to the energy supplied to the load.

Summarizing:

1. Driving torque is proportional to $EI \cos \theta$, where E is the load voltage, I the load current, and $\cos \theta$ the load power factor.
2. Disk load is proportional to $\dfrac{\text{revolutions of disk } (R)}{\text{time } (t)}$.
3. For one load, driving torque = load torque.

Hence,

$$EI \cos \theta = K \frac{\text{revolutions of disk } (R)}{\text{time } (t)},$$

where K is the disk constant, which is the number of watt-hours per revolution. Transposing,

$$EI\, t \cos \theta = KR.$$

The total energy supplied to the load is proportional to the number of revolutions of the disk.

A small copper shading disk (not shown in the figure) is placed under a portion of the potential pole face in an adjustable mounting to develop a torque in the disk to counteract static friction. The disk has the effect of a shading pole and provides a light load adjustment for the meter.

The two drag magnets supply the counter or load torque against which the aluminum disk acts when it turns. The drag is increased (speed of motor reduced) by moving the magnets toward the edge of the disk. Conversely, the drag is decreased and the speed is increased by moving the magnets toward the center of the disk.

INSTRUMENT TRANSFORMERS

It is not usually practicable to connect meters directly to high-voltage a-c circuits. Therefore, meters are coupled to these circuits by means of instrument transformers. These devices permit the use of standard low-voltage meters for all high-voltage or high-current a-c circuits, and at the same time protect the operating personnel from the high-voltage circuits. Meter transformers used to reduce large currents to small proportionate values of current for the operation of ammeters and the current coils of other meters, instruments, and relays are called CURRENT TRANSFORMERS. Meter transformers used to reduce high voltages to small proportionate values of voltage for the operation of voltmeters and the potential coils of other meters, instruments, and relays are called POTENTIAL TRANSFORMERS.

Current Transformer

The primary winding of a current transformer consists of a few turns of heavy wire wound on a laminated iron core, or the primary may be the busbar or cable carrying the line current, as shown in figure 16–6. The primary winding (when a winding is actually used) has a low impedance and is connected in series with the line. The secondary is connected to an ammeter or to the current coils of other instruments or relays.

The flux density in the core is kept low because of the reaction of the secondary ampere-turns. This reaction results from the fact that the instantaneous current in the primary tends to establish the flux in one direction through the core, and the induced

current in the secondary (by Lenz's law) tends to establish it in the opposite direction. The net result is a low flux density established in the direction of the primary ampere-turns working in opposition to the secondary ampere-turns.

With a flux density that varies directly with the load current, the meter current is directly proportional to the load current for all values of meter current not in excess of the transformer volt-

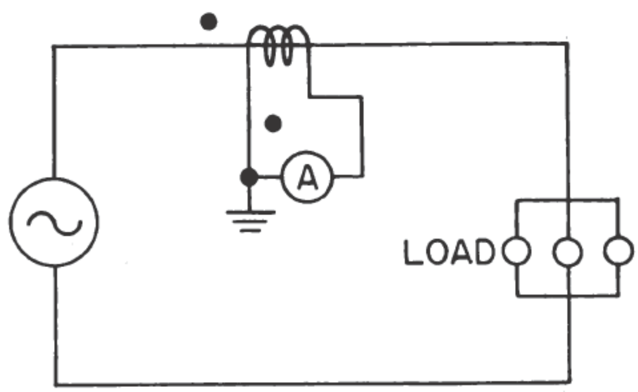


Figure 16–6.—Current transformer.

ampere rating (burden on the transformer). If the core should approach the condition of saturation, the linear relationship would no longer exist, and the meter connected in the secondary circuit would give an incorrect indication.

The secondary winding consists of many turns of relatively small wire, and is usually rated at 5 amperes for full-load primary current.

The secondary circuit of a current transformer is always maintained in a closed-circuit connection. The impedance of this circuit is low—approaching that of a short circuit. Consequently, the secondary voltage of the current transformer is only high enough to cause current to flow in the low-impedance circuit of an ammeter or the current coils of a wattmeter. The low secondary voltage is caused by the low flux density of the transformer core.

THE SECONDARY OF A CURRENT TRANSFORMER SHOULD NEVER BE OPEN-CIRCUITED WHILE THE PRIMARY IS ENERGIZED. The constant-potential transformer has a constant exciting current and a constant flux density irrespective of secondary load. Variations in secondary load vary the load component of primary current in

direct proportion, with little change in total flux or secondary voltage. Unlike the current through the primary of a constant-potential transformer, the current through the primary of the current transformer is determined by the load on the system, not by the burden on the transformer imposed by the meter load connected to its secondary. This meter load is constant; the system load varies. If the secondary were opened while the primary was drawing a heavy current, a high voltage dangerous to life, would be developed across the secondary because of the great increase in flux density of the core and the step-up turns ratio. The flux density would increase because the opposing effect of the secondary ampere-turns would no longer exist. This action would damage the magnetic property of the iron and destroy the calibration. Open-circuited current transformers have been known to develop as much as 60,000 volts across the secondary, which normally would have only a few millivolts on closed circuit.

To prevent injury to the operating personnel or damage to the meter, the secondary must always be shorted before the meter is removed. A shunting switch is provided for this purpose.

Current transformers for use with portable a-c ammeters are constructed of ring-shaped iron cores on which the secondary is toroidally wound. For convenience, the iron core may be hinged so that it can be placed around the primary conductor more conveniently.

Polarity markings are placed on the primary and secondary terminals of current transformers. These usually are in the form of a white dot or **X** mark. There is one on a primary lead and one on a secondary lead. The dot indicates that when the current flowing IN to the primary terminal so marked is a maximum, the current flowing OUT of the secondary terminal having the correspondingly polarity mark will be a maximum.

The following precautions must be taken when working with current transformers:

1. Check to see if the transformer is designed for the meter and for the value of current to be measured. Do not exceed the voltage-ampere rating of the transformer.
2. Make connections only when the circuit is deenergized.
3. Ground one end of the secondary and also the metal case.
4. Never open the secondary while the primary is energized.
5. Always connect the primary in series with the line carrying the current.

Potential Transformer

The potential instrument transformer is similar basically to the constant-potential step-down power transformer except that the volt-ampere rating and size of the potential instrument transformer are very much less. The primary winding of the potential transformer is connected through fuses across the high-voltage line, and the secondary is connected to the meter terminals and ground, as shown in figure 16–7. The secondaries of potential transformers have ratings of 40 to 500 volt-amperes. These windings are required to operate only such small loads as voltmeters, pilot lights, and relays.

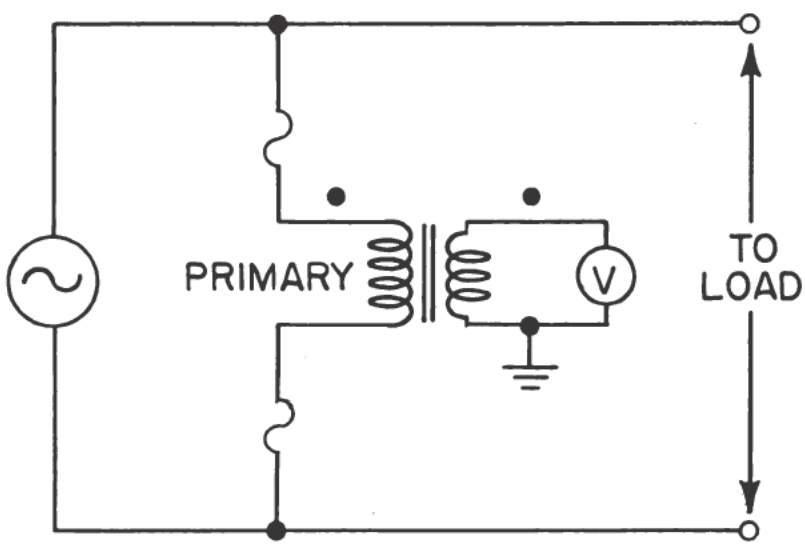


Figure 16–7.—Potential transformer.

Navy-type potential transformers for switchboard mounting are designed for primary voltages of 230 and 460 volts at a rated secondary voltage of 115 volts.

The following safety precautions should be observed when potential transformers are used:

1. Always ground one terminal of the secondary and the metal case.
2. Make sure the transformation ratio is correct for the line voltage and meter used. (Do not exceed the volt-ampere rating of the transformer.)
3. Make connections only when the circuit is deenergized.

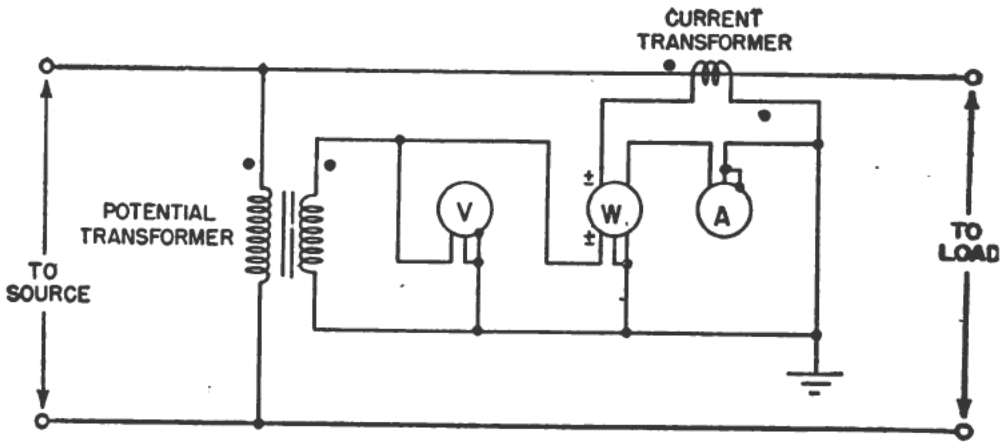


Figure 16–8.—Meter transformer connections.

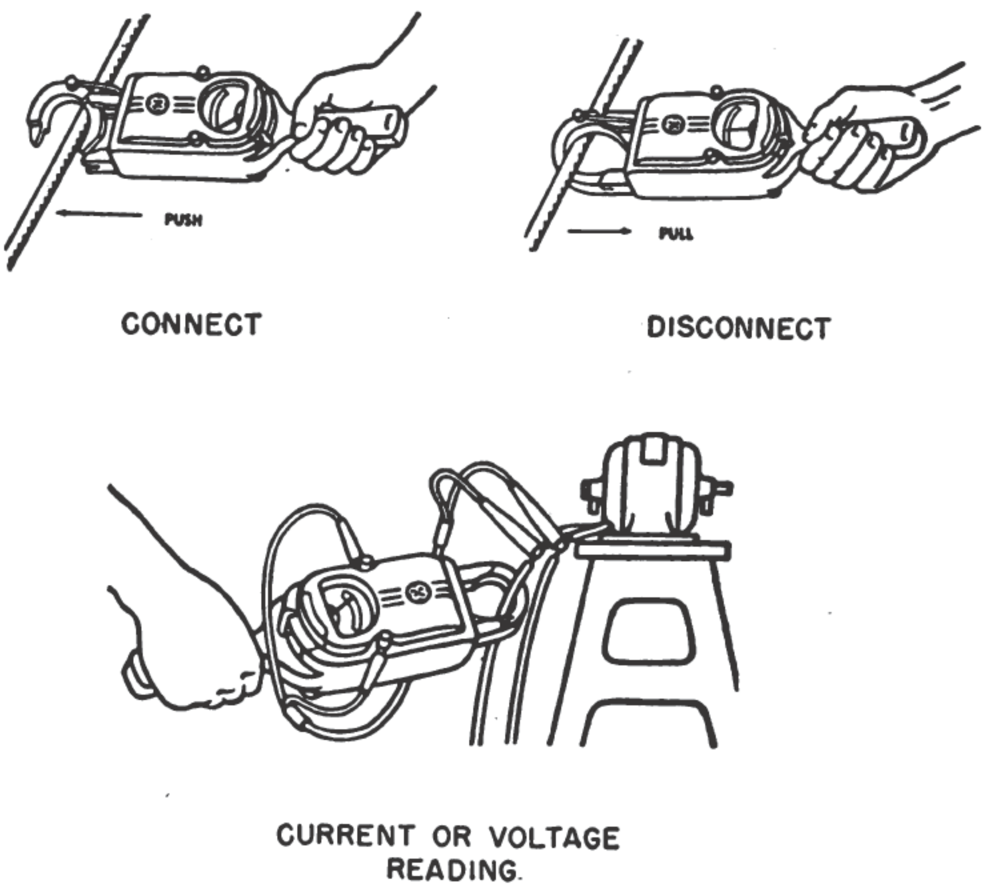


Figure 16–9.—Hook-on type of volt-ammeter.

4. Connect the primary across the voltage source to be measured.
5. Never short-circuit the secondary of a potential transformer.

Potential and current instrument transformers used simultaneously with the appropriate meters to indicate single-phase voltage, current, and power are shown in figure 16–8. The primary and secondary terminals of potential instrument transformers are marked with dots or letters in a similar manner to those of current transformers.

Hook-On Type

The hook-on a-c ammeter consists essentially of a current transformer with a split core and a rectifier-type instrument connected to the secondary. The primary of the current transformer is the conductor through which the current to be measured flows. The split core permits the instrument to be "hooked on" the conductor without disconnecting it. Therefore the current flowing through the conductor may be measured with a minimum of inconvenience, as shown in figure 16–9.

The instrument is usually constructed so that voltages also may be measured. However, in order to read voltage, the meter switch must be set to VOLTS, and leads must be connected from the voltage terminals on the meter to the terminals across which the voltage is to be measured.

BRIDGES

There is a wide variety of a-c bridge circuits that may be used for the precision measurement of resistance, capacitance, inductance, and impedance. The simplest is the resistance-ratio Wheatstone bridge, and a brief description of its operation is included in this chapter. Other types of bridges are treated as needed in the rating texts.

A typical capacitance-inductance-resistance bridge used by the Navy is the ZM–11/U. It is a very flexible instrument, having a wide range of measurements. A person using a bridge such as this will need detailed knowledge of its operation, such as may be acquired in an instruction book.

The following discussion of a-c bridges is simplified and is designed to give some insight into the operation of an elementary a-c bridge.

Resistance Bridge

The Wheatstone bridge is treated in chapter 5 under direct-current bridge circuits. However, alternating current may also be used, as shown in figure 16–10, A. An a-c signal generator having a frequency of 1,000 cps is used as the source of voltage.

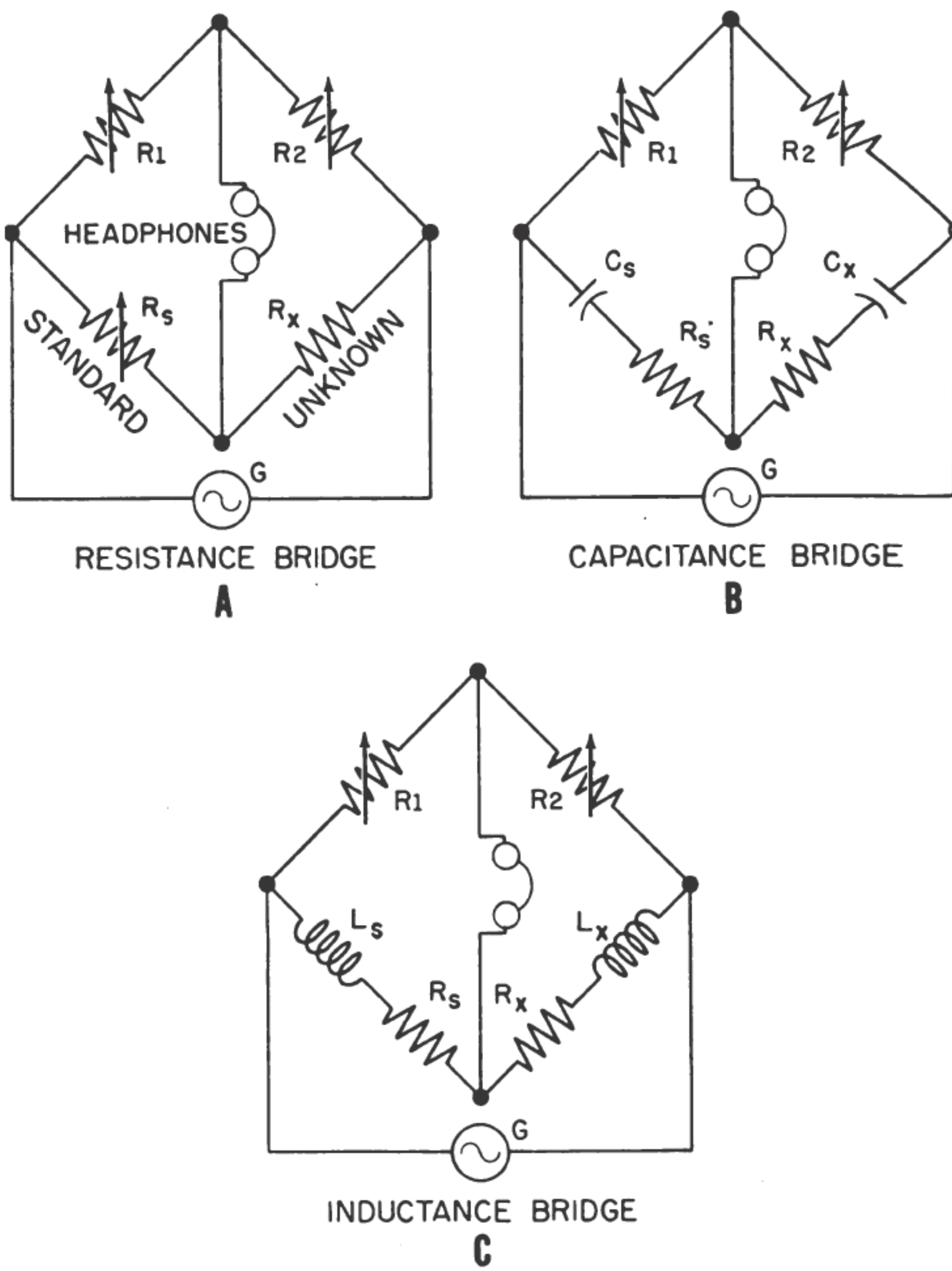


Figure 16–10.—Bridge circuits.

Current from the generator, G, passes through resistors R_1 and R_2, known as the RATIO ARMS, and through R_s and R_x. R_s is an adjustable standard resistance and R_x the unknown resistance. When the voltage drops across R_1 and R_s are equal, the voltage drops across R_2 and R_x are equal, and no difference in potential exists across the headphones, and no hum will be heard. As has been shown in chapter 5, when no voltage appears across the indicating device, a direct proportion exists between the resistances of the four arms of the bridge. Thus,

$$\frac{R_1}{R_2}=\frac{R_s}{R_x},$$

and

$$R_x=\frac{R_2}{R_1}R_s.$$

It is only necessary to select the proper resistance values, so that minimum sound is heard in the headphones; and because R_1, R_2, and R_s are known, R_x can be determined.

Capacitance Bridge

The value of an unknown capacitance, C_x, may be determined by means of the simple bridge circuit shown in figure 16–10, B. The ratio arms, R_1 and R_2, are accurately calibrated resistors. C_s is a standard capacitor whose capacitance is known, R_s is the equivalent series resistance of the standard capacitor, and R_x is the equivalent series resistance of the unknown capacitor.

The a-c signal is applied to the bridge, and R_1 and R_2 are varied until minimum signal is heard in the headphones. Minimum signal indicates that the bridge is balanced except for the phase shift existing between the arms. Because current varies inversely with resistance and directly with capacitance an inverse proportion exists between the four arms of the bridge. Thus,

$$\frac{R_1}{R_2}=\frac{C_x}{C_s},$$

or

$$C_x=\frac{R_1}{R_2}C_s.$$

Because R_1 and R_2 are expressed in the same units, $\frac{R_1}{R_2}$ becomes a

simple multiplying factor, and C_x is determined in the same units as C_s.

If the ratio of reactance to resistance is the same in the arms containing capacitance, the following direct proportion exists between the resistive components:

$$\frac{R_1}{R_2}=\frac{R_s}{R_x},$$

or

$$R_x=\frac{R_2}{R_1}R_s.$$

Thus, the unknown resistance and capacitance, R_x and C_x, can be estimated in terms of the known resistances, R_1, R_2, and R_s, and the known capacitance, C_s. If the ratio of reactances to resistance in the arm containing capacitance is not the same, the relations are only approximate and unless the bridge can be balanced exactly, the unknown cannot be determined except to a rough approximation.

Inductance Bridge

The value of an unknown inductance may be determined by means of the simple bridge circuit shown in figure 16–10, C. The ratio arms, R_1 and R_2, are accurately calibrated resistors. L_s is a standard inductor whose inductance is known, and R_s is the resistance of the standard inductor.

The a-c signal is applied to the bridge, and R_1 and R_2 are varied until minimum signal is heard in the headphones. Minimum signal indicates that the bridge is balanced. Under conditions of balance,

$$\frac{R_1}{R_2}=\frac{L_s}{L_x},$$

or

$$L_x=\frac{R_2}{R_1}L_s;$$

and

$$\frac{R_1}{R_2}=\frac{R_s}{R_x},$$

or

$$R_x=\frac{R_2}{R_1}R_s.$$

The same limitations described in the capacitance bridge apply also to the inductance bridge.

In IMPEDANCE BRIDGES, the relation between reactance and resistance in one or more arms of the bridge is variable in order to adjust the phase of the voltages as well as their magnitudes when obtaining a balance. Impedance bridges permit more accurate measurement of inductance, capacitance, and impedance than the bridge circuits described in this chapter.

FREQUENCY METER

Alternating-current electrical equipments are designed to operate within a given frequency range. In some instances they are designed to operate at one particular frequency, as are electric clocks and time switches.

For example, electric clocks are commonly designed to operate at 60 cps. If the supply frequency is reduced to 59 cps, the clock will lose one minute every hour.

Transformers and a-c machinery are designed to operate at a specified frequency. If the supply frequency falls more than 10 percent from the rated value, the equipment may draw excessive current and dangerous overheating will result.

It is therefore desirable to control the frequency of electric power systems. Frequency meters are employed to indicate the frequency so that corrective measures may be taken if the frequency varies beyond the prescribed limits.

Frequency meters should be designed so that they will not be affected by changes in voltage. Because a-c systems are designed to operate normally at one particular frequency, the range of the frequency meter may be restricted to a few cycles on either side of the normal frequency. There are several types of frequency meters, including the vibrating-reed type, the fixed-coil and moving-coil type, the fixed-coil and moving-disk type, and the resonant-circuit type.

Vibrating-Reed Frequency Meter

The vibrating-reed type of frequency meter is one of the simplest devices for indicating the frequency of an a-c source. A simplified diagram of one type of vibrating-reed frequency meter is shown in figure 16–11.

The current whose frequency is to be measured flows through the coil and exerts maximum attraction on the soft-iron armature TWICE during each cycle (fig. 16–11, A). The armature is attached to the bar, which is mounted on a flexible support. Reeds of suitable dimensions to have natural vibration frequencies

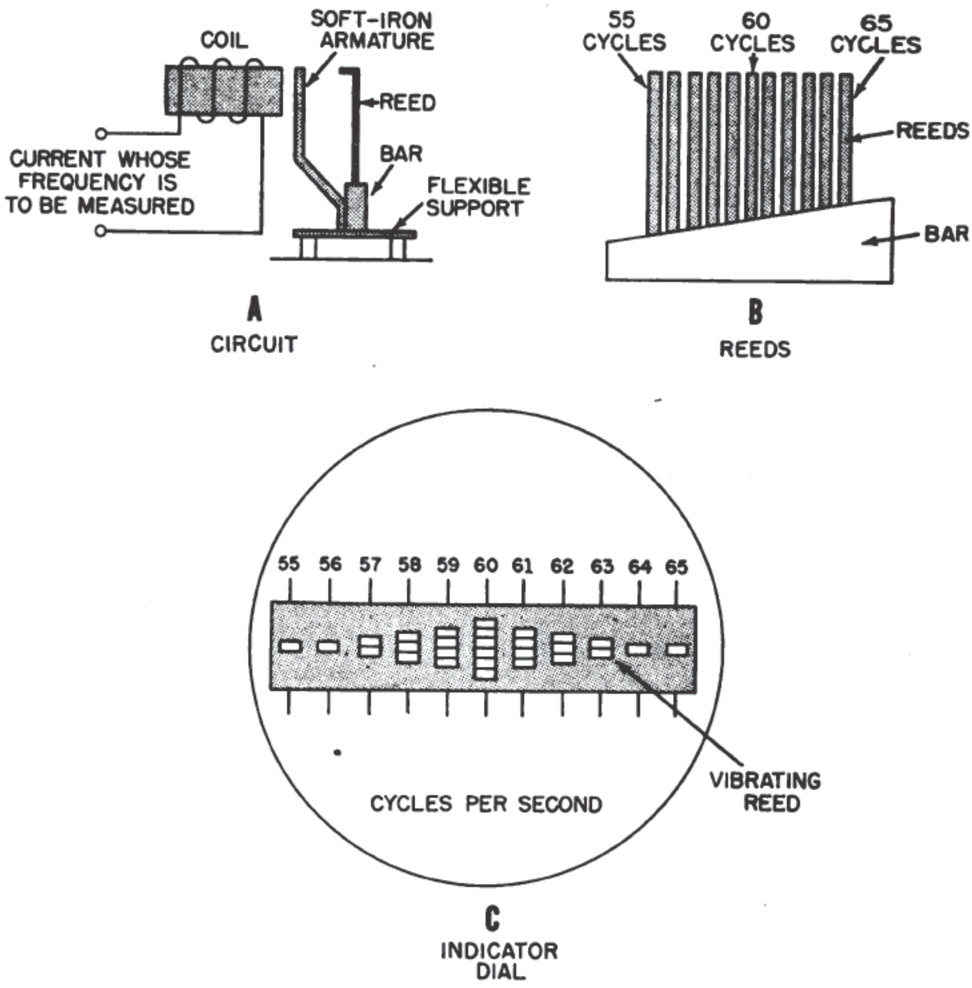


Figure 16–11.—Simplified diagram of a vibrating-reed frequency meter.

of 110, 112, 114, and so forth, up to 130 cycles per second are mounted on the bar (fig. 16–11, B). The reed having a frequency of 110 cycles is marked "55" cycles, the one having a frequency of 112 cycles is marked "56" cycles, the one having a frequency of 120 cycles is marked "60" cycles, and so forth.

When the coil is energized with a current having a frequency between 55 and 65 cycles, all the reeds are vibrated slightly; but

the reed having a natural frequency closest to that of the energizing current (whose frequency is to be measured) vibrates through a larger amplitude. The frequency is read from the scale value opposite the reed having the greatest amplitude of vibration.

In some instruments the reeds are the same lengths, but are weighted by different amounts at the top so that they will have different natural rates of vibration.

An end view of the reeds is shown in the indicator dial of figure 16–11, C. If the energizing current has a frequency of 60 cycles per second, the reed marked "60" cycles will vibrate the greatest amount, as shown.

Moving-Disk Frequency Meter

A moving-disk frequency meter is shown in figure 16–12, A. Each of the two-pole fields (fig. 16–12; B) has a short-circuited turn (shading coil) on one side of the pole face and a magnetizing coil around the associated magnetic circuit. As in a shaded-pole motor, the magnetic field tends to produce rotation toward the shorted turn. One coil tends to turn the disk clockwise, and the other, counterclockwise. Magnetizing coil *A* is connected in series with a large value of resistance. Coil *B* is connected in series with a large inductance and the two circuits are supplied in parallel by the source.

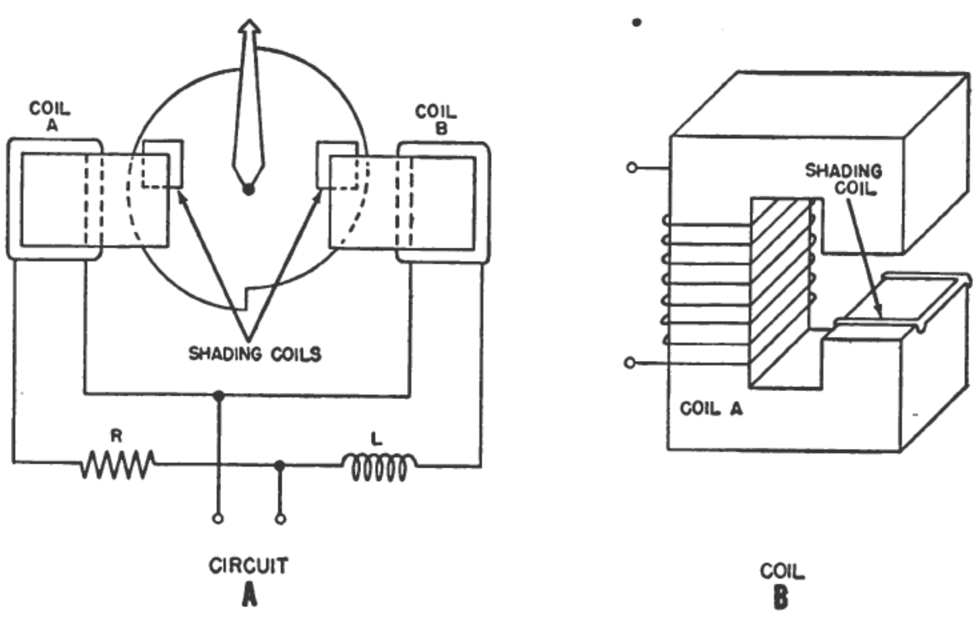


Figure 16–12.—Simplified diagram of a moving-disk frequency meter.

For a given voltage, the current through coil A is practically constant. However, the current through coil B varies inversely with the frequency. At a higher frequency the inductive reactance is greater and the current through coil B is less; the reverse is true at a lower frequency. The disk turns in the direction determined by the stronger coil.

A perfectly circular disk would tend to turn continuously. This is not desirable, and accordingly the disk is so constructed that it will turn only a certain amount clockwise or counterclockwise about the center position, which is commonly marked "60" cycles on commercial equipment. To prevent the disk from turning more than the desired amount, the left half of the disk is so mounted that when motion occurs, the same amount of disk area will always be between the poles of coil A. Therefore, the force produced by coil A to rotate the disk is constant for a constant applied voltage. The right half of the disk is offset, as shown in the figure. When the disk rotates clockwise, an increasing area will come between the poles of coil B; when it rotates counterclockwise a decreasing area will come between the poles of coil B. The greater the area between the poles, the greater will be the disk current and the force tending to turn the disk.

If the frequency applied to the frequency meter should decrease, the reactance offered by L would decrease and the field produced by coil B would increase; the field produced by coil A would remain the same. Thus, the torque produced by coil B would tend to move the disk and the pointer counterclockwise until the area between the poles were reduced sufficiently to make the two torques equal. The scale is calibrated to indicate the correct frequency.

If the frequency should increase, the reactance offered by L would increase and the field produced by coil B would decrease; the field produced by coil A would remain the same. Thus, the decreasing torque produced by coil B would permit a greater disk area to move under the coil, and therefore the disk and the pointer would move clockwise until the two torques were balanced.

If the frequency is constant and the voltage is changed, the currents in the two coils—and therefore the opposing torques—change by the same amount. Therefore, the indication of the instrument is not affected by a change in voltage.

SINGLE-PHASE POWER FACTOR METER

The power factor of a circuit is the ratio of true power to apparent power. It is also equal to the cosine of the phase angle between the circuit current and voltage. A pure resistance has a power factor of unity (the current and voltage are in phase and the true power and the apparent power are the same); a pure inductance has a power factor of zero (current lags the voltage by 90°); and a pure capacitance has a power factor of zero (current leads the voltage by 90°).

The power factor of a circuit may be determined by the use of a wattmeter, a voltmeter, and an ammeter—that is, the power factor may be determined by dividing the wattmeter reading by the product of the voltmeter and ammeter readings. This is inconvenient, however, and instruments have been developed that indicate continuously the power factor and at the same time indicate whether the current is leading or lagging the voltage. An instrument that indicates these values is called a POWER-FACTOR METER.

Crossed-Coil Power Factor Meter

One type of power-factor meter is shown schematically in figure 16–13. The instrument consists of movable potential coils *A* and *B*, fixed at right angles to each other, and stationary current coil *C*. Coils *A* and *B* are pivoted, and the assembly, together with the

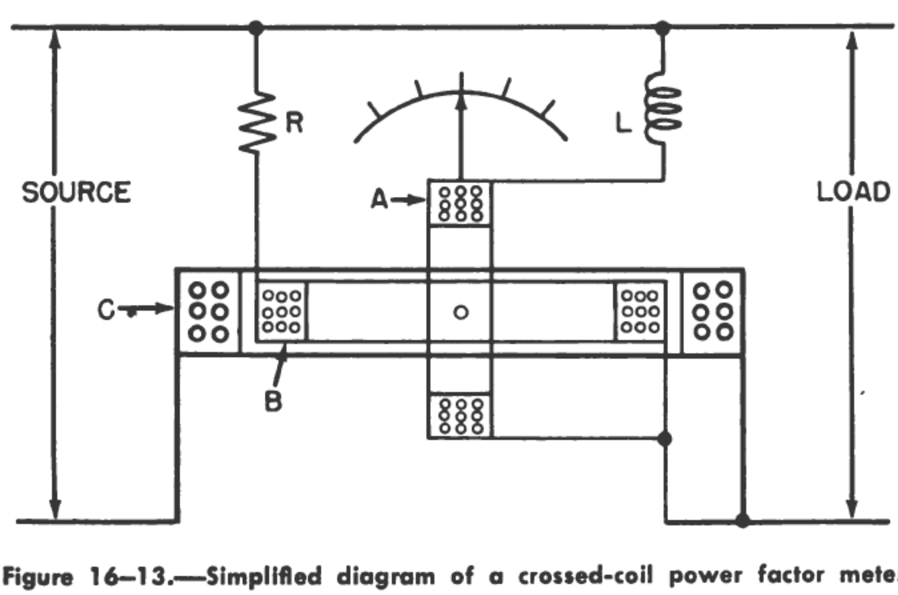


Figure 16–13.—Simplified diagram of a crossed-coil power factor meter.

attached pointer, is free to move through an angle of approximately 90°. Coil A is in series with inductor L, and the combination is connected across the line. Coil B is in series with the noninductive resistor, R, and the combination is also connected across the line.

Circuit continuity to the coils is provided by three spiral springs (not shown in the figure) that exert negligible restraining force on the coils. Therefore, when no current flows through the coils the pointer may come to rest at any position on the dial. Coil C (not drawn to scale) is connected in series with the line. In many switchboard installations the coils are energized by instrument transformers, in which case the currents are proportional to line values but not equal to them.

The current in coil B is in phase with the line voltage. The current in coil A lags the line voltage by 90°. When the line current is in phase with the line voltage the currents in B and C are in phase and a torque is exerted between them that aligns their axes so that the pointer indicates unity power factor. The average torque between A and C is zero because these currents are 90° out of phase when the line power factor is unity.

When the current in coil C lags the line voltage—for example, by 45°—the currents in coil A and coil B both will be out of phase with the current in C. The current in A lags the current in C by 45° and the current in B leads the current in C by 45°. The flux around coil C will therefore react with the resultant of the fluxes around coils A and B, which is in phase with the current of C, and the pointer will be moved to an intermediate position (45°) between zero power factor and unity power factor. In most power-factor meters, lagging current causes the pointer to move to the left of the central position (marked "1" on the scale) and a leading current causes the pointer to move to the right of the central position.

Moving Iron-Vane Power Factor Meter

A diagram of a moving iron-vane type of power factor meter is shown in figure 16–14. In part A of the figure, potential coil A in series with resistor R comprises a resistive circuit, which is connected across the line. Potential coil B in series with inductor L comprises an inductive circuit, which is also connected across the line. Current coil C, having a few turns of large wire, is connected

in series with the line. All coils are fixed in the positions shown, and only the iron vanes are free to move. The curent in B lags the line voltage by 90°. The current in A is in phase with the line voltage. Hence, the current in B lags the current in A by 90° and, since the axes of A and B are displaced 90°, a magnetic revolving field is established by these coils when they are energized. The iron vanes are free to rotate about the axis of coil C. These vanes are magnetized alternately north and south by the line current flowing in coil C.

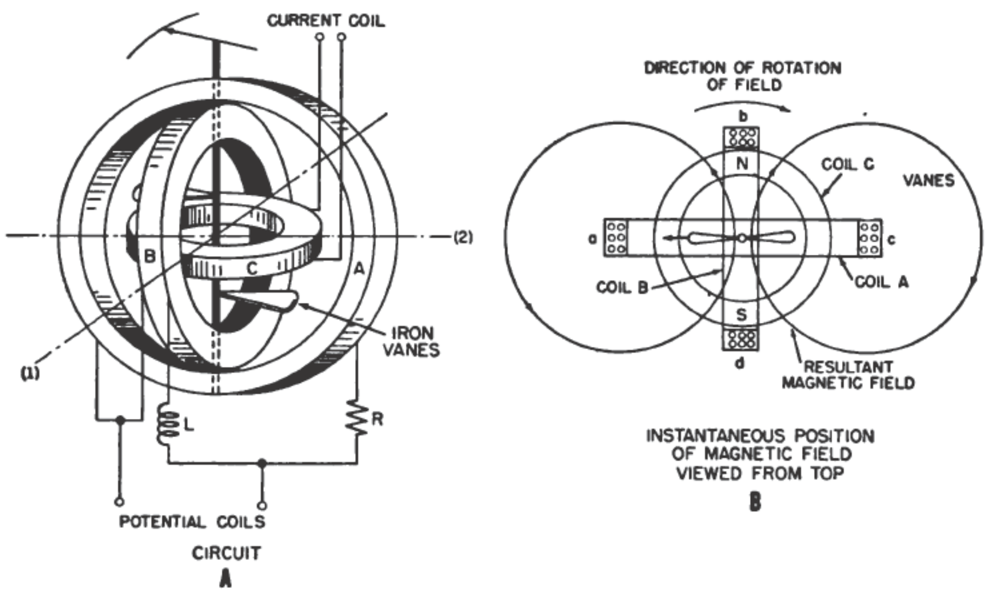


Figure 16–14.—Simplified circuit of a moving iron-vane power factor meter.

None of the moving parts carry current, and therefore springs are not needed. The movement is free to rotate 360°, but is prevented from rotating with the rotating field by an aluminum disk that rotates in a field produced by drag magnets (disk and magnets not shown).

In figure 16–14, B, a two-pole rotating field is assumed to revolve at synchronous speed in a clockwise direction. The vanes are attracted or repelled by this field depending on the instantaneous direction of the resultant flux produced by A and B and on the instantaneous polarity of the iron vanes as determined by the instantaneous direction of the current through C. The vanes will assume a position out of alignment with the field (position of minimum torque) when the line power factor is unity.

If at the instant shown in figure 16–14, B, the current in coil C

is maximum and the resultant flux produced by coils *A* and *B* passes through the movable element at right angles to the vanes (and has no effect on them), the pointer will indicate unity power factor, as shown. Ninety degrees later, the resultant north-south field will pass through the iron vanes in alignment with them. At this instant the current in magnetizing coil *C* is zero and the torque on the vanes is again a minimum. If the current through *C* leads the line voltage by 45° (line power factor 70% leading) the vanes will move 45° to a new position of minimum torque. The current in *C* will become maximum earlier in the cycle and the revolving field will occupy a new position, again out of alignment with the vanes at this instant.

If the line current lags the line voltage the pointer will come to rest on the opposite side of the unity power factor mark.

SYNCHRONIZING A-C GENERATORS

It is a relatively simple matter to connect two d-c generators in parallel. The precautions are much the same as those for connecting batteries in parallel—that is, polarities and voltages must be the same. Additional factors, however, must be taken into consideration when a-c generators (alternators) are connected in parallel. The frequencies and voltages of the two alternators must be equal. Likewise, the voltages and currents of the two alternators must reach their respective maximum and minimum values at the same time—that is, they must be synchronized and in the proper phase—and in a polyphase system, the two alternators must have the same phase sequence.

Because the two alternators would very likely be designed to operate at the same frequency, the small adjustment (varying the rpm of the incoming alternator) needed to bring them to the same speed could be made with the aid of a FREQUENCY METER. The rms voltage of the generators may be determined by means of a VOLTMETER.

The proper phase relations may be determined by means of SYNCHRONIZING LAMPS or a SYNCHROSCOPE. By observing the synchroscope while making the necessary adjustments, the proper phase relations between the voltage and current of the incoming alternator and those of the running alternator may be established. In other words, the two alternators may be brought into synchronism.

Synchronizing Lamps

One method of determining if two alternators are properly synchronized (the two voltages reaching maximum and minimum values in step) is by the use of SYNCHRONIZING LAMPS, as shown by the simplified arrangement in figure 16–15.

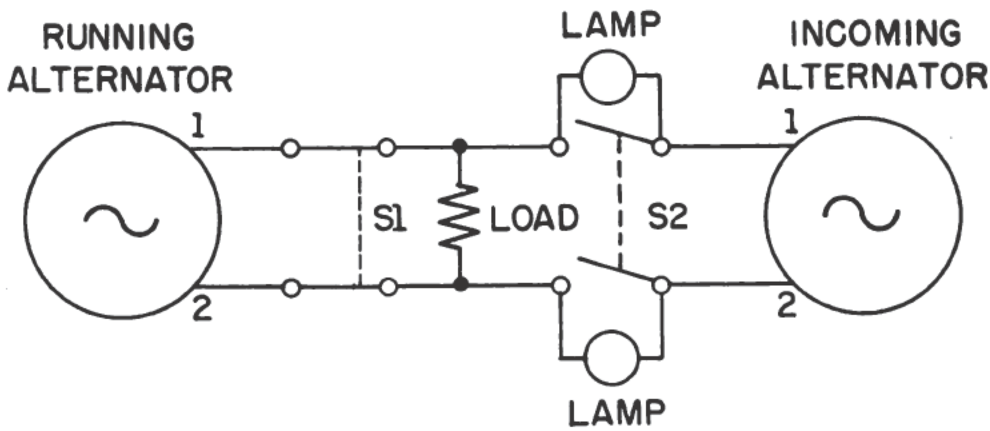


Figure 16–15.—Synchronizing lamps.

Assume, initially, that $S2$ is open and that the incoming alternator is running and developing normal voltage. At a given instant, the polarity of terminal 1 of the running alternator is maximum negative. Assume that at the same instant, terminal 1 of the incoming alternator is maximum positive. If the two voltages have the same frequency, current will flow through the lamps which will glow with maximum brilliance because the two voltages are in series addition. Under these circumstances, the alternators are out of phase by 180°. Current circulates through the two generators, and the lamps, in series. If, however, terminal 1 of the running alternator and terminal 1 of the incoming alternator become maximum positive (or maximum negative) at the same instant, the lamps will go out because the two voltages are equal and in opposition with respect to each other.

When the lamps go out, the alternators are in synchronism and $S2$ may be closed, thus permitting both alternators to supply the load.

Synchroscope

Synchronization by means of lamps is a simple and convenient method. However, greater accuracy can be obtained by the

use of a synchroscope, especially when the phase difference is very slight.

CROSS-COIL SYNCHROSCOPE.—One type of synchroscope employs cross-coils, similar in some respects to the crossed-coil type of power factor meter. A simplified circuit of this type of meter is shown in figure 16–16. The stator consists of a 2-pole laminated

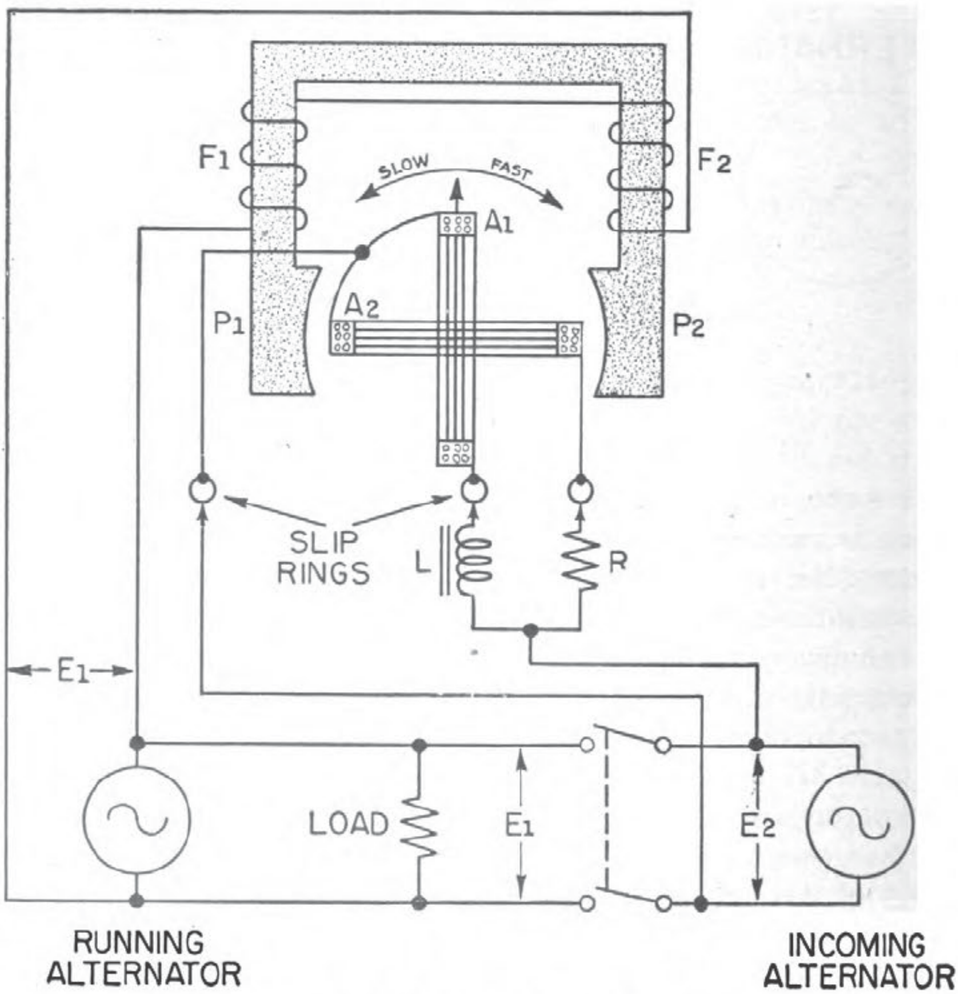


Figure 16–16.—Simplified circuit of crossed-coil synchroscope.

field structure energized by field coils $F1$ and $F2$. The rotor contains a 2-phase winding supplied through slip rings and brushes. Current from the running alternator energizes field $F1F2$, and current from the incoming alternator energizes rotor coils $A1$ and $A2$. This current divides a part of it flowing through R and $A2$ and the remainder flowing through L and $A1$.

The current in $F1$ and $F2$, and therefore the flux passing between the poles, $P1P2$, lags the voltage, E_1, by 90° because of the large self-inductance of $F1F2$. Likewise, the current in rotor coil $A1$ lags E_2 by 90° because of the self-inductance of L. The current in rotor coil $A2$ is in phase with E_2 because of the high resistance of R. Therefore, when E_1 and E_2 are in the proper phase alignment, the current in $A1$ is in phase with the flux across $P1$ and $P2$, and the axis of $A1$ aligns itself with the field across $P1$ and $P2$. Also, when E_1 and E_2 are in proper phase alignments, the current in rotor coil $A2$ is 90° out of phase with the field across $P1P2$, and no torque is exerted by this coil to turn the rotor. Under these circumstances, a pointer, rigidly attached to the shaft, will point vertically to the center of the dial, thus indicating that the incoming alternator is in phase with the running alternator.

If E_2 of the incoming alternator is 90° behind E_1 of the running alternator, the current through $A2$ is likewise 90° behind E_1. Because the flux across $P1P2$ lags E_1 by 90°, the flux across $P1P2$ is now in phase with the current through $A2$. The rotor turns until the axis of coil $A2$ is in alignment with the field across $P1P2$. The current in coil $A1$ is 90° out of phase with the field flux and no torque is exerted on the rotor by this coil. The direction in which the rotor turns depends on the relative speed and frequency of the incoming machine. If clockwise rotation occurs when the incoming generator is fast, counterclockwise rotation will occur when it is slow.

Thus, the synchroscope gives a more complete indication than synchronizing lamps can give. It indicates (1) the exact point of synchronism, (2) whether the incoming machine is running too fast or too slow, and (3) the amount by which the incoming machine is running too fast or too slow.

Moving iron-vane synchroscope.—A schematic diagram of the moving iron-vane type of synchroscope is shown in figure 16–17, A. The current in coil A is in phase with the incoming generator voltage E' because of resistor R, and the current in coil B lags E' by 90° because of inductor L. The current in coil C is in phase with the running machine voltage, E, because of resistor R_L.

When the voltages E and E' are in proper phase alignment for paralleling, the current in A and the current in C are in phase as indicated in the vector diagram of figure 16–17, B. The current

in B is 90° out of phase with the currents in A and C. Thus, a torque is exerted on the iron vanes, which aligns them with the axis (No. 1) of coil A. No torque is exerted to align the vanes with the axis of B (No. 2) because of the 90° phase shift.

When the two machines are 90° out of phase the incoming generator voltage, E', may lag the running machine voltage, E, by

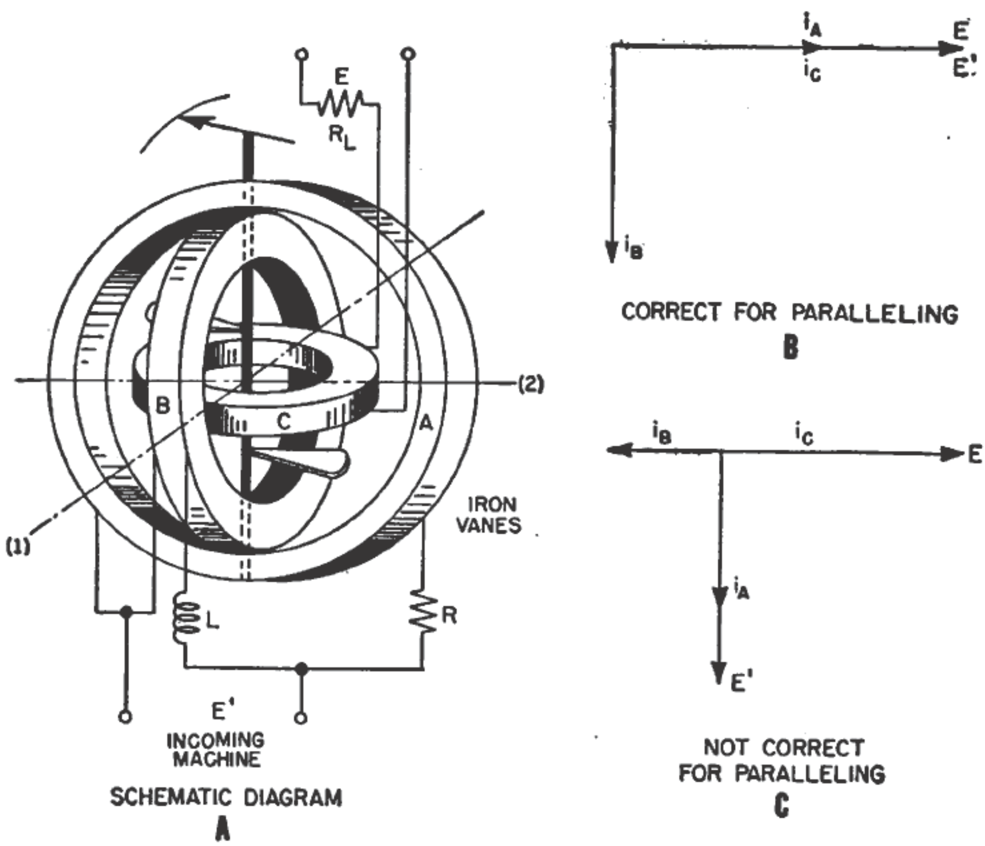


Figure 16–17.—Simplified diagram of a moving iron-vane synchrescope.

90°, as indicated in figure 16–17, C. In this case the current in C is in phase opposition to the current in B and the vanes turn 90° until they are in alignment with the axis of B (No. 2). (See fig. 16–17, A.) No torque is exerted on the vanes by coil A in this case because of the 90° phase shift between the current in A and the current in C.

The function of coil C is to establish the instantaneous polarity of the iron vanes and thus indicate whether the incoming machine is running fast or slow. As the voltage, E', of the incoming machine gains or loses phase with respect to the voltage, E, of the

running machine, the iron vanes turn clockwise or counterclockwise to indicate whether the incoming machine is running too fast or too slow.

This type of synchroscope is of a size suitable for mounting on small instrument panels at each alternator. It is rugged and relatively trouble free because there are no moving coils or slip rings.

SUMMARY

In general, the basic d-c indicating instrument incorporates the D'Arsonval-type movement and the basic a-c indicating instrument incorporates some form of moving iron vane.

The moving iron vane and the electrodynamometer instruments may be used with either a-c or d-c. Specialized instruments employing the principles of induction are used only on a-c.

The D'Arsonval-type instrument is more sensitive than the moving iron-vane instrument, and therefore some of the more sensitive a-c ammeters and voltmeters contain the D'Arsonval movement and a suitable rectifier to change the meter current from a-c to d-c. Such an instrument is called a RECTIFIER-TYPE a-c meter. Full-wave rectifiers are commonly used because they are more efficient.

WATTMETERS employ the electrodynamometer movement, which in effect multiples the instantaneous current through the load by the instantaneous voltage applied across the load, taking into consideration the phase angle between the voltage and the current. Care must be used in making wattmeter connections. The current coils are connected in series with the load and the potential coil is connected in parallel with the load.

WATT-HOUR METERS sum up all the instantaneous rates of power consumption and indicate the total energy consumed during a given period of time.

The THOMPSON WATT-HOUR METER is essentially a series motor (may be used on a-c or d-c) with a provision for recording the number of revolutions made by the armature. The current that flows through the load also flows through the field (current) coils, and the voltage that is applied to the load is also applied to the armature (potential) coils. The speed of the armature is therefore determined by the current and voltage applied to the load. The number of revolutions that the armature makes in a

given time interval is a measure of the energy consumed by the load during the time interval.

The INDUCTION WATT-HOUR METER is used only on a-c. The induction-type meter does not use a commutator and brushes and is more reliable than the Thompson watt-hour meter. The induction watt-hour meter is essentially an induction-type motor (used only on a-c) with a provision for recording the number of revolutions made by the armature. The speed of the armature depends on the current through the current coils (the load current) and on the voltage applied across the potential coil (the load voltage) and takes into account the phase angle between them. The number of revolutions that the armature (disk) makes in a given time interval is a measure of the energy consumed by the load during the time interval. As in making wattmeter connections, care must be exercised in making watt-hour meter connections.

INSTRUMENT TRANSFORMERS are used to transform high voltages to low voltages so that measurements may be made safely and conveniently with ordinary instruments or to transform high currents to low currents so that the average type of instrument may be used. The instrument in either case is calibrated to indicate the correct values.

The HOOK-ON AMMETER is a current transformer conveniently arranged so that it may be hooked over a current-carrying conductor, which then becomes the primary of the current transformer. The ammeter is connected across the secondary terminals.

BRIDGES are used for precision measurement of resistance, capacitance, inductance, and impedance. The a-c exciting voltage is applied across two terminals of the bridge and the headphones, or other indicating device, are connected across diagonally opposite terminals. The adjustable arms of the bridge are then varied until minimum response is obtained in the indicating device. The unknown value (connected in one arm of the bridge) may then be obtained from a simple calculation involving the known resistance values of the two ratio arms and the value of the standard unit used in the remaining arm of the bridge.

FREQUENCY METERS are used to indicate the frequency of the current in a circuit or device. Because some equipments are designed to operate on one particular power frequency, it is desirable to control the frequency of the generator, or alternator, within prescribed limits. The vibrating-reed type of frequency

meter is one of the simplest. However, the moving-vane meter has certain advantages and is widely used.

POWER-FACTOR METERS are used to indicate the power factor of a circuit. The power factor is the ratio of the true power to the apparent power and is mathematically equal to the cosine of the phase angle between the current and the voltage. The power-factor meter provides an instantaneous indication of the circuit power factor.

In order to connect two a-c generators in parallel they must have the same frequency, the same voltage, and they must be in the proper phase alignment. In the case of polyphase generators they must have the same phase sequence; in other words, they must be synchronized. Alternators may be synchronized by means of SYNCHRONIZING LAMPS. A more accurate method of synchronizing generators is by the use of a SYNCHROSCOPE.

QUIZ

1. What is needed to convert a D'Arsonval-type meter for use on a-c?
2. What essentially is the function of a copper-oxide meter rectifier?
3. What are two advantages of using the D'Arsonval-type instrument with a rectifier for making a-c measurements compared with the iron-vane type of instrument?
4. Why are rectifier-type meters not suitable for use on frequencies higher than the audio range?
5. The force acting on the moving coil of a wattmeter is proportional to the product of what three values?
6. For a given line voltage and current, what power factor permits the maximum deflecting force in a single-phase wattmeter?
7. At what phase angle between current and voltage will a single-phase wattmeter indicate zero power?
8. Under what conditions may the elements of a wattmeter be overloaded without the needle indicating a maximum value?
9. What is the essential difference between the wattmeter and the watt-hour meter?
10. What is the function of the aluminum disk and permanent magnets shown in figure 16–3?
11. What component of the watt-hour meter shown in figure 16–3 functions to overcome static friction?
12. In the curves of figure 16–5, I_{cc}, and E_{line} are in phase. If the load were purely inductive, what would be the relation between I_{cc} and E_{line}?

13. In the single-phase induction watt-hour meter, the driving torque is proportional to what three electrical values?
14. How is bearing friction counteracted in an induction watt-hour meter?
15. How is the drag increased in an induction watt-hour meter?
16. What device is used to change large alternating currents to smaller proportionate values to operate ammeters and the current coils of other instruments?
17. What device is used to reduce high voltage to smaller proportionate values of voltage to operate voltmeters and the potential coils of other meters?
18. Why must the iron of a current transformer be operated below saturation?
19. What are two harmful effects that would be produced if the secondary of a current transformer should become open-circuited (load is removed) while the primary is energized?
20. Why should the secondary of a potential transformer never be short-circuited when the load is removed?
21. The hook-on a-c ammeter consists of essentially what two components?
22. In the bridge circuits of figure 16–10, how is the approach to a state of balance determined?
23. In the capacitance bridge shown in figure 16–10, B, how is the accuracy of the result affected if an inequality exists between the ratios of reactance to resistance in the two arms containing capacitance?
24. If the frequency of the current supplied to an electric clock designed to operate on 60-cycle current is increased to 61 cycles, how will the accuracy of the clock be affected during a 24-hour period?
25. In the vibrating-reed type of frequency meter, why is the soft-iron armature attracted by a maximum amount twice during each cycle of applied voltage?
26. Why is the operation of the moving-disk frequency meter not affected by a change in applied voltage?
27. Why is the average torque zero between coils A and C in the power factor meter of figure 16–13 when the line current and line voltage are in phase?
28. What four factors must be considered in synchronizing polyphase generators?
29. When synchronizing lamps are used as in figure 16–15, what will be the indication on the lamps at the time the switch is to be closed?
30. What three things does the synchroscope indicate?

CHAPTER 17

SYNCHROS AND SERVOMECHANISMS

SYNCHROS

Introduction

It is necessary in many Navy applications involving electromechanical systems to transmit the angular position of a shaft from one location to another without the benefit of an actual mechanical linkage. A widely used method employs small a-c machines that operate as single-phase transformers. These machines are called SYNCHROS. The system is called a SYNCHRO TRANSMISSION SYSTEM. The simplest synchro transmission system contains a transmitter and a receiver, as shown in figure 17–1. Synchros are also widely used in naval aviation equipment for transmitting various kinds of position data.

The SYNCHRO TRANSMITTER is a transformer with a rotatable primary that converts a mechanical input into an electrical signal, and transmits the signal to the receiver. The SYNCHRO RECEIVER, similar in construction to the transmitter, receives the signal and converts it into a mechanical output. Thus, the synchro system transmits information electrically in the form of an angular displacement from one part of the ship to another. However, the mechanical load on the synchro must be very light if the output is to be an accurate reproduction of the input because the synchro develops a relatively weak torque.

Synchro systems aboard ship include the pitometer log system, the wind direction and intensity indicator system, and the shaft revolution indicator system. The ship's order and indicating systems including the engine order system, the propeller order system, and the rudder order system likewise employ synchros. Fire control and computer systems, as well as antenna and search-

light control and gyrocompass follow-up systems, employ numerous synchro transmitters and receivers.

Because synchros have such wide and varied uses in the Navy, it is necessary that the interested technician acquire a fundamental knowledge of the principles of synchro operation, and an understanding of the applications of synchros in indicator and control systems.

The first chapters of this text treat the basic principles of electric

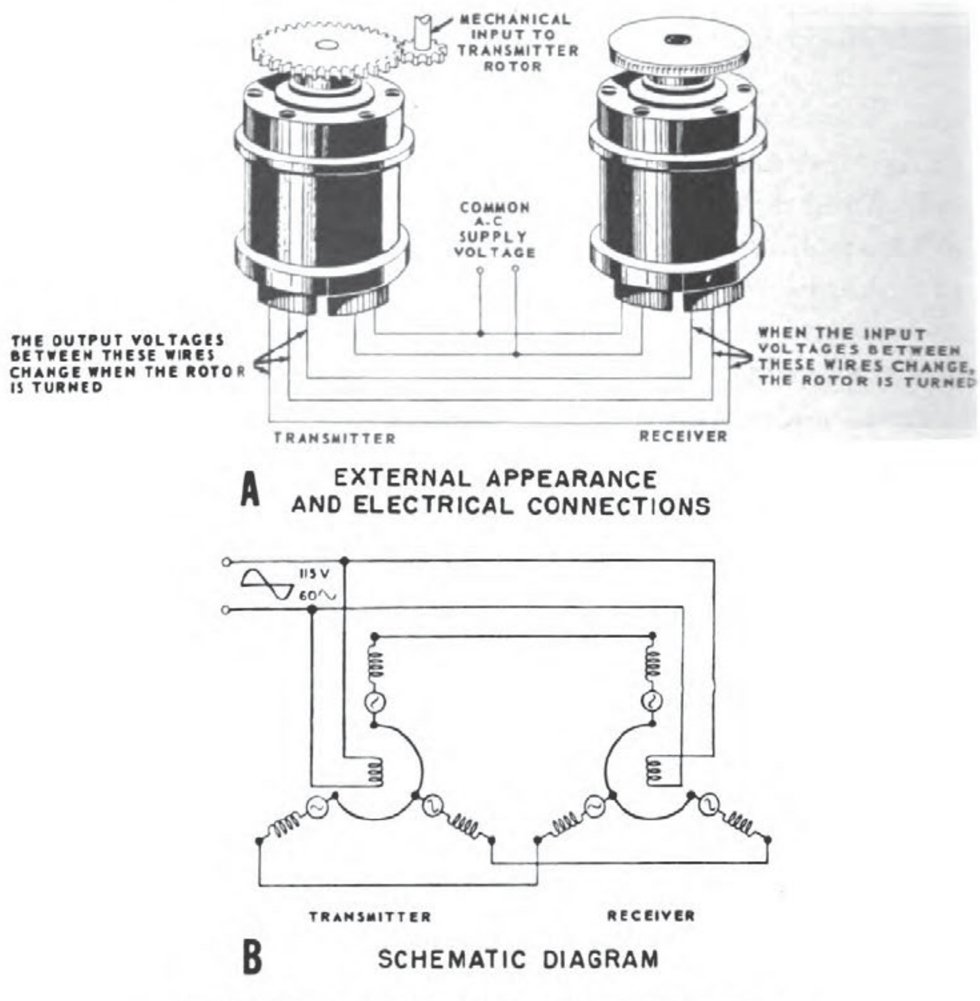


Figure 17–1.—Simple synchro transmission system.

circuits. Subsequent chapters describe the operating principle of transformers, generators, motors, and indicating instruments. This chapter in effect combines the principles of transformers, generators, motors, and indicating instruments into one system.

Synchro Construction

The simplest synchro transmission system consists of a self-synchronous generator called a TRANSMITTER and a self-synchronous motor called a RECEIVER, connected as shown in figure 17–1. These units are self-synchronous because, when they are properly connected and energized, any angular displacement introduced into the transmitter rotor produces an equal angular displacement in the receiver rotor—that is, when the transmitter is rotated a certain number of degrees, the receiver rotor will rotate through the same number of degrees.

The construction (fig. 17–1, A) of the synchro transmitter or receiver (they are the same except that a form of damping device is used in the receiver) is similar to that of a miniature 2-pole alternator, and the operation is basically that of a single-phase transformer. In the transmitter, the rotor winding is the primary. The secondary has three windings which are distributed uniformly around the stator. In most Navy applications these windings are wye-connected as shown in figure 17–1, B. The 3-winding stator of the receiver is connected to the 3-winding stator of the transmitter, and both rotors are connected to the same single-phase source.

When the rotors of the transmitter and receiver are in correspondence (for example, when they have the same relative positions with respect to their stators, one position of which is shown in fig. 17–1, B), the voltages induced in the secondary windings of the transmitter and receiver by transformer action are equal and opposite and no current flows in the lines connecting their corresponding stator coils. Both rotor windings draw only a small magnetizing current from the single-phase supply. When the transmitter rotor is turned through any angle, the rotors are out of correspondence, the voltages induced in the distributed windings of the transmitter and receiver are unbalanced (not equal), and a load component of current flows between the transmitter and receiver windings. Power to turn the receiver rotor is supplied through the transmitter stator, and the receiver rotor moves through the same angle as the transmitter rotor.

Thus, to transmit a signal the transmitter rotor is positioned mechanically and the receiver rotor follows almost simultaneously through the same angle.

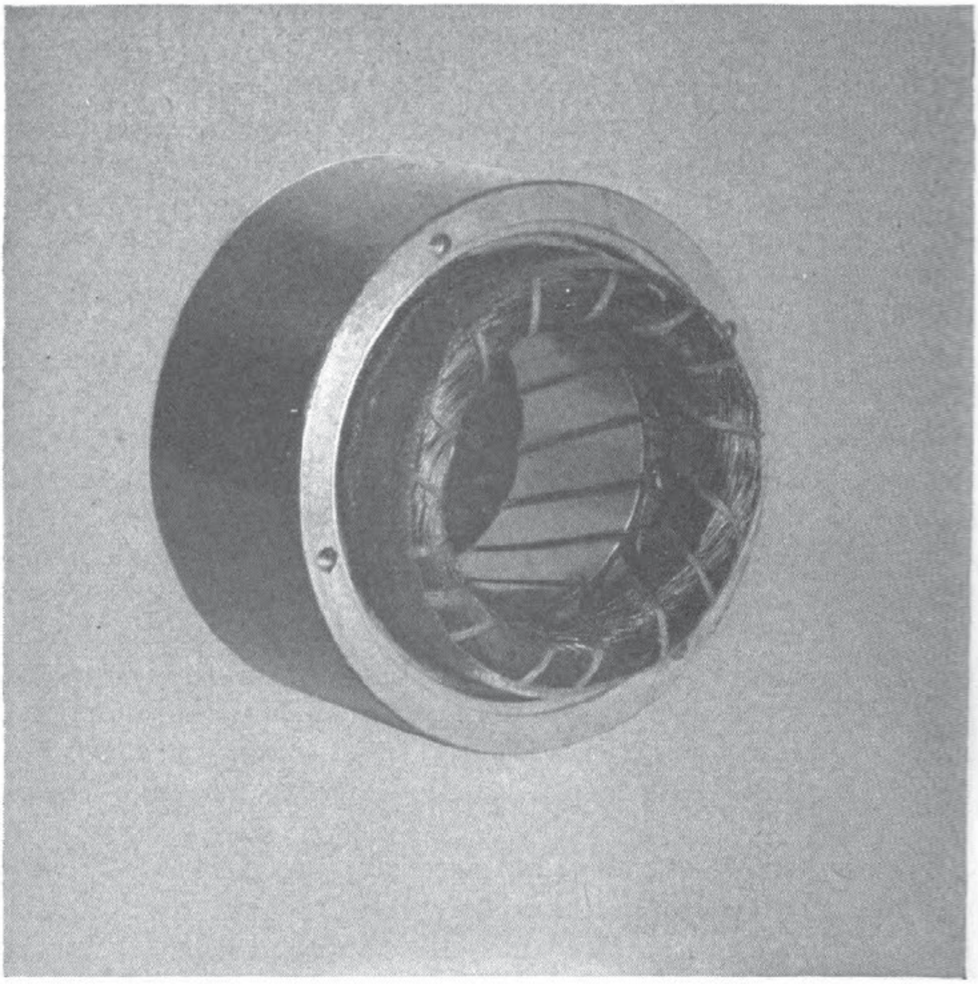


Figure 17–2.—Bureau of Ordnance synchro stator.

There are two types of Navy synchros—Bureau of Ordnance and Bureau of Ships synchros. The Bureau of Ordnance synchro has been adopted as the Navy Standard synchro and is used in essentially all new installations. Therefore only the BuOrd synchros are described in this chapter. A comparison between the two types is given later in the text.

The STATORS of the BuOrd synchro transmitter and receiver are identical electrically. Each stator (fig. 17–2) consists of a steel shell containing a laminated-steel field structure similar to that of an ordinary a-c motor. The windings in the stator slots consists of three groups of coils, the axes of which are 120 electrical degrees apart. The three groups of coils are wye-connected to form a 3-circuit distributed 2-pole winding. The three free

ends of the windings are attached to the synchro terminals. These leads are marked "$S1$," "$S2$," and "$S3$." The stator coils are referred to as the secondary winding.

The ROTORS of the transmitter (fig. 17–3, A) and the receiver (fig. 17–3, B) are also electrically alike. Each rotor consists of a laminated-steel core having two salient poles secured to the shaft by a collar. The rotor winding has one coil wound around the salient poles. The two free ends of the winding are connected to two insulated slip rings mounted on the shaft. The rotor coil is the primary winding and is energized through brushes and slip rings from a single-phase 115-volt 60-cycle power supply.

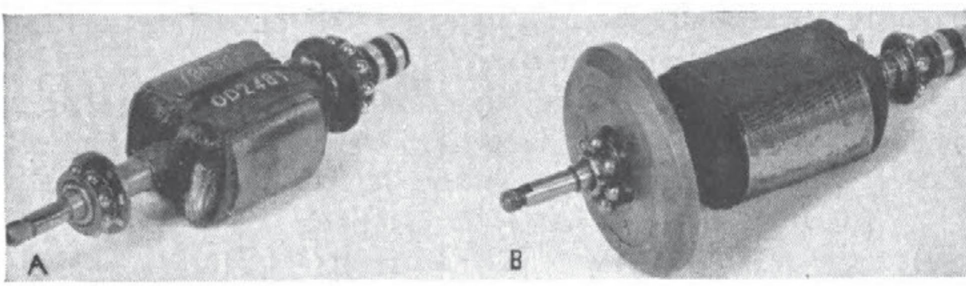


Figure 17–3.—Bureau of Ordnance synchro rotors.

The transmitter and receiver rotors are mechanically similar except that the receiver rotor has an inertia damper. The inertia damper consists of a metal flywheel mounted on ball bearings and provided with a keyed bushing and a friction-disk assembly. The purpose of the damper is to apply a braking effect on the receiver

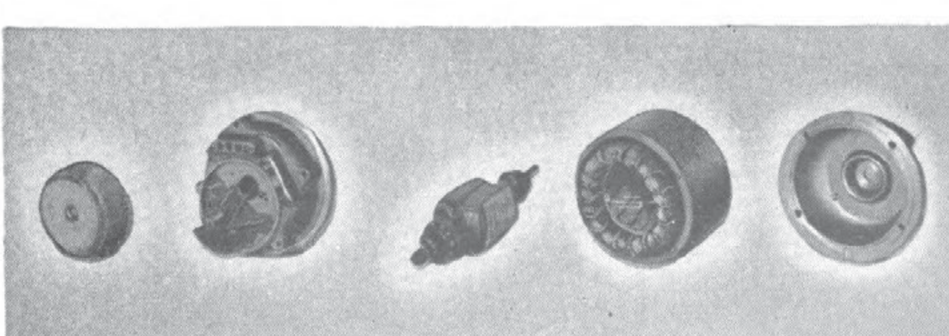


Figure 17–4.—Bureau of Ordnance synchro transmitter.

rotor to prevent it from oscillating or spinning. An exploded view of a Bureau of Ordnance transmitter is shown in figure 17–4.

Synchro Operation

A cross-sectional view of a synchro transmitter is shown in figure 17–5. In some types of installations the rotor does not turn continuously, and unless it does there is no magnetic revolving field produced around the stator. Hence, the action is that of a single-phase transformer with a single-winding primary (the rotor) and a 3-winding secondary (the stator). The transformer-induced voltages in the 3 secondary windings are either in phase or 180° out of phase with each other. If the rotor turns continuously, rotationally induced voltages 120° out of phase with each other are also developed in the three secondaries.

The turns ratio between the rotor and stator is such that when single-phase 115-volt power is applied to the rotor, the highest value of effective voltage that will be induced in any one stator

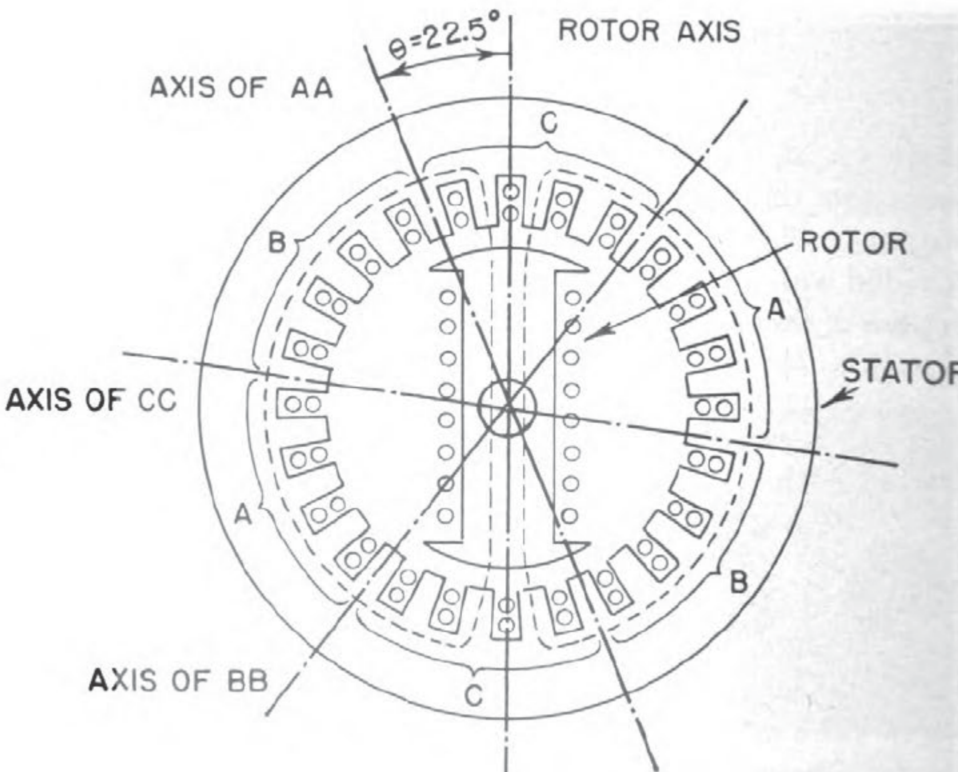


Figure 17–5.—Cross section of a synchro transmitter.

coil will be 52 volts. For example, the highest effective voltage that can be induced in stator coil A will occur when the rotor is turned through angle θ. In this position the rotor axis is in alignment with the axis of winding A and the magnetic coupling between the primary and secondary coil A is maximum.

The effective voltage, E_s, induced in any one of the secondary windings A, B, or C is approximately equal to the product of the effective voltage, E_p, on the primary; the secondary-to-primary turns ratio, N; and the magnetic coupling between primary and secondary which depends upon the cosine of the angle, θ, between the rotor axis and the axis of the corresponding secondary winding. Expressed mathematically,

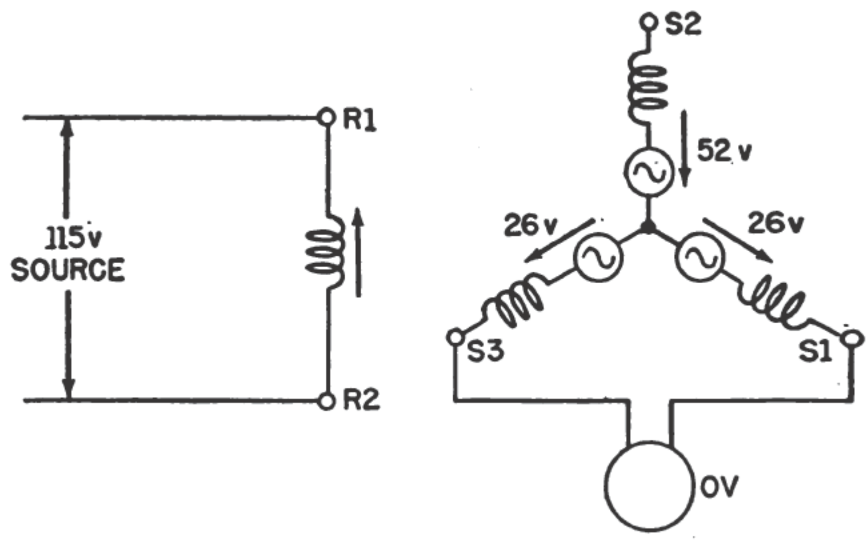
$$E_s = E_p \times N \times \cos \theta.$$

Therefore, because the effective primary voltage and the turns ratio are constant, the effective secondary voltage varies with angle θ.

In the example of figure 17–5, the axes of the rotor coil and stator coil AA are 22° apart; and the cosine of 22° is 0.928. Because $E_p \times N$, in this case, equals 52 volts, the effective voltage induced in A is 52×0.928, or 48.2 volts. When the rotor is turned so that its axis is brought into alignment with a particular stator coil, the voltage induced in that coil will have its greatest value. Turning the rotor 180° from this position will reverse the polarity of the voltage induced in this stator coil. The voltage relations are the same for synchro transmitters and receivers that are properly connected on the same system because they are identical electrically and because their rotors are energized from the same source.

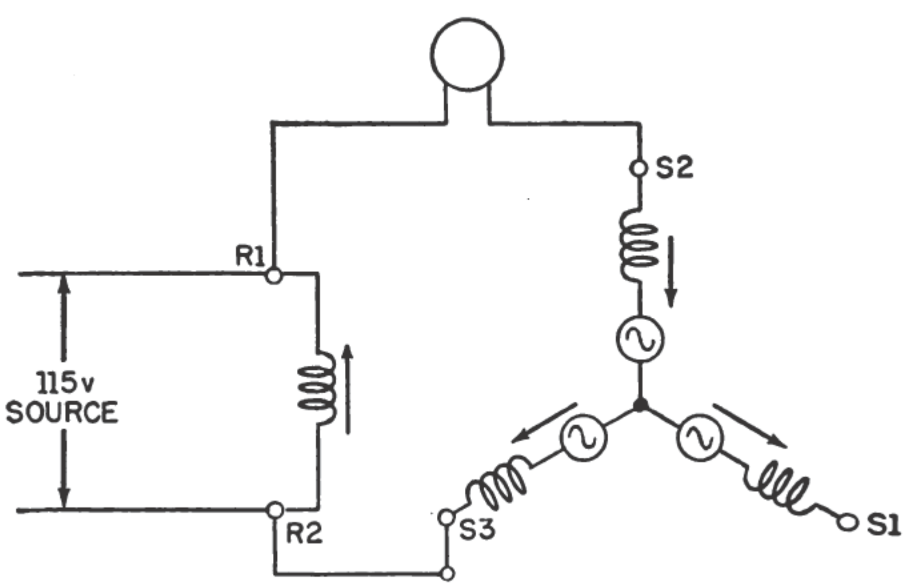
A REFERENCE POSITION is necessary if the angular position of a synchro rotor is to have meaning. In DIAGRAMS, a convenient reference, or zero degree, position is one in which the axis of the rotor coincides with the axis of the $S2$ coil, with the $R1$ terminal adjacent to the $S2$ coil. However, in practical applications the reference position must be determined electrically.

In order to properly align a synchro transmitter with its associated receiver, reference points are established on the rotor and stator and when these points are brought into alignment, the voltage distribution for the ELECTRICAL ZERO or reference position should be the same for all similar synchro transmitters and receiv-



VOLTAGE BETWEEN S1 AND S3 IS ZERO

A



VOLTAGE BETWEEN R1 AND S2 IS ADDITIVE
WHEN R2 AND S3 ARE JOINED

B

Figure 17-6.—Voltage distribution of a synchro transmitter in electrical zero position.

ers. The electrical test connections that are necessary to indicate the zero reference are shown in figure 17–6.

Electrical zero is taken as the position in which there is no voltage between terminals *S*1 and *S*3, (fig. 17–6, A) and in which the rotor and stator voltages are additive between *R*1 and *S*2 when *R*2 and *S*3 are connected together (fig. 17–6, B). The angular position of the rotor is measured clockwise from the position in which electrical zero is attained when the synchro is viewed from the shaft end. The connections indicated in this figure are used only to determine the electrical zero position.

Simple Synchro System

A simple synchro transmission system consists of a transmitter connected to a receiver, a shown in figure 17–7. The *R*1 transmitter and *R*1 receiver leads are connected to one side of the a-c supply line and the *R*2 transmitter and *R*2 receiver leads are connected to the other side of the supply line. The stators of both the transmitter and the receiver are connected *S*1 to *S*1, *S*2 to *S*2, and *S*3 to *S*3 so that the voltage in each of the transmitter stator coils opposes the voltage in the corresponding coils of the receiver stator. The voltage directions are indicated by arrows for the instant of time shown by the dot on the sine wave of the rotor supply voltage. The a-c generator symbol inserted in series with each of the synchro windings indicates the presence of a transformer-induced voltage.

The field of the transmitter is aligned with the axis of *S*2 (fig. 17–7, A) because the transmitter rotor axis is aligned with *S*2. Assume for the moment that the receiver rotor is not connected to the single-phase supply. Under this condition the voltage induced in the transmitter stator windings will be impressed on the receiver stator windings through the three leads connecting the *S*1, *S*2, and *S*3 terminals. Exciting currents that are proportional to the transmitter stator voltages will flow in the receiver stator windings, and the magnetomotive forces produced by these currents will establish a 2-pole field that orients itself in the receiver stator in exactly the same manner that the transmitter field is oriented in its own stator. Thus, if the transmitter field is turned by turning the transmitter rotor, the receiver field will also turn in exact synchronism with the transmitter rotor.

A soft-iron bar placed in a magnetic field will always tend to align itself so that its longest axis is parallel to the axis of the field. Thus, the salient pole receiver rotor will also align itself with the receiver field even though the receiver rotor winding is open-circuited. Operation with an open-circuited rotor is not desirable, however, because for a given position of the transmitter rotor and field there are two positions, 180° apart, in which the salient

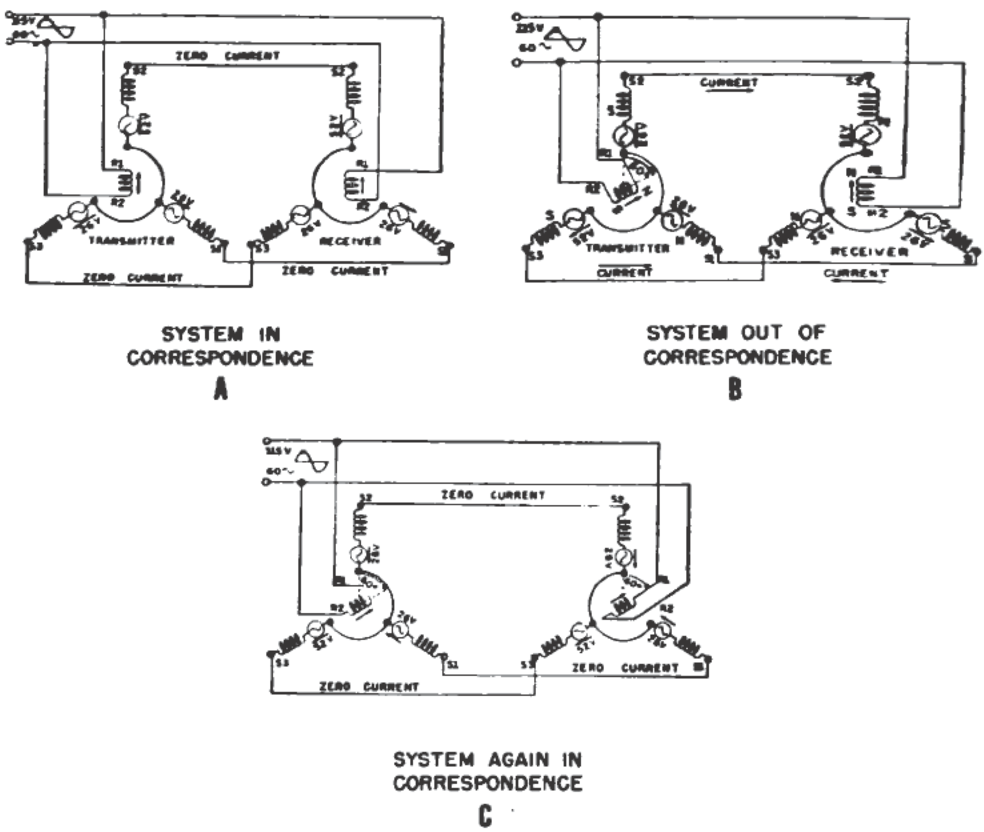


Figure 17–7.—Simple synchro system operation.

pole receiver rotor may come into alignment with the receiver field. This difficulty is eliminated by energizing the receiver rotor with alternating current of the same frequency and phase as that supplied to the transmitter rotor. Now the receiver rotor electromagnet comes into alignment with the receiver field in such a position as to always aid the magnetomotive forces of the three windings of the receiver stator. A rotor position in which the magnetomotive force of the rotor opposes the magnetomotive forces of the stator would not be stable. For example, a compass

needle always aligns itself with the earth's field so that the north pole of the needle points toward the geographic north—not toward the south.

When both rotors are displaced from zero by the same angle, they are properly aligned with their respective stator fields. Under this condition they are in correspondence and the induced voltages in each of the three pairs of corresponding stator coils are equal and in opposition. Hence, there is no resultant voltage between corresponding stator terminals and no current flows in the stator coils.

The angle through which a transmitter rotor is mechanically rotated is called a SIGNAL. For example, the angle that the transmitter rotor is displaced (fig. 17–7, B) is 60°. As soon as the transmitter rotor is turned, the transmitter *S*2 coil voltage decreases, the *S*1 coil voltage reverses direction, and the *S*3 coil voltage increases. Current immediately flows between the transmitter stator and the receiver stator in the direction of the stronger voltages. The unbalanced voltages are absorbed in the line drop and in the internal impedances of windings.

At the instant represented by the dot on the sine wave at the common input, the stator currents in the synchro receiver establish a north pole at the bottom of *S*2, a south pole at the top of *S*1, and a north pole at the top of *S*3. This polarity produces a torque on the receiver rotor and causes it to turn. The receiver rotor turns until the stator voltages in the transmitter and receiver are again equal and the two rotors come into correspondence, as shown in figure 17–7, C. Thus, the receiver rotor has turned 60°—that is, the same angle that the transmitter rotor has been turned.

As a receiver approaches correspondence, the stator voltages of the transmitter and receiver approach equality. This action decreases the stator currents and produces a decreasing torque on the receiver that reduces as the position of correspondence is reached. At correspondence, the torque on the receiver rotor is only the amount that is caused by the tendency of the salient pole rotor to align itself in the receiver field. Hence, a receiver can position only a very light load.

Synchro transmission systems are used in a great many over-all systems, as stated in the early part of this chapter. However, when heavier drive power than required for a dial or pointer is

necessary, some method of amplifying the synchro's weak torque must be employed. Two common methods used to provide this amplification are the SERVO SYSTEM and the AMPLIDYNE.

Many installations connect one transmitter to several receivers in parallel. This procedure requires a transmitter large enough to carry the total load current of all the receivers. All the $R1$ rotor leads are connected to one side of the a-c power supply, and all the $R2$ rotor leads are connected to the other side of the supply. The receiver stator leads are connected lead for lead to the transmitter stator leads.

The sizes of Navy synchro transmitters are designated by numbers. Synchro transmitters in general use are sizes 1, 5, 6, and 7. Synchro receivers in general use are sizes 1 and 5. A size-1 transmitter can control one or two size-1 receivers; a size-5 transmitter can control two size-5 receivers; a size-6 transmitter can control as many as nine size-5 receivers; and a size-7 transmitter can control as many as eighteen size-5 receivers.

Synchros employed in airborne equipments are usually designed for 400-cycle operation, and as a result, are considerably lighter in weight and smaller in size than those that operate with 60-cycle line voltages.

Differential Synchros

CONSTRUCTION.—The differential synchro is similar to the synchros described thus far except that the rotor has, instead of a salient pole concentrated winding, a three-coil distributed winding similar to that of the stator. The rotor core is slotted to accommodate the three distributed windings and, with no rotor current (fig. 17–8), has no torque to align it with its stator field. This action is in contrast with that of the salient-pole rotor of the synchro motor, which develops a rotor torque even with zero rotor current.

The salient-pole rotor tends to align itself in the rotor field so that its axis is parallel to that of the field. The circuit for the magnetic flux is through the path of least reluctance. The flux lines always act like stretched rubber bands tending to shorten themselves, and thus the salient pole rotor is pulled into alignment with the stator field axis. When this action is accomplished, the rotor air gap presented to the stator field flux is a minimum.

The differential synchro is designated as a transmitter or as a

receiver depending on its circuit function. The differential transmitter has two inputs (one mechanical and one electrical) and one electrical output. The differential receiver has two inputs (both electrical) and one mechanical output. Mechanical inputs or outputs are shaft positions and electrical inputs or outputs are stator or rotor voltages. The two differential synchros are identical, except that the differential receiver has a mechanical damper like that of an ordinary synchro receiver.

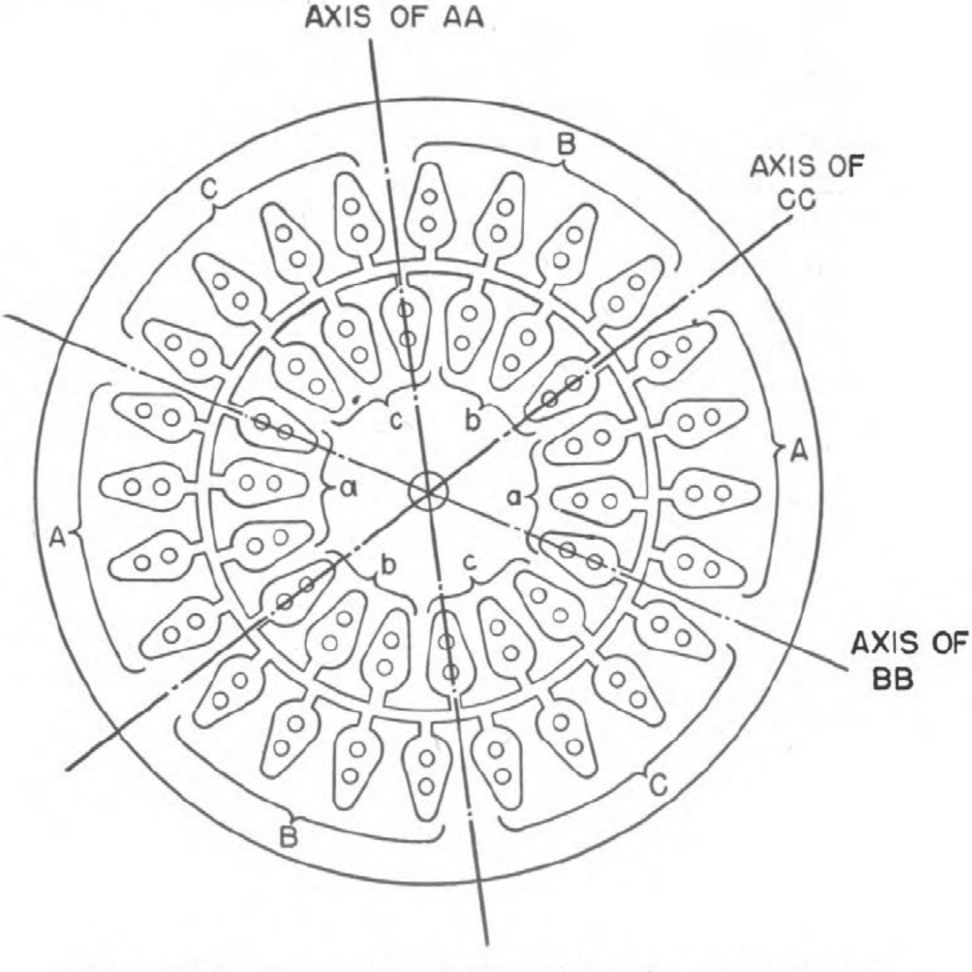


Figure 17–8.—Cross section of differential synchro rotor and stator.

The synchro DIFFERENTIAL TRANSMITTER signals come from two separate sources. One is cranked mechanically into the rotor of the differential and the other is fed electrically into the stator windings of the differential. The differential transmitter can be connected to transmit either the difference or the sum of these signals as an electrical output to a synchro receiver.

One of the synchro DIFFERENTIAL RECEIVER electrical inputs is fed into the rotor from one synchro transmitter and the other is fed into the stator from another synchro transmitter. The mechanical output is determined by the movement of the differential rotor caused by the interaction of the rotor currents with the stator field. As in the case of the differential transmitter, the differential receiver can be connected to indicate either the difference between two signals or the sum of two signals.

OPERATION.—The standard connection for a DIFFERENTIAL TRANSMITTER to produce the DIFFERENCE between two signals (differential subtracting) is to connect the stator leads of the differential transmitter to the respective stator leads of the synchro transmitter, as shown in figure 17–9. The rotor leads of the differential transmitter are connected to the respective stator leads of the synchro receiver. Note that the positions of the three rotors are at electrical zero in figure 17–9, A, and that all the signals are zero. The induced voltages in the transmitter stator coils produce a magnetizing current, I_{mag}, that flows through the stator coils of the differential. This magnetizing current establishes an alternating 2-pole field flux across the differential rotor, which induces an emf in the differential rotor coils. The voltages induced in the differential rotor coils are equal and opposite to the voltages induced in the corresponding stator coils of the receiver. Hence, no current flows in the receiver stator coils.

If a signal is put into the system by turning the transmitter rotor (while the differential rotor is held in the electrical zero position), the electrical signal is transmitted unchanged through the differential to the receiver and the receiver rotor follows the signal of the transmitter.

For example, if the transmitter rotor (fig. 17–9, B), is turned clockwise through an angle of 60°, the effective voltage in $S3$ becomes a maximum because the transmitter rotor axis is aligned with $S3$. The field of the differential will now turn clockwise 60° and align itself with $S3$.

The effective voltage in $S3$ increases to a maximum and causes the effective current in the receiver stator coil $S3$ to increase to a maximum value. Thus, the field of the receiver turns clockwise 60° and comes into alignment with $S3$. Because the salient pole rotor of the receiver is aligned with the field of that machine at all

times, the rotor turns 60° in a clockwise direction. The receiver thus follows the transmitter signal for any angle of the transmitter rotor either clockwise or counterclockwise.

Turning the differential rotor through an angle of 60° (fig. 17–9, C) moves rotor coil $R2$ into alignment with the differential field, and the effective voltage in $R2$ increases to a maximum value.

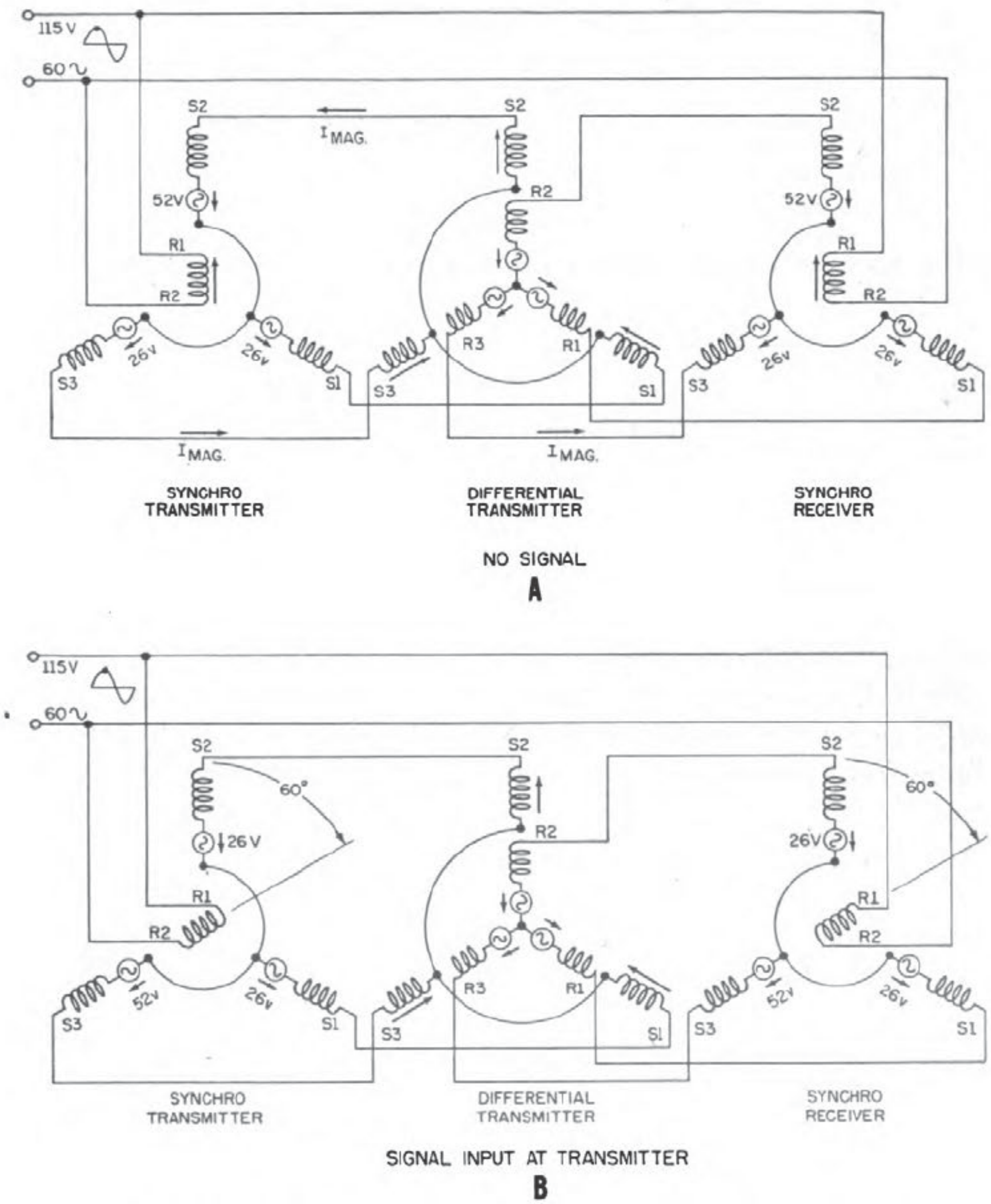


Figure 17–9.—Differential transmitter action in a synchro system.

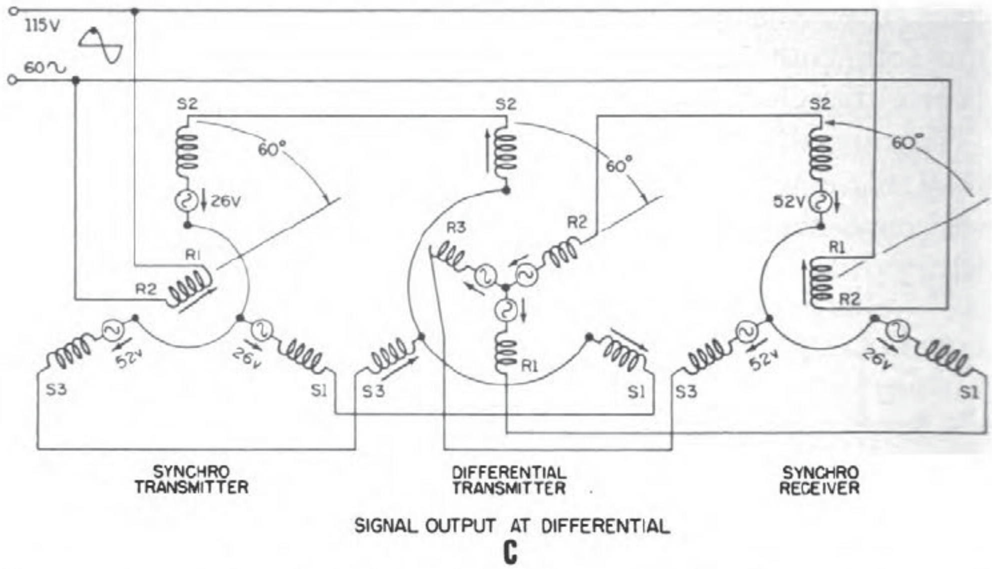


Figure 17–9.—Differential transmitter action in a synchro system—Continued.

The currents in the receiver stator coils depend on the differential rotor voltages, and the field of the receiver orients itself with respect to the STATOR coils of that machine in the same manner that the differential field is oriented with respect to the ROTOR coils of the differential transmitter.

Therefore, as the voltage in differential rotor coil $R2$ increases, the effective current in the receiver stator coil $S2$ increases. The receiver field and rotor turn 60° in a counterclockwise direction and come into alignment with $S2$.

Note that the receiver rotor follows the differential transmitter signal in the reverse direction. The differential subtracts its own signal from the signal at the transmitter and transmits the difference of the two signals to the receiver. When the receiver rotor has rotated 60° counterclockwise, its stator voltages again match the differential rotor voltages.

A positive (clockwise) signal input to the synchro transmitter shaft positions its FIELD with respect to its stator; whereas, a positive (clockwise) signal to the differential transmitter shaft positions its SECONDARY (rotor coils) clockwise with respect to its stator field. A clockwise movement of the differential rotor is equivalent to a counterclockwise movement of the differential field with respect to the differential rotor. Hence, the receiver follows the differential transmitter signal in the reverse direction.

The foregoing paragraphs describe the differential action with only one signal at a time, either at the transmitter or at the differential. The action is similar, however, if two signals are put in simultaneously—one at the transmitter rotor and one at the differential rotor.

The general equation for obtaining the differential action for a standard-connection differential transmitter system is

$$T-DT=R,$$

where T, DT, and R are the respective shaft positions of the transmitter, differential transmitter, and receiver.

To reverse the differential action so that ADDITION of the two input signals results, reverse the $S1$ and $S3$ leads and the $R1$ and $R3$ leads of the differential transmitter. The general equation for obtaining the additive action when the leads are reversed is

$$T+DT=R.$$

In a synchro DIFFERENTIAL RECEIVER system (fig. 17–10) two ordinary synchro transmitters, $T1$ and $T2$, feed signals θ_1 and θ_2 respectively to the differential receiver. Signal θ_1 goes to the differential stator, and signal θ_2 goes to the differential rotor. The differential subtracts θ_2 from θ_1 and produces $\theta_1-\theta_2$ at the differential rotor shaft. The standard connection for subtraction is to connect the $R1$ leads of $T1$ and $T2$ to the same side of the single-phase 115-volt power supply and the $R2$ leads to the other side. The three stator leads of $T1$ are connected to the respective stator leads of the differential receiver, and the three differential rotor leads are connected to the respective stator leads of $T2$, as shown in figure 17–10, A.

The equation for obtaining the differential action of a standard-connected differential receiver system is

$$\theta_1-\theta_2=\theta_3,$$

where θ_1, θ_2, and θ_3 are the respective shaft displacements of $T1$, $T2$, and DR.

For differential action to be reversed so that addition of the two input signals results, the $R1$ and $R3$ leads of the differential

receiver must be reversed, as shown in figure 17–10, B. The equation for obtaining the additive action is

$$\theta_1+\theta_2=\theta_3.$$

Synchro Control Transformers

Unlike the synchro, which is used to transmit angular motion, the synchro control transformer is used to indicate by means of

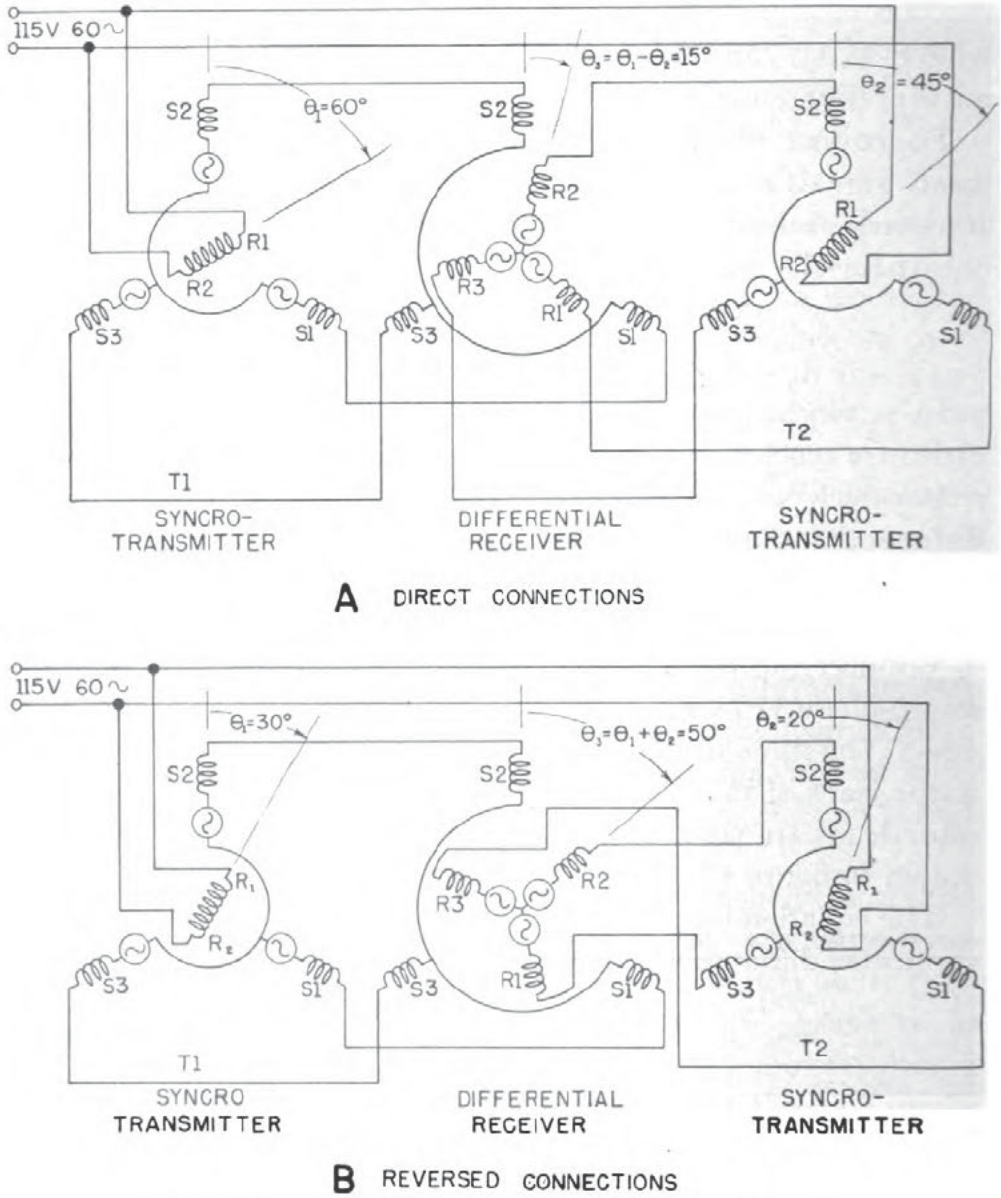
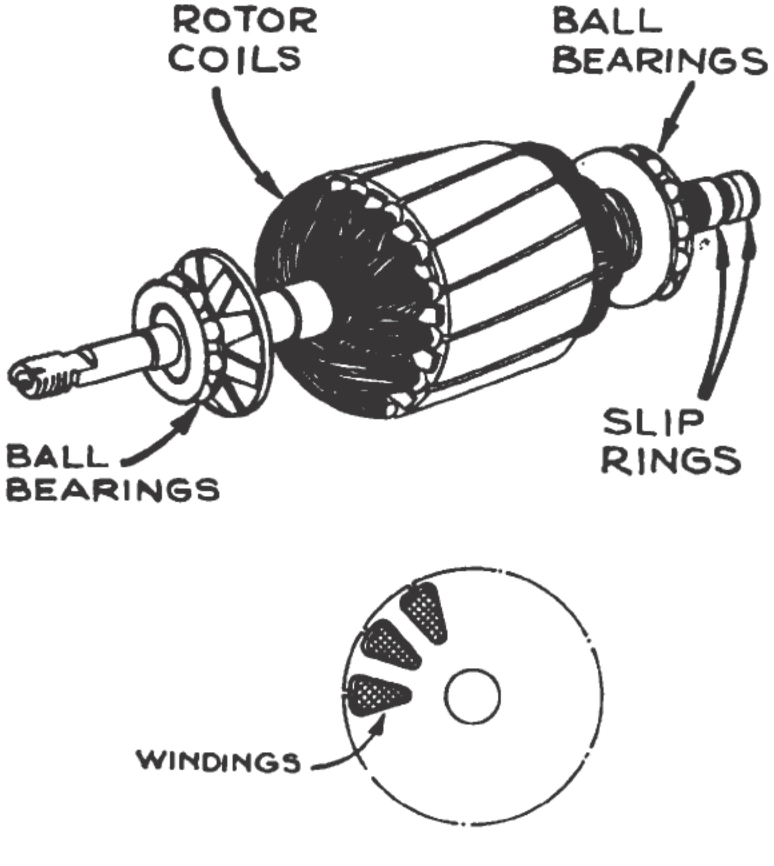


Figure 17–10.—Differential receiver action in a synchro system.

a voltage, the error or difference in the angular positions of the input and output shafts of a servomechanism (treated later).

The synchro control transformer receives an electrical signal from a synchro transmitter or a differential transmitter in the same manner that a synchro receiver receives such a signal. The synchro control transformer differs from the synchro receiver in that the output of the control transformer (*CT*) is not a movement of a shaft position but an error voltage, from its rotor winding. Unlike other synchros, the *CT* rotor is not connected to the a-c supply. The control transformer is used to convert the mechanical input of a synchro transmitter into an electrical output for an amplifier. The electrical output of the control transformer, after being properly amplified, is used to control the power drive for a gun, radar antenna, and so forth.

Construction.—The external appearance of a control trans-



Cross Section of Rotor

Figure 17–11.—Synchro control transformer rotor.

former is similar to that of the ordinary synchro, but the windings are different. The stator and rotor windings of a control transformer have a relatively high impedance. The stator-to-rotor voltage transformation ratio is approximately 3 to 2. The stator is the primary and has a 3-circuit 2-pole distributed wye-connected high-impedance winding having many turns of fine wire. The rotor is the secondary and consists of a single distributed 2-pole winding. This winding is placed in slots uniformly distributed around the periphery of the smooth-surface rotor, and the two free ends are connected to two slip rings (fig. 17–11). For zero error, the axis of the synchro control transformer field is in quadrature with the axis of the rotor winding, and the voltage induced in the rotor is zero. Irrespective of the rotor position no appreciable torque is developed, and the rotor current has negligible effect on the primary currents. The shaft of the control transformer is always turned mechanically and does not require the use of an inertia damper.

The electrical-zero position for a control transformer is different from that of an ordinary synchro transmitter or receiver. In a control transformer it is the position where a minimum voltage is induced in the rotor by the *S*2 stator coil.

OPERATION.—A simple control transformer circuit is shown in figure 17–12. The rotor of the control transformer is in the electrical-zero position. The output of the control transformer rotor is zero because the rotor axis is displaced exactly 90° from the transmitter rotor and is therefore at right angles to the control transformer field established by the stator coil currents from the transmitter. Hence, the transmitter and the control transformer rotors are in correspondence.

If the transmitter rotor is turned in one direction, the two rotors are no longer in correspondence and a voltage is induced in the rotor of the control transformer because the stator field links the rotor winding of the control transformer. This output voltage is represented by the solid-line sine wave (fig. 17–12), and is assumed to be in phase with the source voltage.

If the transmitter rotor is displaced in the opposite direction, the induced voltage output is represented by the broken-line sine wave, and is 180° out of phase with the source voltage. The PHASE of the control transformer output voltage reverses with a reversal in the DIRECTION of the transmitter rotor displacement

from its electrical-zero position. The MAGNITUDE of the control transformer output voltage varies with the amount by which the shafts of the control transformer and the synchro transmitter are out of correspondence.

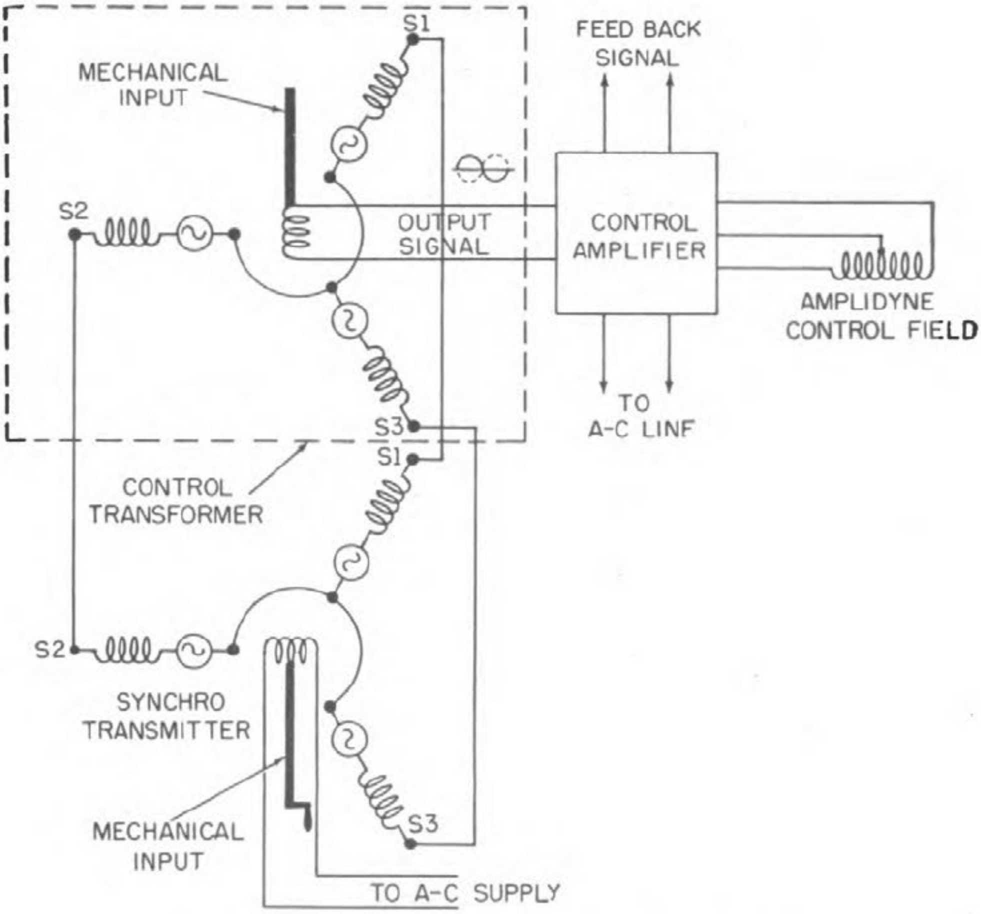


Figure 17–12.—Synchro control transformer circuit.

The *CT* output signal, acting through the control amplifier and amplidyne network, causes the load drive motor (not shown) to drive the load and the *CT* rotor to the position of correspondence with the synchro transmitter.

Comparison of Bureau of Ordnance and Bureau of Ships Synchros

The BuOrd and BuShips synchros differ in some respects, although they operate on the same principle. The differences

are in the shaft and mounting dimensions, winding arrangements, and connection markings.

The physical appearance of the BuShips synchro is similar to that of the BuOrd (already discussed). The main difference between the two is the location of their primary and secondary windings. The BuShips synchro is the reverse of the BuOrd type—that is, the rotor is the secondary and the stator is the primary. As with the BuOrd type, the BuShips synchro transmitter and receiver are identical electrically.

The capacities of Bureau of Ordnance and Bureau of Ships synchros of comparable size are approximately the same. The corresponding size designations of these two types of synchros are listed as follows:

Bureau of Ordnance	*Bureau of Ships*
5G transmitter	Type-A transmitter
6G transmitter	Type-B transmitter
5F receiver	Type-M receiver (standard)
1F receiver	Type-M receiver (midget)

Flange- and Bearing-Mounted Synchros

FLANGE-MOUNTED SYNCHROS.—Thus far, only the flange-mounted synchro has been treated. The method of mounting (by means of a flange) is shown in figure 17–13. Only the rotor moves; the field coils and the frame on which they are mounted do not move.

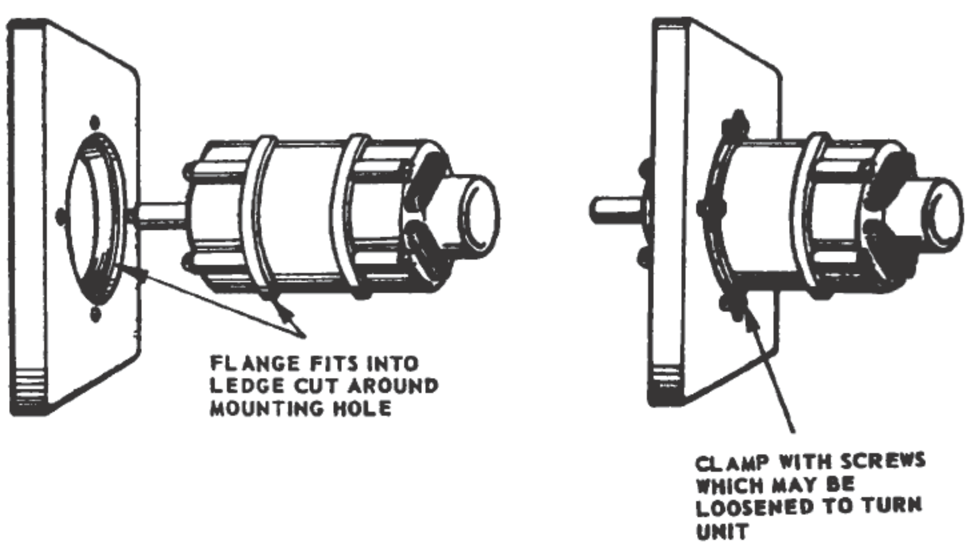


Figure 17–13.—Flange-mounted synchro.

BEARING-MOUNTED SYNCHROS.—Electrically, the bearing synchro is exactly like the ordinary synchro receiver and transmitter, but mechanically the operation is different. The method of mounting (by means of bearings) is shown in figure 17–14. The bearings permit the stator to be turned mechanically. The signal is transmitted from or received in the stator just as it is in the flange-mounted synchro. However, in the bearing-mounted synchro a mechanical signal is also applied to the stator. In the bearing-mounted synchro, the stator leads are brought out through slip rings and brushes.

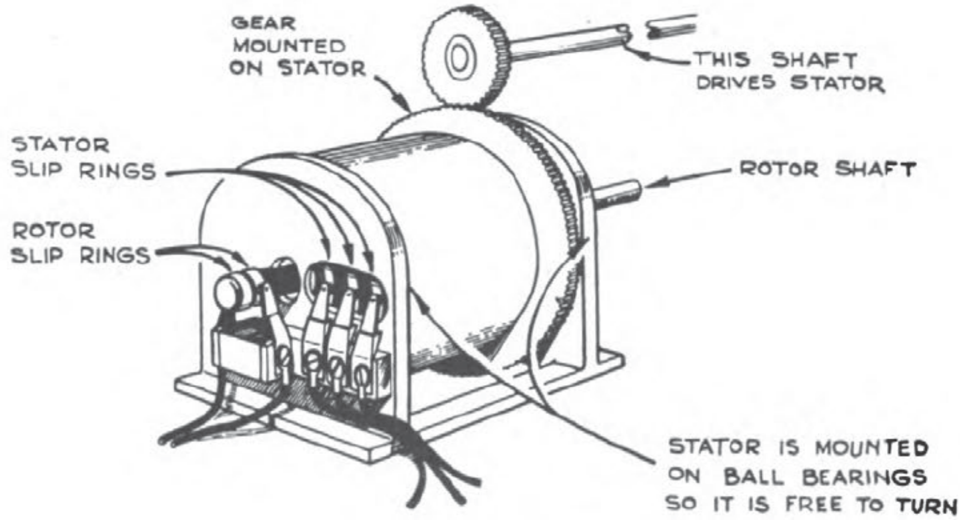


Figure 17–14.—Bearing-mounted synchro.

The electrical signal is received (receiver operation) via the three stator leads. This signal causes the rotor to rotate within the stator. The mechanical signal is received via the drive shaft and gears, thus causing both the stator and the rotor to rotate. The rotor turns with the stator because the stator flux holds the rotor in position with respect to the stator.

Bearing-mounted synchros are usually used to operate a set of electrical contacts. The contacts control the current to a SERVOMOTOR (described later) which drives a load. A system of cams and linkages causes the rotor to move in such a direction as to reduce the synchro signal. The elements of these systems are treated as needed in the rating texts.

SERVOMECHANISMS

Introduction

The synchro systems considered thus far do not amplify the receiver torque, and therefore the receiver can move only light loads such as indicator dials. When torque, or power, amplification is needed, some type of control system capable of supplying the necessary power is used. In general, there are two types of control systems, the open loop and the closed loop.

The OPEN-LOOP control system is shown in figure 17–15, A. If the handwheel, H, is moved through an angle, θ_i, the power from the external power source is controlled by the amplifier (a device for increasing the power delivered to the load) so that the motor rotates the load shaft, L, through an angle, θ_o. At all times, θ_o should be equal to θ_i. In order that this equality may exist constantly, the output of the power should not vary and the characteristics of the amplifier and the motor should be constant, even under varying load conditions. Otherwise some type of compensation must be employed to counteract the variations in the operating characteristics. In general, these requirements cannot be met in the open-loop control system.

The simple CLOSED-LOOP control system shown in figure 17–15, B, differs from the open-loop control system in that the output angle θ_o, is subtracted from the input angle, θ_i, to obtain the error signal, e. The error signal is then used to control the amplifier output. This error signal represents inverse, or negative, feedback—that is, the error signal always acts in such a way as to reduce the error. In contrast to the open-loop system, it is not necessary to use a compensated amplifier or a motor insensitive to load variations.

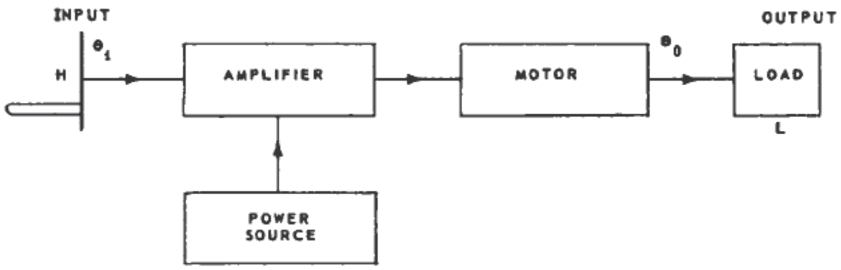
Thus, it is almost always more advantageous—that is, cheaper and simpler—to use a closed-loop servomechanism in place of an open-loop control system when tight control of an output action is demanded.

A SERVOMECHANISM is a special type of closed-cycle automatic control system. In this system the signals fed to the controller may change rapidly and often, and the function of the system is to cause the controlled device to "follow" these signals. If the controlling signals change slowly or not at all, the problem is to

prevent other factors from influencing the controlled element, and the system is called a REGULATOR.

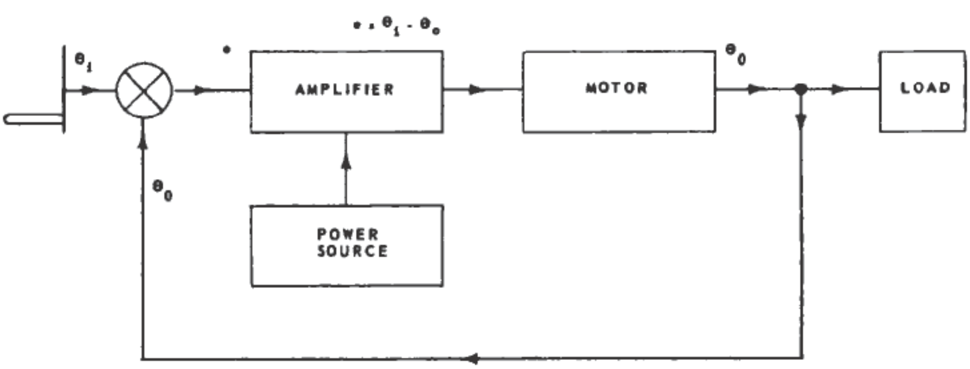
Servomechanisms that are used to drive small loads are commonly called INSTRUMENT SERVOMECHANISMS. The load in this instance is the gear train and the bearing friction of the system. This type of servomechanism has wide application in computing systems, where the required power output is only a fraction of a horsepower.

Servomechanisms having larger power output are used for steering ships, controlling airplanes, positioning guns, and antennas; and in many other applications employing automatic control where accuracy of reproduction is the primary objective.



A

OPEN-LOOP CONTROL SYSTEM



B

CLOSED-LOOP CONTROL SYSTEM

Figure 17–15.—Automatic control systems.

A complete servomechanism involves many components all functioning together to control a certain quantity or device. Because servomechanisms, as well as indicator systems, have a wide and varied use in the Navy, it is necesary that the technician broaden his knowledge to include at least some of the more elementary principles employed in servomechanisms. Toward this end a simplified servo and indicator system is first described. The components are then discussed and a diagram of a complete system is given together with a description of the function of each part in the over-all system. Finally the double-speed transmission system is briefly described.

It should be emphasized that relatively complex systems are used in fire control, radar, interior communications, and other systems. None of the mechanical linkages are described in this chapter; however, in a practical system the mechanical aspects are perhaps as important as the electrical aspects. Also, some systems combine electrical control with mechanical and hydraulic control. In this chapter only the more basic electrical concepts are treated. The more complex systems are treated as required in the rating texts.

Simplified Servo and Indicator System

A simplified servo and indicator system is shown in figure 17–16. The objective is to position the heavy antenna by turning the handwheel (which requires a minimum of force) at a location remote from the antenna and at the same time to indicate the antenna bearing on a dial so that it may be observed by the operator.

In the INDICATOR SYSTEM, synchro transmitter *TR*1 converts the position of the shaft attached to the antenna into an equivalent electrical signal. This signal is transmitted by a 3-wire transmission line to synchro receiver *REC*, which converts the electrical signal back into shaft rotation. The shaft is attached to a pointer which indicates the antenna position on a dial. For every position of the antenna the direction of the beam in horizontal angle is thus indicated on the dial. A similar arrangement can also be applied if desired to an elevation indication (vertical angle) or to any other data-transmission problem.

In the SERVO SYSTEM, synchro transmitter *TR*2 converts the position of the handwheel into an electrical signal that is trans-

mitted to control transformer *CT*. The control transformer has two inputs, one mechanical from the antenna and one electrical from *TR*2. It compares the relative position of the antenna shaft (mechanical input) with the position of the handwheel (electrical input) and develops an ERROR voltage proportional to the difference between the two positions. The error voltage is applied to the control amplifier which supplies the field current of the

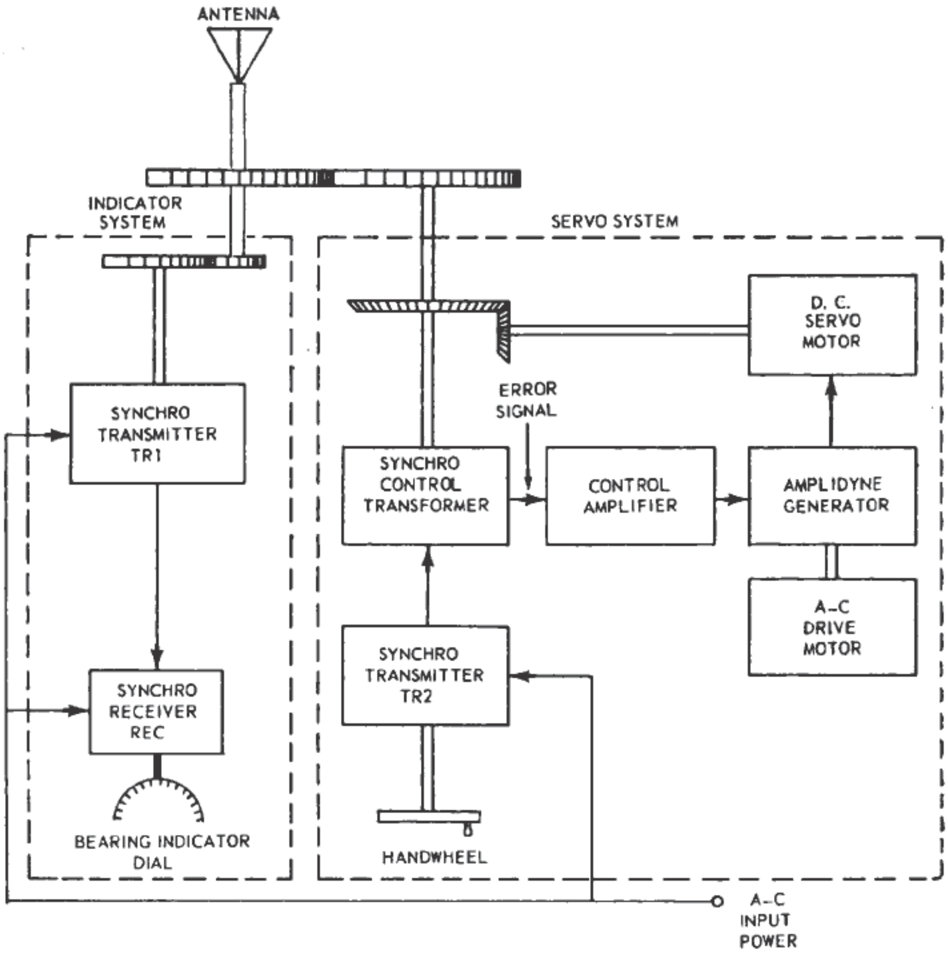


Figure 17–16.—Block diagram of a simplified servo and indicator system.

amplidyne generator. The output of the amplidyne generator drives the d-c servomotor which rotates the antenna. Whenever an error voltage is present, the antenna is rotated in such a direction as to reduce the error voltage. Thus, when the antenna position corresponds to the handwheel position, the error voltage is reduced to zero and the antenna comes to a stop.

The principal means of providing the electromechanical conversions necessary for the operation of both the position indicator and the servo system is by the use of synchros.

Amplidyne

INTRODUCTION.—As has been stated earlier in this chapter, SYNCHROS are used for the transmission of angular motion without developing a large amount of torque.

DIFFERENTIAL SYNCHROS are used for combining, in the desired manner angular motion from two different sources, again without developing a large amount of torque.

SYNCHRO CONTROL TRANSFORMERS are used to produce an output voltage that is proportional to the angular difference between the input and output shafts of a servomechanism. The error voltage from the control transformer is fed to a CONTROL AMPLIFIER (amplifies the output of the control transformer) which increases the amplitude of the error signal. In an amplidyne power drive the output of the control amplifier supplies the field coils of an amplidyne generator. A small variation in the strength of the current in the field coils of the amplidyne generator causes a great variation in its output power. In this respect the amplidyne is a d-c amplifier. Thus, the signal developed in the control amplifier can cause the amplidyne generator to supply enough output power to operate the d-c servomotor which has sufficient torque to move a heavy load.

The amplidyne power drive, as commonly used, consists of an amplidyne generator, a driving motor, and a servomotor, as shown in simplified form in figure 17–17. The amplidyne generator has two sets of brushes on the commutator. One set is short-circuited and the other set supplies the armature of the servomotor. The field of the servomotor and the control field of the amplidyne are separately excited. The amplidyne control field has a split winding, one for each polarity of the applied signal. The series compensating winding is discussed later in the text. The amplidyne drive motor is ordinarily a 3-phase a-c induction motor.

OPERATION.—The amplidyne generator is a small separately excited direct-current generator having a control field that requires a low power input. The armature output voltage may be 100 volts and the output load current 100 amperes, giving an output power of 10 kilowatts. The amplidyne is essentially a control

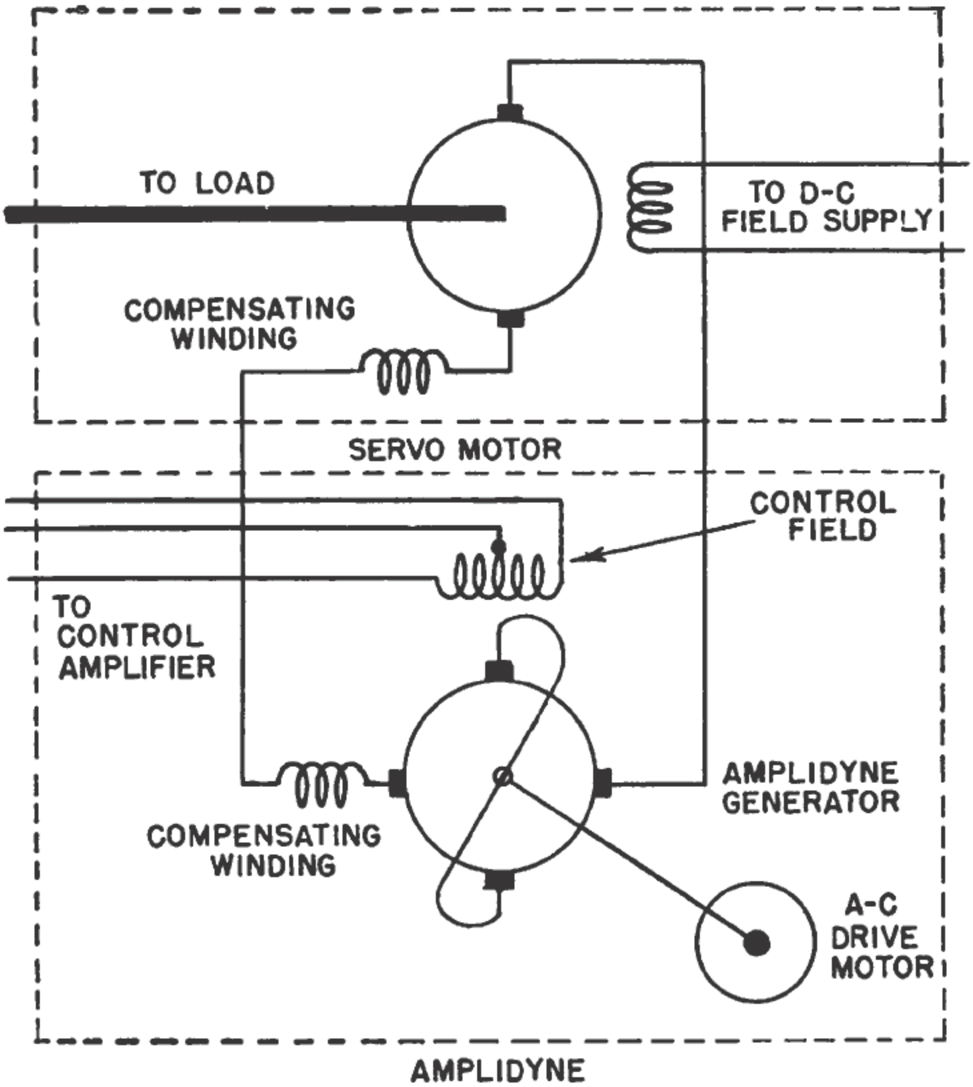


Figure 17–17.—Basic amplidyne drive.

device that is extremely sensitive. For example, an increase in control field power from zero to 1 watt can cause the generator output power to increase from zero to 10 kw. It is thus a power amplifier that increases the power by an amplification factor of 10,000. The following analysis shows how this high amplification is accomplished.

In an ordinary 2-pole generator (fig. 17–18) the separately excited control field may take an input power of 100 watts in order to establish normal magnetism in the field poles and a normal armature voltage of 100 volts. When this voltage is impressed on a 1-ohm load the armature will deliver 100 amperes,

or an output power of 10 kw. The circles with dots represent armature conductors carrying current toward the observer; those with crosses represent electron flow away from the observer. The control field winding develops a north pole (N) on the left-hand field pole and a south pole (S) on the right-hand field pole.

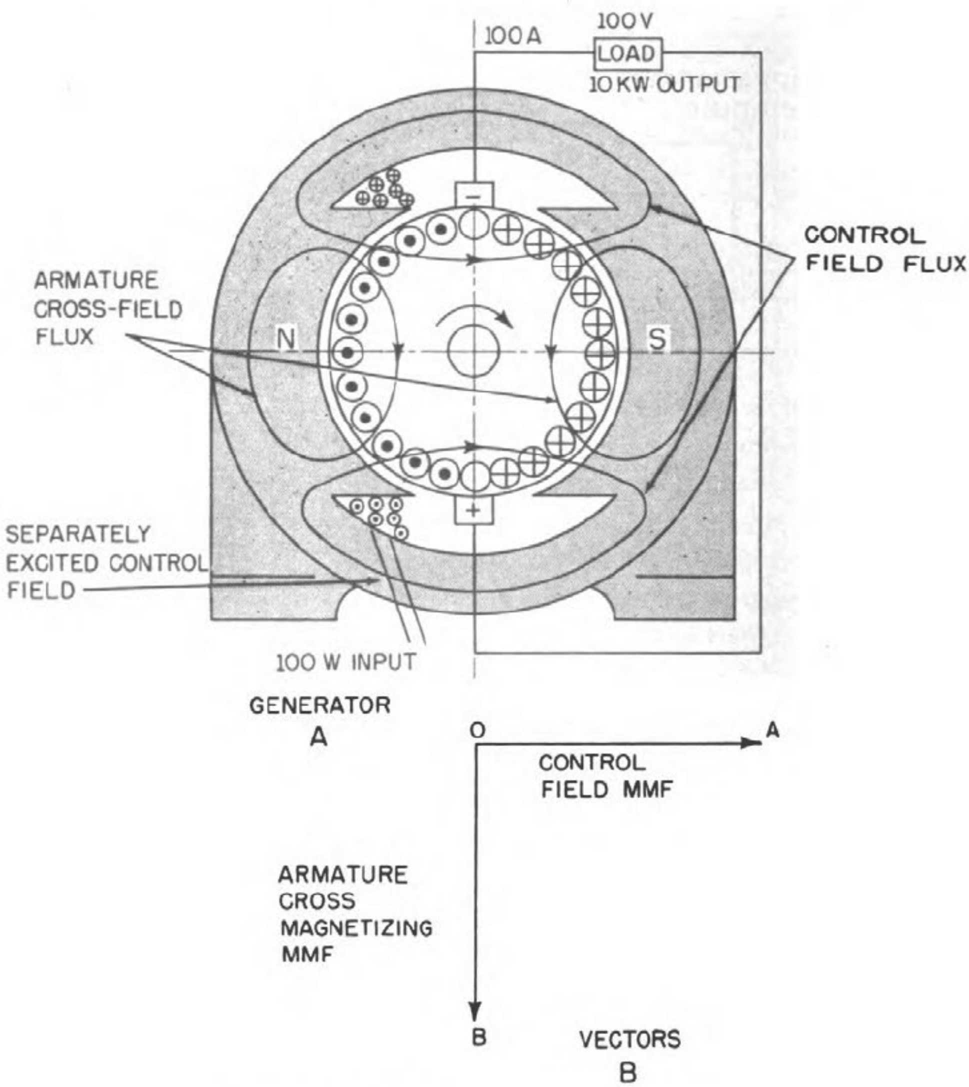


Figure 17–18.—Ordinary d-c generator.

The relative strength of the control field is represented by vector OA in figure 17–18, B. The armature acting as an electromagnet establishes a quadrature, or cross, magnetomotive force having an axis that coincides with that of the brushes. The relative strength and position of this armature cross magnetizing

force is indicated by vector OB in figure 17–18, B. It has about the same length as the control field vector, OA, and is perpendicular to it.

If the strength of the control field in figure 17–18, A, is reduced to 1 percent of normal, the armature generated voltage will fall to 1 percent of normal, or to about 1 volt in this example. The output current from the armature will fall to 1 ampere through the 1-ohm load.

Normal armature current may be restored by short-circuiting the brushes, as shown in figure 17–19, A. One volt acting

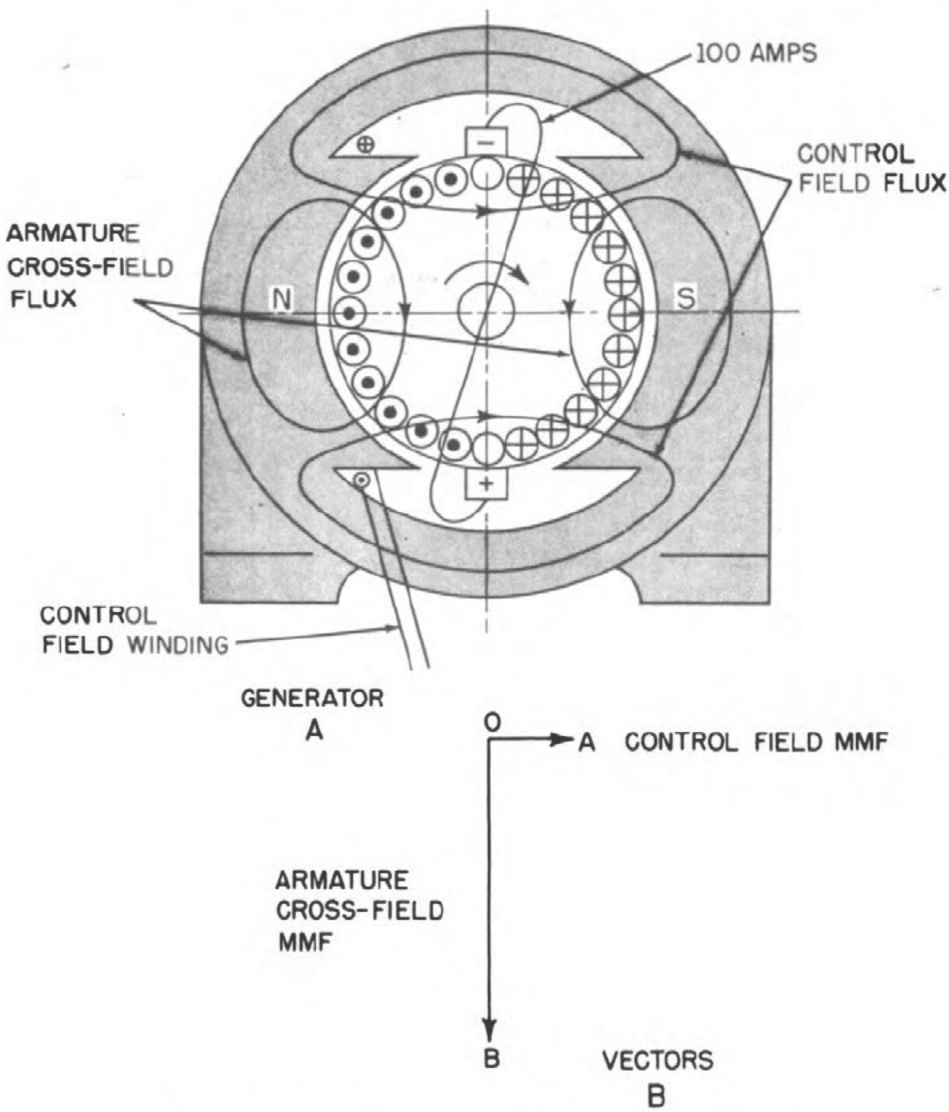


Figure 17–19.—Generator with short-circuited brushes.

through an internal armature resistance of 0.01 ohm will circulate 100 amperes between the brushes. The control field strength is still 1 percent of its normal value (vector *OA*, fig. 17–19, B), but the armature cross-field mmf has been restored to its normal value, as indicated by vector *OB*.

The armature cross field is cut by the armature conductors and may be regarded as being responsible for normal load voltage, which is obtained across an additional pair of brushes, as shown in figure 17–20, A. The axis of these brushes is perpendicular to that of the shorted brushes. The load is connected in series with these new brushes and a compensating winding shown schematically around the south field pole, *S*.

Armature load current is toward the observer around the upper half of the armature windings and away from the observer around the lower half. The armature acts like an electromagnet, which creates an mmf directly opposed (in this case) to the control field mmf and in the same axis as that of the control field.

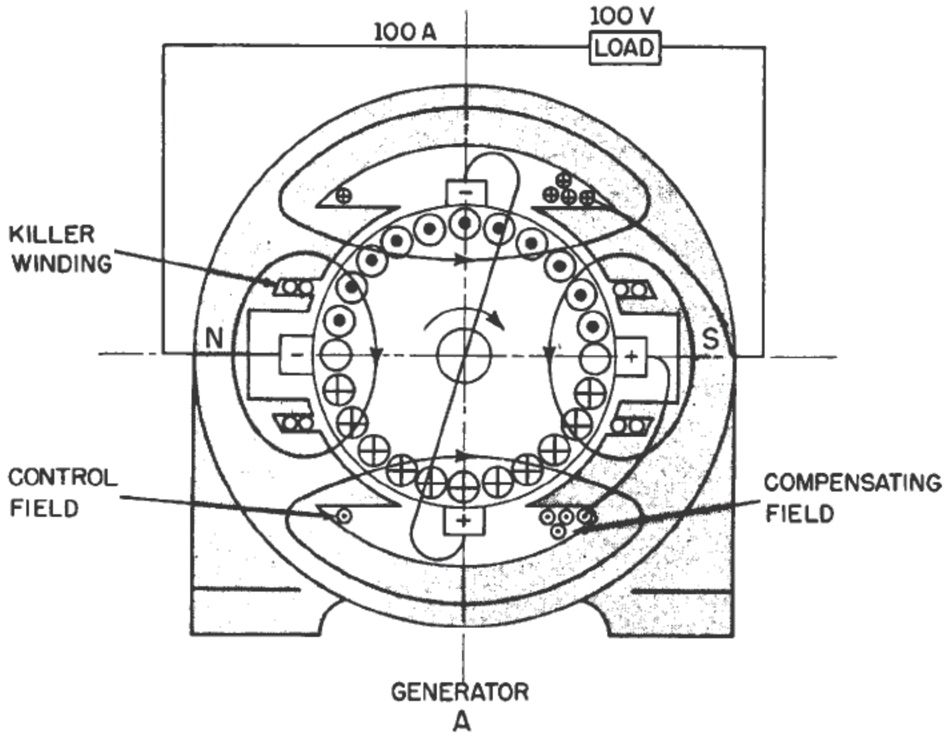
The purpose of the compensating winding is to create an opposing mmf along this same axis to counterbalance the armature load current mmf. This counterbalance is made automatic for any degree of load by connecting the compensating winding in series with the load brushes and the load. Vector *OC* (fig. 17–20, B) represents the relative magnitude and direction of the compensating winding mmf with respect to the control field mmf, *OA*; the armature load current mmf, *OD*; and the armature cross-field mmf, *OB*.

The main field poles are shown slotted along the load-brush (horizontal) axis to indicate a method of achieving satisfactory commutation in the coils being shorted by these brushes. A restriction in the size of the amplidyne is the self-induced voltage in the coils being commutated and the resultant sparking at the commutator.

Because any residual magnetism along the axis of the control field would have an appreciable effect on the amplidyne output, it is necessary to demagnetize the core material when the control field winding is deenergized. This demagnetization is accomplished by means of a small a-c magneto generator (mounted on the amplidyne frame) which supplies a suitably placed demagnetizing winding, known as a KILLER WINDING (fig. 17–20, A).

The action of the amplidyne is summarized as follows:

1. A very low power (1 watt) input to the control field creates an armature short-circuit current of 100 amperes, thus producing a relatively strong armature cross field. This is responsible, in turn, for the generation of a normal output voltage of 100 volts across the output load brushes. The output load of 1 ohm is supplied with a current of 100 amperes and an output power of 10,000 watts.



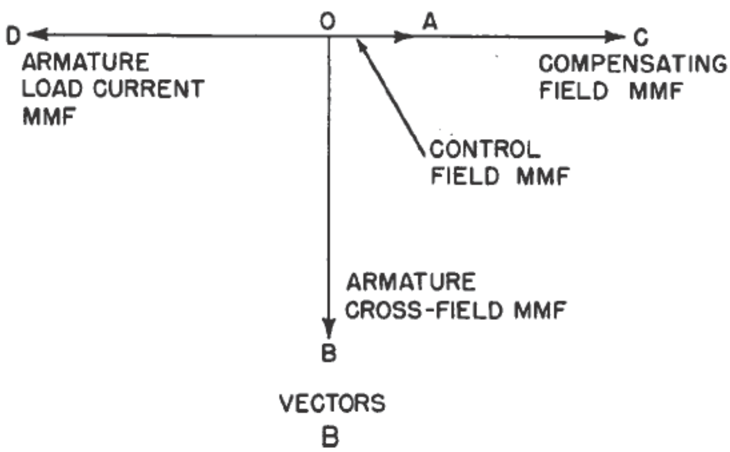


Figure 17–20.—Amplidyne generator.

2. In larger size amplidyne generators, a control field input power of 4 watts will develop an output voltage of 200 volts and an armature load current of 200 amperes, or 40,000 watts output.
3. Because of the high power amplification, any residual magnetism in the poles would greatly interfere with the proportion between the input and output. Residual magnetism is removed by means of a small a-c magneto generator which supplies a demagnetizing winding having the same axis as the control field winding.
4. The laminated armature and stator cores are worked at a low flux density in order to maintain a straight-line proportion between input and output.
5. Amplidyne generators are driven at relatively high speeds (1,800 rpm to 4,000 rpm) in order that they may be of small size and light weight.
6. The amplidyne generator has a quick response to changes in control field current. The lapse in time between a change in the magnitude of the control field current and the load output response is about 0.1 second.

Control Amplifier

The output of the CONTROL TRANSFORMER is fed to the input of the CONTROL AMPLIFIER (fig. 17–12). The control amplifier utilizes vacuum tubes in the proper circuit arrangement to amplify, or build up, the strength of the input signal so that it will be sufficient to energize the control field windings of the amplidyne generator. The input of the control amplifier is an a-c voltage, but the output is direct current that varies in polarity and magnitude with the relative polarity and magnitude of the input signal.

The detailed circuit operation of the control amplifier is not included in this text. Amplifiers are treated in *Basic Electronics,* NavPers 10087, and the details of control amplifiers will be treated as needed in the rating texts.

Antihunt Considerations

One problem in the application of the servo system is that of hunting. Hunting refers to the tendency of a mechanical system to oscillate about a normal position. Thus, in figure 17–21 the

steel ball, if depressed from its normal position and suddenly released, oscillates vertically because of its inertia and the forces exerted by the springs. In time, the oscillation is damped out by the frictional losses in the oscillating system.

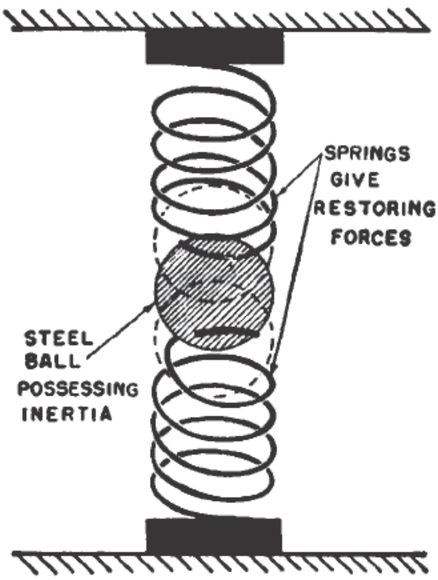


Figure 17–21.—Mechanical demonstration of hunting.

Although other factors are involved, a somewhat similar effect is observed in radar fire control, and other equipments employing servo-drive. Because the connection between the handwheel or other device controlling the position of the equipment (antenna, gun, etc., to be moved) and the equipment itself is somewhat elastic (because of the action of the electrical and magnetic circuits involved), the inertia of the equipment causes it to overtravel its required position. An error voltage is developed in the servo system in the opposite direction and the equipment reverses. Successive overtravels by the equipment would be less and less, and the mechanical oscillation would die out except for one factor: there may be a time lag in the servo system, which causes reinforced oscillations. In such a case, the equipment would continue to oscillate or HUNT about its normal position. This action would cause harmful mechanical vibration of the entire rotating system.

In order to eliminate hunting, an ANTIHUNT device or circuit is introduced. This device commonly consists of arrangements to

slow up the drive motor as the equipment to be moved approaches its final position. If the drive power is reduced soon enough, the inertia of the moving part causes it to coast into its final position without any overtravel.

The antihunt circuit commonly connects the servo or follow-up motor to the control amplifier. A feedback signal is supplied by means of this circuit to the control amplifier where the hunting problem is handled electrically.

Servomotors

The output drive motor in a servo system should have the required power, be easily reversible, and be capable of speed control over a fairly wide range. The d-c motor has characteristics that make its use advantageous. However, under certain circumstances the a-c motor has distinct advantages.

THE D-C SERVOMOTOR.—The d-c servomotor used with the amplidyne generator must have a constant magnetic field if its output is to be proportional to the voltage applied across its armature. This voltage is supplied by the controlled output of the amplidyne generator. A constant field can be produced by a field winding supplied by a constant-voltage d-c source, or supplied by means of a suitable rectifier and an a-c source. A simpler method is to use permanent magnets that are capable of establishing the necessary flux. These are restricted to small-size, low-horsepower motors.

A disadvantage in using permanent magnets is that they may become demagnetized by armature reaction. However, this condition may be counteracted by the use of a compensating field winding. This winding is connected in series with the armature and so placed around the pole faces that the armature reaction effect is completely eliminated.

THE A-C SERVOMOTOR.—As has been stated, the output drive motor in a servo system should be easily reversible and should be capable of speed control over a fairly wide range. Ordinarily, an a-c motor cannot fulfill the requirements of a servo drive motor as completely as a d-c motor because the range of speed control is limited to a greater degree. However, the use of an a-c motor may provide a much simpler drive system, especially where a-c power is the only source available and where some sacrifice in range of speed control can be made.

An a-c motor that can be adapted for servo-system use is the single-phase induction motor. This motor contains 2-phase stator windings with starting and running windings displaced 90° and containing currents that are displaced in time phase to produce a magnetic revolving field to start the motor. The rotor may be either a WOUND rotor or a CAGE rotor. The latter type is the most common. It consists of heavy conducting bars embedded in the armature slots and connected at the ends by conducting rings. The revolving magnetic field cuts the rotor windings at start and induces voltages in them. The resulting rotor currents react with the field to start the motor in the direction of the revolving field. When the motor comes up to speed, the slip current in the rotor constitutes one of two out-of-phase magnetomotive forces, the other being the running winding current. These two magnetomotive forces cooperate to maintain the revolving field about the stator when the starting winding is disconnected by a centrifugal starting switch.

The starting winding is frequently designed to operate continuously with a capacitor to increase the phase displacement between the currents at start and to improve the motor power factor when the motor comes up to speed. When this arrangement is used it is called a CAPACITOR MOTOR. In order to reduce the size of the phase-splitting capacitor required for induction motor applications (and to produce essentially the same result), a high voltage capacitor of smaller capacitance and a small autotransformer are used. The operation of the capacitor motor and the method of employing the capacitor are explained in chapter 15.

In order to give the necessary range of speed control and higher starting torque for certain applications, it is possible to modify the two-phase motor in other ways. These include increasing the resistance of the rotor bars and end rings and the use of a tapped autotransformer, one arrangement of which is shown in figure 17–22. In this circuit the phase-splitting capacitor and autotransformer are placed in parallel with one coil, and the combination is placed in series with the second coil. The current through coil 1 is the vector sum of the lagging current through coil 2 and the leading current through the primary of the autotransformer. Because the current through the capacitor is leading the current through coil 2, the total current through coil 1 leads that through

coil 2. The capacitor is chosen to give approximately a 90° phase shift between the currents in coils 1 and 2. Thus, the desired rotating magnetic field is produced. However, the efficiency of this type of motor is relatively poor because of the high resistance rotor.

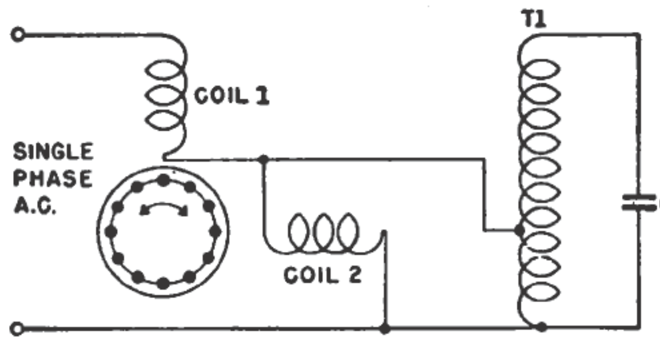


Figure 17–22.—Single-phase capacitor motor with series stator coil connections.

The DIRECTION of rotation of the capacitor motor is reversed either by reversing the connections to one stator coil or by shifting the capacitor from one coil to the other. The SPEED of the motor is varied over a limited range by changing the voltage applied to the motor. The voltage may be changed by placing a variable impedance in series with one or both phases. The effect of such an impedance is to lower the voltage, and hence the current, input to the windings. This action weakens the field and lowers the induced voltage in the rotor bars, hence reduces the motor torque and speed. The basic requirements of this system are: (1) a means of using an error signal to vary the impedance in series with the motor in order to control the speed, and (2) a means of comparing the phase of the error signal with a reference voltage in order to control the direction of rotation of the motor.

The block diagram of a servo system in which a capacitor motor is used to provide the output power is shown in figure 17–23. A synchro control transformer is used to provide an error signal that is proportional to the difference between the actual antenna position and the desired position, represented by the position of the handwheel. The rotor of the control transformer is geared to the load so that rotation of the load turns the rotor toward the position in which the error voltage is zero. The antenna is rotated by turning the handwheel connected to the rotor of the

synchro transmitter. The turning of the rotor shifts the position of the stator field of the control transformer and thus causes an error voltage to be induced in its rotor. The error voltage is fed to the control amplifier where it is amplified and used to lower the impedance in series with the a-c motor in order to start the motor, and to control (within limits) the speed of rotation. The

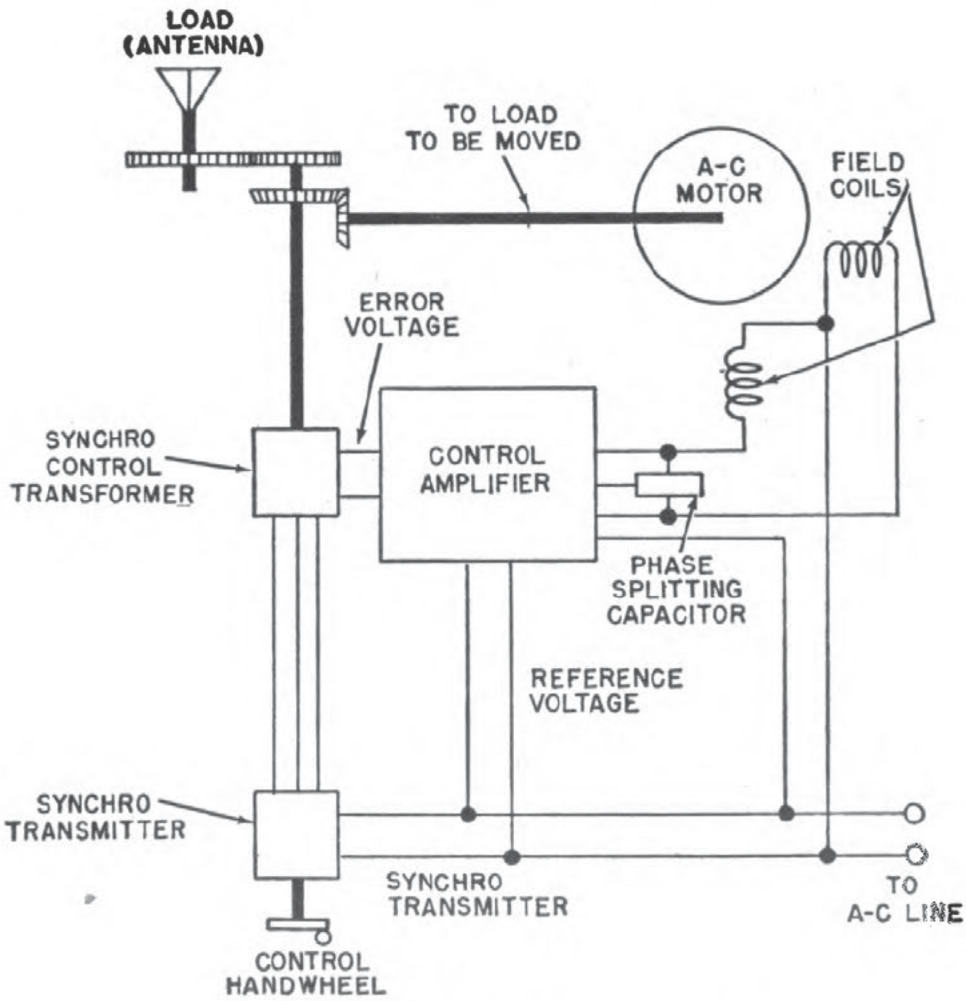


Figure 17–23.—Servo system using capacitor motor.

error voltage is simultaneously fed to another section of the amplifier where its phase is compared with the phase of a reference voltage in order to control the direction of rotation. The output of this section of the amplifier is fed to a relay which selects the stator winding with which the phase-splitting capacitor is to be connected in series.

Summary of Indicator and Servo System Operation

The components that make up a simple remote INDICATOR SYSTEM have been described in the preceding paragraphs. Likewise, the components that make up a simple SERVO SYSTEM have been described. Figure 17–24 shows how the various components of a combined indicator and servo system may be connected. This example employs an amplidyne generator and d-c servomotor. Although it has been discussed in the text, the differential synchro is not included in figure 17–24. The control amplifier is shown in block form, and is discussed only in general terms in the text. Amplifiers are treated in detail in *Basic Electronics,* and a study of their basic principles should precede the study of the control amplifier. Control amplifiers will be treated as needed in the rating texts.

The antenna (or other load that is to be rotated) is assumed to be in the position of correspondence (the desired position) because the transmitter rotors are all in the null position—that is, lined up with their respective $S2$-stator coils. No signal is being produced in the synchro transmitters, no input is fed to the control amplifier, and the servomotor is not energized. The arrows on the signal circuits indicate the direction in which the signals are transmitted when they exist in the circuits. The power circuits are drawn with heavy lines, and the gears and shafting are arranged to show the mechanical linkage, which in a practical system may be entirely different in arrangement.

A practical system may be more complex than the one shown. However, this figure illustrates some of the basic functions that must be performed by a simple servo and indicator system.

Assume that it is desired to turn the antenna 60° in a certain direction. The following sequence of events takes place:

1. The hand crank is turned through a 60° angle in the desired direction.
2. The rotor of $TR2$ is turned through a 60° angle because it is mechanically coupled to the hand crank. The $TR2$ field axis is shifted 60° and comes into alignment with $S3$. Signal currents in the CT stator windings shift the field of the CT through a 60° angle into alignment with $S3$.
3. The rotor of the control transformer is no longer 90° out of phase with the field. A signal (error) voltage is induced

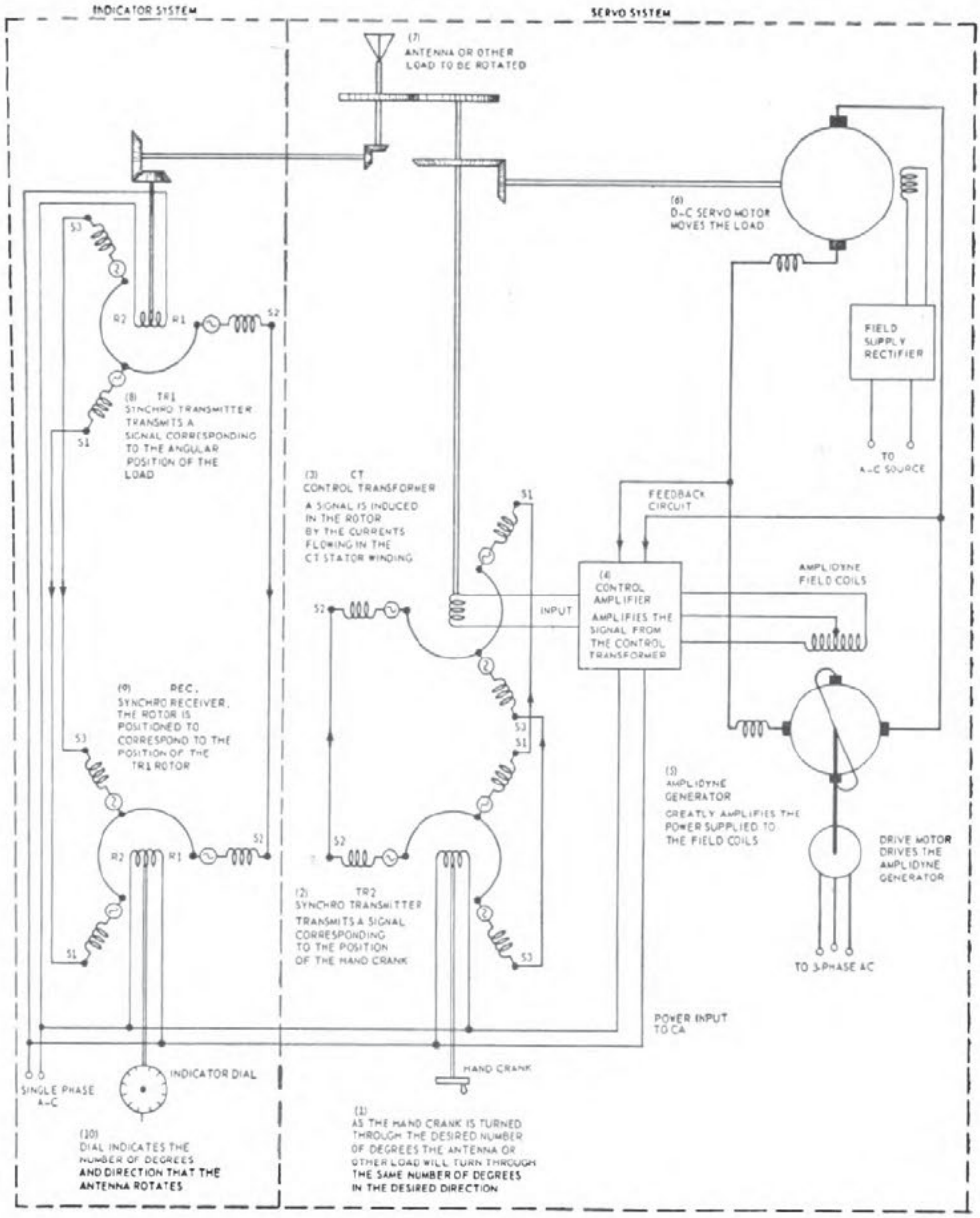


Figure 17–24.—Indicator and servo system.

in the rotor windings. This error voltage is fed to the input of the control amplifier.

4. The control amplifier amplifies, or increases the strength of, the signal from the control transformer. This is accomplished by the use of electron tubes and the proper circuitry supplied by power from the a-c line. The feedback circuit is a part of the antihunt system used to prevent the servomotor from overshooting the mark or driving the antenna

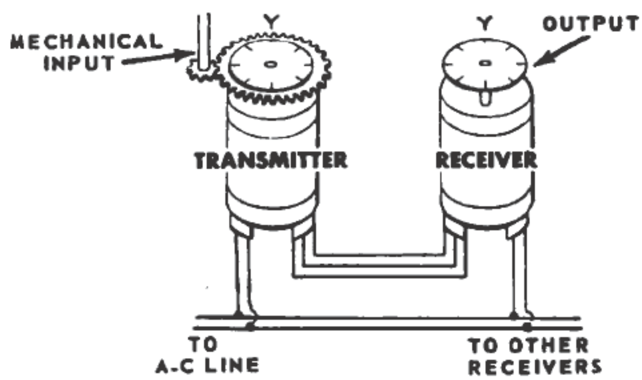
too far. The direct-current output from the control amplifier is of sufficient amplitude to energize the field coils of the amplidyne generator.

5. The amplidyne generator is so constructed that a relatively small current flowing in its field coils will produce a large power output. The extra power is supplied by the amplidyne drive motor. The output polarity of the amplidyne generator is determined by the direction of the current through the field coils, and this in turn is determined initially by the direction in which the handwheel is turned. Power from the amplidyne is fed to the d-c servomotor.
6. The d-c servomotor develops the torque that, by means of the proper gearing, positions the antenna or other load to be rotated. The field of the servomotor is energized by a separate field-supply rectifier that changes the alternating current from the line to direct current.
7. As the antenna moves toward the 60° position, the shaft connected to the control transformer rotor, and therefore the *CT* rotor, moves 60° bringing it into the null position, or 90° with respect to the *CT* field. At this position, no voltage is induced in the rotor, no voltage is applied to the input of the control amplifier, and therefore the output of the amplidyne generator falls to zero. Consequently the voltage applied to the servomotor armature is zero and the antenna stops rotating at the 60° position.
8. In the meantime, the rotor of synchro transmitter, *TR*1, which is geared to the antenna, has been moving through the same angle as the antenna. Signal currents are therefore induced in its stator windings. These signal currents also flow through the stator windings of the synchro receiver.
9. The signal currents flowing through the stator windings of the synchro receiver cause the rotor to move into alignment at the 60° position, as indicated on the indicator dial.

It should be emphasized that the needs of a system might dictate an entirely different arrangement of the components. A differential generator might be needed, other devices than the amplidyne might be used, and the servomotor might be operated on alternating current. Hydraulic methods of position control are also widely used, as are combinations of electric and hydraulic systems.

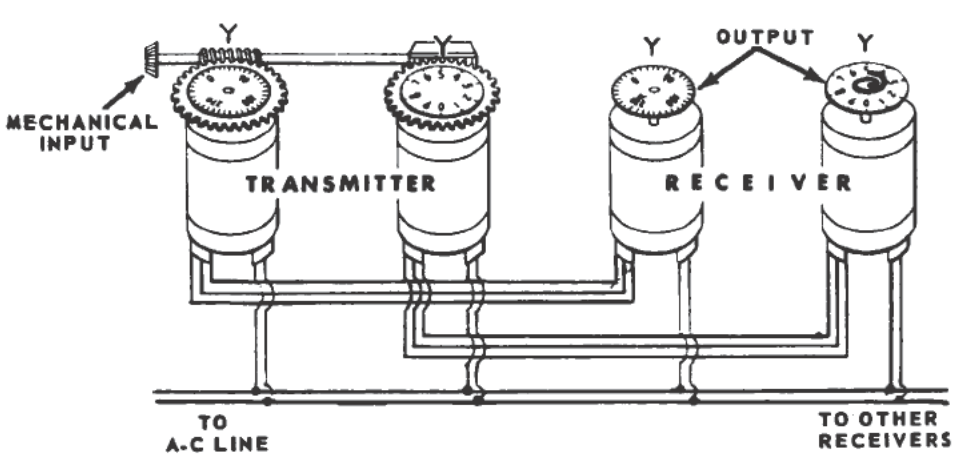
Double-Speed Transmission

Thus far, only SINGLE-SPEED TRANSMISSION has been considered. A single-speed transmission system consists of a single synchro transmitting a signal to single synchro receivers (one or more), as shown in figure 17–25, A. The single transmitter is known as a SINGLE-SPEED TRANSMITTER, and each receiver as a SINGLE-SPEED RECEIVER. In many cases control transformers, differentials, amplifiers, servos, and other devices are used.



A

SINGLE-SPEED SYNCHRO SYSTEM



B

DOUBLE-SPEED SYNCHRO SYSTEM

Figure 17–25.—Single- and double-speed synchro systems.

Because of the low torque developed by the synchro receiver as it approaches the null position, bearing friction may cause slight errors (static errors) in the positioning of the receiver rotor when a single-speed transmission system is used. For certain types of data transmission this error can be tolerated.

DOUBLE-SPEED INDICATOR SYSTEM.—When great accuracy is required, a DOUBLE-SPEED TRANSMISSION system is used. This system consists of a pair of transmitters geared together and transmitting to one or more pairs of synchro receivers, as shown in figure 17–25, B. The "coarse" transmitter is usually worm driven and rotates once for 36 revolutions of the "fine" transmitter. The fine transmitter is geared to the mechanical input through a 1-to-1 gear ratio.

The two transmitters thus geared together are known as a DOUBLE-SPEED TRANSMITTER, and the two companion receivers are known as a DOUBLE-SPEED RECEIVER. The dial of the coarse transmitter has graduations from 0° to 360° so that one revolution of the dial equals 360° of bearing. The dial of the fine transmitter has graduations from 0° to 10° so that one revolution of the dial equals 10° of bearing. The gearing between the two generators is such that while the coarse dial is revolving once, the fine dial must turn 36 times. The two dials and the gearing are shown in figure 17–26.

Thus, if 65° of bearing is cranked into the transmitter, the coarse dial having 360 divisions moves through $\frac{60}{360} \times 36$, or 6 divisions (60°), and a fraction that is hard to estimate on the dial. The fine dial having 10 division moves through $\frac{65}{10}$, or 6½

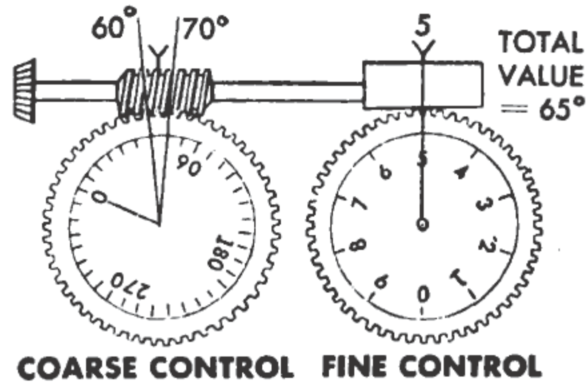


Figure 17–26.—Coarse and fine transmitter dials.

revolutions, stopping at the 5° mark on the dial. Thus, 60° is indicated on the coarse dial and 5° is indicated on the fine dial, making a total of 65°. The information is presented on the two-speed receiver in exactly the same way, and is read as 65°.

In a single-speed follow-up system any error in the synchro transmitter would be transmitted to the control transformer and the load would be out of correspondence by the amount of the error. For example, an error of 36 minutes in the synchro transmitter would mean that the control transformer and load would be about 36 minutes out of correspondence. If the synchro transmitter is made to revolve 36 times for each revolution of the load, each revolution of the transmitter represents only $\frac{360}{36}=10°$ of movement of the load. Therefore, the synchro's error of 36 minutes now represents only $\frac{36}{36}=1$ minute of error in the load. The 36-to-1 gearing reduces the electrical error by 36 to 1. This arrangement is called FINE CONTROL.

Fine control by itself is not adequate because any one position of the synchro transmitter can correspond to 36 different positions of the load. The load could synchronize at any one of the 36 different positions. Thus, misalignment could occur if the originating signal and the load were more than 10° out of synchronism. The coarse control has the full 36 minutes of error but it has only one point of correspondence. Hence, the fine control synchro transmitter is geared 36 to 1 to the input (fig. 17–26), and the fine-control control transformer to which it is connected (not shown in the figure) furnishes one input to the load device. There is also a coarse control synchro transmitter geared 1 to 1 to the input (fig. 17–26), and a coarse-control control transformer connected to this transmitter. The coarse control furnishes a second input to the load device.

This system is described more in detail in the advanced rating texts.

QUIZ

1. Synchros transmit information in what basic form?
2. How does the synchro receiver differ physically from the synchro transmitter?
3. When the synchro transmitter and receiver rotors are in correspondence, why is there no current in their corresponding stator coils?

4. What type of synchro has been adopted as the Navy standard synchro?
5. How is the rotor coil of a BuOrd synchro energized?
6. What is the purpose of the damper used on a synchro receiver?
7. What is the relation between the secondary effective voltage of a synchro winding and the angle formed between the axes of the primary and this winding?
8. In the electrical zero position of a synchro, figure 17–6, A, what is the value of the voltage between terminals $S1$ and $S3$?
9. What is the disadvantage of operating a synchro receiver with its rotor winding on open circuit?
10. Why are the servo system and/or the amplidyne sometimes used in conjunction with the synchro transmission system?
11. How does the differential synchro rotor differ from the receiver synchro rotor?
12. Compare the input and output (electrical and mechanical) of the differential synchro transmitter and receiver.
13. If in figure 17–9, B, the differential transmitter rotor is held stationary, through what angle will the receiver rotor move when the transmitter rotor is moved through 45°?
14. What adjustments are necessary to reverse the differential action of a differential transmitter?
15. What adjustments are necessary to reverse the differential action of a differential receiver?
16. What is the function of a synchro control transformer?
17. For zero error, what is the position of the axis of the synchro control transformer field with respect to the axis of the rotor winding?
18. What is the electrical-zero position for a control transformer?
19. Upon what does the magnitude of the control-transformer output voltage depend?
20. What is the main difference between the BuOrd and the BuShips synchros with respect to the way the windings are arranged?
21. How are the electrical signals fed to the rotor and the stator of a bearing-mounted synchro?
22. Bearing-mounted synchros are usually used for what purpose?
23. What is the advantage of a closed-loop control system over the open-loop system?
24. What two functions are performed by the servo and indicator system shown in figure 17–16?
25. In the amplidyne servo system shown in figure 17–16, the control transformer error voltage is proportional to the difference between what two values?

26. When the antenna position (fig. 17–16) corresponds to the hand-wheel position, what is the error voltage fed to the control amplifier?
27. How is the power output of an amplidyne generator controlled?
28. As commonly used, what three units make up the amplidyne power drive?
29. What relative amount of power is required by the control field of an amplidyne compared to that of a separately excited d-c generator of the same rating?
30. What establishes the quadrature, or cross-magnetizing force in an ordinary d-c generator?
31. What is the purpose of the compensating winding used in the amplidyne generator?
32. Why is the compensating winding of an amplidyne generator connected in series with the load?
33. What is the purpose of the killer winding in an amplidyne generator?
34. Why are the laminated armature and stator cores of an amplidyne generator worked at low flux density?
35. What is the function of the control amplifier as used in figure 17–12?
36. What is the name of a device or circuit (in a servo-drive unit) that is commonly used to prevent an equipment from oscillating about the desired stop position?
37. What are three desirable features of the output drive motor in a servo system?
38. Why does the d-c servo motor have a constant field?
39. What type of a-c motor can be adapted for servo-system use?
40. In a capicitor-type servo motor how are the following changed:
    a. Direction of rotation?
    b. Speed?
41. What are the two control functions of the control amplifier in figure 17–23?
42. What source supplies most of the power delivered to the servo motor by the amplidyne generator in figure 17–24?
43. What is the advantage of a double-speed transmission system compared to a single-speed system?
44. In the coarse and fine transmitter dials shown in figure 17–26, through how many degrees is the coarse transmitter dial moved when the fine transmitter dial is moved through one revolution?
45. In the two-speed transmission system, the 36-to-1 gearing reduces the electrical error by what ratio?

APPENDIX I

ANSWERS TO QUIZZES

CHAPTER 1

FUNDAMENTAL CONCEPTS OF ELECTRICITY

1. The molecule.
2. The atom.
3. The nucleus.
4. They are equal.
5. One proton and one electron.
6. 96 protons and 96 electrons.
7. Weight $=9\times10^{-28}$ gram, charge $=1.6\times10^{-19}$ coulomb.
8. About $\frac{1}{1,845}$ of the weight of the proton.
9. Unequal.
10. A small particle of matter or group of such particles that have a net positive or negative charge.
11. The movement of free electrons in the wire.
12. A good conductor has a large number of free electrons; a good insulator has very few free electrons.
13. No.
14. More or less than the normal number.
15. (a) Negatively, (b) positively.
16. (a) Positively, (b) negatively.
17. It is one-fourth as much.
18. The electric, electrostatic, or dielectric field.
19. By the direction in which a small positive charge would move along the line.
20. They are imaginary lines along which a real force acts.
21. 2 amperes.
22. The coulomb.
23. 4 amperes.
24. Doubled.
25. Decreased.
26. Decreased.

CHAPTER 2

BATTERIES

1. The elements of a secondary cell can be restored to their original condition by charging from an electric supply circuit; the elements of a primary cell cannot.
2. Electrode size (surface area below liquid), distance between electrodes, and the resistance of the solution.
3. (a) Mixture, (b) compound, (c) compound.
4. Zinc.
5. The formation of hydrogen bubbles on the surface of the positive carbon electrode while current is flowing.
6. Increases the effective internal resistance, and reduces the terminal voltage and output current.
7. A depolarizing agent.
8. Wasting away of zinc on open circuit due to impurities in zinc.
9. By treating the zinc with mercury to remove impurities.
10. Sal ammoniac (NH_4Cl).
11. Positive electrode, carbon; negative electrode, zinc.
12. 15 volts.
13. Batteries.
14. 350 amperes.
15. Reserve cell.
16. Lead-acid cell and nickel-iron-alkaline cell.
17. Positive plate active material consists of lead peroxide and the negative plate consists of pure sponge lead.
18. It decreases.
19. No.
20. As submarine batteries.
21. To allow the gas to escape that forms in the cell during charging.
22. Electrolyte.
23. Less.
24. Twice the length of time.
25. Added.
26. To mix electrolyte and thus ensure a true reading.
27. Pour acid slowly into water while stirring.
28. Capacity.
29. 10, ampere hour.

30. 18.5 amperes.
31. 80 percent.
32. 50 points.
33. 1.180.
34. A long, low-rate charge given to new batteries before being placed in service to convert the plates from the uncharged condition in which they are shipped to the charged condition.
35. A routine charge given during the ordinary cycle of operation to restore the battery to its charged condition.
36. All sulfate is driven from the plates and that all cells are restored to a maximum specific gravity.
37. Voltage.
38. Name plate.
39. Too high a charging rate.

CHAPTER 3

THE SIMPLE ELECTRIC CIRCUIT

1. Volt, ampere, ohm, and watt.
2. $I=\frac{E}{R}$.
3. 1.75 ampere.
4. A decrease of 0.5 ampere.
5. An increase of 15 volts.
6. (a) $E=IR$, (b) $R=\frac{E}{I}$.
7. Power is the rate of doing work $\left(\frac{w}{t}\right)$.
8. (a) $P=EI$, (b) $P=\frac{E^2}{R}$, (c) $P=I^2R$.
9. 180 watts.
10. 80 watts.
11. An increase of 25 watts.
12. (a) $W=EIt$, where E is volts, I is amperes, and t is hours. (b) $W=\frac{E^2}{R}t$, (c) $W=I^2Rt$, (d) $W=QE$.
13. 172.5 kw-hrs.
14. 10 amperes.
15. 12 ohms.
16. 240 watt hours.

17. 112 volts.
18. 0.5 ampere.
19. 22 kw.
20. The filament resistance increased and prevented the current from varying directly with the voltage.
21. (a) $I=\frac{E}{R}$, $R=\frac{E}{I}$

(b) $I=\frac{P}{E}$, $E=\frac{P}{I}$

(c) $I=\sqrt{\frac{P}{R}}$, $R=\frac{P}{I^2}$

(d) $E=\sqrt{PR}$, $R=\frac{E^2}{P}$.

CHAPTER 4

DIRECT-CURRENT SERIES AND PARALLEL CIRCUITS

1. A series circuit has only one possible path for current flow; a parallel circuit has two or more paths supplied by a common voltage source.
2. The algebraic sum of the voltages around the circuit is equal to zero.
3. The sign preceding the source emf is positive if the first sign encountered in the trace through the source is positive. The sign is negative if the first sign encountered is negative. The sign preceding the voltage drop across a load is negative if the direction of the trace is the same as that of the electron flow through the load. The sign is positive if the direction of the trace is opposite to that of the electron flow through the load.
4. The magnitude of the current will not be affected; however, the sign preceding the current will be negative.
5. $E_1=2.5$ volts, $E_2=20$ volts, and $E_3=7.5$ volts.
6. Power in $R1=1.25$ watts, in $R2=10$ watts, and in $R3=3.75$ watts.
7. 0.5 ampere.
8. Zero.
9. 5.76 amperes.
10. 0.25 ampere.
11. 80 volts.
12. Zero.
13. No.
14. (a) 580 ohms; (b) 13.05 watts.

15. They are reduced.
16. In the internal resistance of the source.
17. When the load resistance is equal to the internal resistance of the source.
18. When the load resistance is much larger than the resistance of the source.
19. Decreased.
20. 7.5 amperes.
21. 225 watts.
22. 1 ohm.
23. 4 ohms.
24. At any junction of conductors the algebraic sum of the currents is zero.
25. 10.9 volts.
26. 8.25 watts.
27. To keep line losses to a minimum by the use of high voltage and low current.
28. Parallel.

CHAPTER 5

DIRECT-CURRENT COMPOUND AND BRIDGE CIRCUITS

1. (a) 4 amperes, (b) 10 amperes.
2. (a) 15 amperes, (b) 50 amperes, (c) 20 amperes.
3. (a) (1) 1 ampere, (2) 1 ampere, (3) 0.75 ampere, (4) 0.25 ampere. (b) (1) 4 amperes, (2) 1.6 amperes, (3) 2.4 amperes.
4. (a) 25 amperes, (b) 2 ohms, (c) (1) 10 amperes, (2) 200 watts.
5. (a) 5 milliamperes, (b) 50 volts.
6. (a) 5 K-ohms, (b) 3.33 K-ohms, (c) 4 K-ohms, (d) 2.22 K-ohms.
7. (a) (1) 7 milliamperes, (2) 11 milliamperes, (3) 17 milliamperes. (b) (1) 70 volts, (2) 180 volts, (3) 350 volts.
8. To reduce the voltage, current ,and power delivered to a load without changing the ratio of voltage to current at the input terminals (input resistance).
9. Pads.
10. (a) They are equal. (b) No. (c) (1) 160 ohms, (2) 50 ohms.
11. (a) They are equal. (b) (1) Each 20 ohms, (2) 80 ohms.
12. (a) (1) 3 amperes, (2) 2 amperes.
    (b) (1) 7 amperes, (2) 1 ampere.

13. (a) They are equal, (b) 50 ohms.
14. (a) 400 ohms, (b) 200 ohms.
15. $I_2=21$ amperes.
16. (a) 3,250 watts, (b) 1,570 watts.
17. (a) 107.5 volts, (b) 122.5 volts.
18. (a) 4 amperes, (b) 1 ampere, (c) 9 amperes, (d) 5 amperes, (e) 14 amperes, (f) 0 ampere, (g) 14 amperes, (h) 117.2 volts, (i) 116 volts, (j) 115.6 volts, (k) 115.3 volts, (l) 111.4 volts.

CHAPTER 6

ELECTRICAL CHARACTERISTICS OF CONDUCTORS

1. A round conductor 1 foot in length and 1 mil in diameter.
2. 0.001 inch.
3. 875,000 square mils.
4. 1 circular mil.
5. 10,000 circular mils.
6. (a) 1,125,000 sq. mils.
   (b) 1,432,000 circular mils.
   (c) 143.
7. (a) 10.37 ohms per circular mil-foot.
   (b) 17.02 ohms per circular mil-foot.
   (c) 660 ohms per circular mil-foot.
8. $R=\rho\frac{L}{A}$.
9. 2.52 ohms.
10. (a) 5,000 circular mils.
    (b) 1,000 circular mils.
    (c) 100,000 circular mils.
11. (a) 1,000 ohms.
    (b) 20 ohms.
    (c) 0.1 ohm.
12. 5.0 ohms.
13. (1) Allowable power loss (I^2R).
    (2) Permissible voltage drop (IR drop).
    (3) Current-carrying ability of the line.
14. 0.00427 ohm.
15. 56.2 ohms.
16. 66° C.

17. 18.6 ohms.
18. 38° (approx.).
19. Insulation resistance.
20. Dielectric strength.
21. A thin coating of tin is used, or a winding of cotton threads between the copper and the rubber.
22. High voltage generator coils and leads, and also transformer leads.
23. High voltage power cables.
24. Coils of meters, relays, small transformers, and electromagnets.
25. Subway-type electric cables or wires continually subjected to water.
26. Western Union splice.
27. Fixture joint.
28. To protect from corrosion, to provide increased area of contact, and to increase the mechanical rigidity of the joint.
29. Rubber tape provides a great degree of electrical insulation. Friction tape provides a protective covering, and a minor degree of electrical insulation.
30. Plastic electrical type.

CHAPTER 7

MAGNETISM AND MAGNETIC CIRCUITS

1. Magnetism—that is, the power to attract such substances as iron, steel, nickel, or cobalt.
2. The magnetic poles.
3. Imaginary lines that form closed loops along which magnetic forces act.
4. The north pole.
5. A complete path through which magnetic lines of force may be established under the influence of a magnetizing force.
6. The permanent magnetic is composed of hardened steel or some alloy such as alnico; the electromagnetic has a soft-iron core.
7. (1) By inserting the bar in a coil of insulated wire and passing a heavy direct current through the coil, (2) by stroking it with a bar magnet.
8. Residual magnetism.
9. Because of the random arrangement of the molecules, the magnetism of each of the molecules is neutralized by that of adjacent molecules.
10. A group of perhaps 10^{14} to 10^{15} atoms that have their poles orientated in the same direction.

11. Because essentially all of the domains have been orientated in the direction of the external magnetizing force.
12. The lines leave the north pole, passing through the surrounding space, enter the south pole, and return through the bar magnet to the north pole.
13. $B=\frac{\Phi}{A}$.
14. (a) Repel, (b) attract.
15. One sixteenth as great.
16. 4 times as great.
17. South.
18. Clockwise.
19. (a) Repel, (b) attract.
20. The coil is grasped in the left hand, with the fingers pointing in the direction of the current flow across the face of the coil; the thumb extended longitudinally along the coil will point in the direction of the flux and the north pole.
21. By increasing the number of turns, increasing the current through the coil, or using a better (more permeable) core material.
22. 100 ampere-turns.
23. The instrument is shielded by placing it in a soft iron core that diverts the flux around the instrument.
24. It will be doubled.
25. The maxwell is one magnetic line of force.
26. 1,257 gilberts.
27. 125.7 gilberts per centimeter of length.
28. (a) Directly, (b) inversely, (c) inversely.
29. 230.7 ampere turns.
30. 1,500.
31. The ability of a magnetic substance to retain its magnetism after the magnetizing force has been removed.
32. Molecular friction.
33. The hysteresis energy loss per cycle of operation.
34. Increase.
35. It would lower the flux and lower the force acting on the bell armature.
36. A relay.
37. The solenoid is strengthened, the carbon pile resistance is increased, and the output voltages is prevented from rising with the speed.

CHAPTER 8

INDUCTANCE AND CAPACITANCE

1. Inductance is that property of an electric circuit that opposes any change of current in the circuit and enables energy to be stored in a magnetic field.
2. Capacitance is that property of an electric circuit that opposes any change in voltage in the circuit and enables energy to be stored in an electrostatic field.
3. Zero.
4. The thumb, index finger, and second finger of the left hand are extended at right angles to each other. The thumb points in the direction of motion of the conductor with respect to the field, the index finger points in the direction of the field, and the second finger points in the direction of the induced emf.
5. (a) 50 millihenries, (b) 50,000 microhenries.
6. The induced emf in any circuit is always in such a direction as to oppose the effect that produces it.
7. Resistance.
8. 7.1 millihenries.
9. (a) Maximum, (b) zero, (c) 2 seconds.
10. Flux, turns.
11. $K=0.3$.
12. $M=K\sqrt{L_1L_2}$.
13. 30 henries.
14. 5 henries.
15. 0.002 farad.
16. (a) 20, (b) 2,000.
17. (1) Area of plates, (2) distance between the plates, and (3) dielectric constant of the material between the plates.
18. 1,000 micromicrofarads.
19. (a) 2 seconds, (b) zero, (c) maximum, (d) 400 micromicrofarads.
20. 66.7 micromicrofarads.
21. 300 micromicrofarads.
22. 50.
23. Type, thickness.
24. (1) Paper, (2) mica, (3) oil, (4) electrolytic.
25. The thinness of the dielectric and its insulation strength permit closer spacing of the plates.

CHAPTER 9

BASIC ELECTRICAL INDICATING INSTRUMENTS

1. A wattmeter.
2. A watt-hour meter.
3. The D'Arsonval movement.
4. (1) Supports the moving coil, (2) provides a conducting path for the coil current, and (3) provides the restoring force against which the driving force is balanced to obtain a measurement of current strength.
5. $F=\dfrac{8.85\ BLI}{10^8}$.
6. (a) Down, (b) up.
7. To concentrate the magnetic flux in the space through which the coil and bobbin move.
8. By the motion of the aluminum bobbin on which the coil is wound. The aluminum frame cuts the field and the induced currents oppose the motion, thereby bringing the bobbin quickly to rest.
9. (1) Increased flux density, (2) lighter weight assembly, and (3) jewel bearings.
10. It extends the range of the meter.
11. The resistance is substantially unaffected by the temperature increase.
12. 50 millivolts.
13. 0.00002 ohm.
14. 0.005 ohm.
15. As a shunt-connected meter, it would short-circuit the source and burn up the meter. It is connected in series with the load in order to indicate the magnitude of the load current.
16. Much larger.
17. 150 k-ohms.
18. 5 megohms.
19. 14,950 ohms.
20. (a) High, (b) so as not to disturb the current and voltage of the circuit when the meter is connected.
21. 20,000 ohms per volt.
22. 100,000 ohms per volt.
23. Less.
24. In series with the source and load.
25. In shunt with the source and load.

26. Because of low current, the deflection will be less, thereby indicating a high value of resistance.
27. To compensate for changes in battery voltage.
28. The megger.
29. A hand-driven d-c generator.
30. None.
31. (a) 500 volts, (b) an increase in voltage is necessary.
32. To magnetize the iron vanes.
33. Less.
34. A thermocouple-type r-f ammeter.

CHAPTER 10

DIRECT-CURRENT GENERATORS

1. Mechanical energy is converted into electrical energy.
2. Mechanical support and it serves as a portion of the magnet circuit to provide the necessary flux across the air gap.
3. The function is to establish the generator field flux.
4. Because of the small residual flux.
5. To prevent arc-over between the commutator segments.
6. The strength of the magnetic field and the speed of rotation of the conductor.
7. By the use of more coils and commutator bars.
8. Because in the closed circuit formed by the two paths, the induced voltages are equal and in opposition.
9. The distance between the two sides of the coil.
10. (1) Improved magnetic circuit, (2) use of preformed coils, (3) shorter end-connections.
11. Because the coil-side voltages do not reach their maximum values at the same time.
12. 52.8 volts.
13. They are numerically the same.
14. Two.
15. The current per brush is reduced.
16. 800 volts.
17. (a) I^2R, (b) eddy-current, (c) hysteresis.
18. The allowable current rating of the generator armature.
19. To reduce the eddy-current losses.
20. Lowers the loss to one fourth the original value.

21. Magnetic hysteresis loss.
22. It distorts the field in the direction of rotation.
23. By the use of high flux density in the pole tips, a compensating winding, and commutating poles.
24. The compensating winding is connected in series with the armature and is arranged so that the ampere-turns are equal in magnitude and opposite in direction to those of the armature.
25. Commutation.
26. It will be burned and pitted.
27. The brushes must be shifted with each change in load.
28. By means of commutating poles, the windings of which are connected in series with the armature, and by operating the interpole iron at flux densities well below saturation.
29. Twice as much.
30. The residual flux of the main poles.
31. Because of partial saturation of the steel portion of the magnetic circuit.
32. To permit the field current to be varied through a relatively wide range.
33. 300 ohms.
34. The armature *IR* drop is small compared with the generated voltage and the armature reaction voltage loss is small; also the shunt field current is reduced only slightly.
35. Armature *IR* drop, armature reaction voltage drop, and drop due to decreased field current.
36. The field flux and generated voltage increase with load.
37. (a) Full-load voltage is less than no-load voltage, (b) full-load voltage is equal to no-load voltage, (c) full-load voltage is greater than the no-load voltage.
38. 4.16 percent.
39. Flat-compound and shunt.
40. It is increased.
41. It is increased.
42. The lever weight immediately overcomes the solenoid and tilts the plates in a direction to reduce the field circuit resistance, thus increasing the field current, and checking the fall in terminal voltage.
43. *L*2 opposes *L*1 and the magnetic pull of *L*1 is partially neutralized.
44. To protect the generator.
45. To automatically disconnect the battery from the generator when the generator voltage is lower than that of the battery. This action prevents motoring the generator.

CHAPTER 11

DIRECT-CURRENT MOTORS

1. In parallel with the armature.
2. In series with the armature.
3. (a) It has more starting torque than a shunt motor.
   (b) It has less variation in speed than the series motor.
4. (1) Natural circulation, (2) self-ventilation, (3) separately ventilated.
5. (a) Below the right-hand conductor and above the left-hand conductor, and (b) above the right-hand conductor and below the left-hand conductor.
6. Field strength, active length of the conductor, and current flowing through the conductor.
7. 0.443 pounds.
8. Torque=16 pound-feet.
9. $T=K_t\Phi I_a$.
10. To reverse the current in the armature coils at the proper time to maintain the current flow in the same direction in all conductors under a given pole.
11. They are equal.
12. (a) 1,980,000 foot-pounds, (b) 550 foot-pounds per second.
13. 10 hp.
14. Speed and strength.
15. Backward.
16. (a) Backward, (b) forward.
17. On the no-load neutral plane.
18. By connecting the interpole field winding in series with the armature and working the interpole iron below saturation; the interpole flux is made to vary directly with the armature current and load.
19. No.
20. Commutation will be impaired and the motor speed-torque characteristics will be altered.
21. The speed falls slightly.
22. 5.88 percent.
23. The armature current varies almost directly with the load.
24. The torque doubles.
25. There is a relatively small decrease in field strength.
26. Because armature reaction weakens the field slightly, causing a decrease in generated voltage in the generator and increased speed in the motor.

27. Constant speed and variable torque.
28. (a) Shunt motor, (b) because of its constant-speed variable-torque characteristics.
29. The torque is proportional to I_a^2.
30. Variable speed and variable torque.
31. Traction equipment and hoists.
32. (a) 2,970 rpm, (b) 1,470 rpm.
33. Because the torque varies directly as the product of the armature current and field flux, and field flux is increased due to the series field.
34. Because the series field opposes the shunt field, and the series field becomes stronger as the armature current increases.
35. Without starting resistance, the low-resistance armature would draw too much current before the counter voltage could build up.
36. The size of the orifice in the dash pot.
37. Because the load delays acceleration of the armature, which delays the rise in counter emf. This action delays the rise in voltage on the operating coils of the accelerating contactors.
38. The starting resistance is removed from the armature circuit only after the speed has built up, and the motor current is reduced to a safe value irrespective of line-voltage variations.
39. The initial rise of flux cuts through the damper winding and induces a current in it. The resulting counter mmf diverts the flux away from this part of the magnetic circuit, causing it to cross an auxiliary air gap and lock the contactor open.
40. 80%.
41. (a) The speed increases, (b) the speed decreases.
42. The rheostatic losses are too large at low speeds, and the armature voltage varies with the load.
43. It eliminates the armature rheostat and its associated losses; it maintains a constant armature voltage with variable load and thus eliminates unstable speed characteristics.

CHAPTER 12

INTRODUCTION TO ALTERNATING-CURRENT ELECTRICITY

1. The principle of electromagnetic induction.
2. Three.
3. 60 cycles per second.
4. (a) 0.0167 second, (b) the period of the alternator.
5. 120 volts.

6. D-c generators called exciters.
7. Magnitude and direction.
8. 90°.
9. 108 volts.
10. 778 volts.
11. 7.07 amperes.
12. 14.14 amperes.
13. The form factor of the wave.
14. (a) Either the effective value or the positive maximum value as desired, (b) the time difference between the positive maximum values.
15. 1,414 volts.
16. (a) Because they are out of phase, (b) 163 volts, (c) 0.0021 second.
17. Zero.
18. Topographic vectors are arranged end to end; polar vectors extend from a common pole.
19. Because of the opposition developed by the inductive emf.
20. The counter emf.
21. Counter emf in volts to current in amperes.
22. (a) $X_L=2\pi fL$, (b) $X_L=\omega L$.
23. The var.
24. 600 watts.
25. (a) 5 ohms, (b) 53.1°.
26. 600 watts.
27. $I=2\pi fCE$.
28. 331 ohms.
29. Displacement current is the movement of bound electrons; conduction current is the movement of free electrons.
30. They are equal.
31. (a) 56.5 ohms, (b) 2.37 amperes, (c) 95 volts, (d) 95 volts, (e) 70.7 percent, (f) 318 volt-amperes, (g) 225 watts, (h) 225 vars.

CHAPTER 13

BASIC ALTERNATING-CURRENT CIRCUIT THEORY

Slide Rule Answers

1. (a) 50 ohms, (b) 500 ohms.
2. (a) 1 ampere, (b) 100 ohms, (c) 1 ampere, (d) 1.41 amperes, (e) 100 watts, (f) 100 vars, (g) 141.4 volt-amperes, (h) 70.7 ohms, (i) 0.707, (j) 45°.

3. (a) 6,000 ohms, (b) 60 ohms, (c) 600,000 ohms, (d) 89.4°.
4. (a) 25 ohms, (b) 25 ohms, (c) 10 amperes, (d) 10 amperes, (e) 19.8 amperes, (f) 0.61, (g) 2,500 volt-amperes, (h) 2,500 volt-amperes, (i) 3,010 watts, (j) 12.6 ohms, (k) 7.7 ohms, (l) 10 ohms.
5. (a) 125 ohms, (b) 2 amperes, (c) 2 amperes, (d) 2.83 amperes, (e) 500 watts, (f) 500 vars, (g) 707 volt-amperes, (h) 0.707, (i) 88.4 ohms, (j) 62.5 ohms, (k) 62.5 ohms, (l) 51 microfarads.
6. (a) 50 Ω, (b) 1 Ω, (c) 50 Ω, (d) 5 amperes, (e) 0.1 ampere, (f) 5 amperes, (g) 2,500 Ω, (h) 25 watts, (i) 1,250 vars, (j) 1,250 volt-amperes.
7. (a) 6 amperes, (b) 60 ohms, (c) 100 ohms, (d) 1.2 amperes, (e) 7 amperes, (f) 36.9°, (g) 5.9°, (h) 17.14 Ω, (i) 1.77 ohms, (j) 225 μf, (k) 840 volt-amperes, (l) 835 watts.
8. (a) 10 amperes, (b) 10 ohms, (c) 20 ohms, (d) 12.50 amperes, (e) 30°, (f) 10.83 amperes, (g) 6.25 amperes, (h) 17.52 ohms, (i) 17.52 ohms, (j) 14.3 amperes, (k) 22.3 amperes, (l) 21.2°, (m) 11.2 ohms, (n) 10.4 ohms, (o) 4.06 ohms, (p) 5,200 watts, (q) 5,570 volt-amperes.
9. (a) 7,500 watts, (b) 2,500 watts, (c) 75%, (d) 40 amperes, (e) 89.5%.
10. (a) 62.5%, (b) 0.625, (c) 15.6 amperes, (d) 12.5 amperes.
11. (1) Ohmic-resistance loss, (2) skin-effect loss, (3) eddy-current loss, (4) dielectric loss, (5) magnetic-hysteresis loss, (6) corona loss, and (7) radiation loss.
12. By inserting in the coil a laminated core of soft silicon steel.
13. Letters *A*, *B*, and *C*.
14. $\sqrt{3}$, or 1.73.
15. $\sqrt{3}$, or 1.73.
16. $P=3\times E_{\text{phase}}\times I_{\text{phase}}\times\cos\theta_{\text{phase}}$.
17. (a) 265 volts, (b) 250 amperes, (c) 199 kva, (d) 149 kw.
18. (a) 460 volts, (b) 530 amperes, (c) 414 kva, (d) 323 kw.

CHAPTER 14

ALTERNATORS AND TRANSFORMERS

1. The field rotates; the armature is stationary.
2. Alternators are rated in terms of armature load current and voltage output or kva output at a specified frequency and power factor (usually 80 percent lagging).
3. 170 volts.

4. 208 volts.
5. $f = 400$ cps.
6. 220 volts.
7. (1) Armature resistance drop, (2) armature reactance drop, and (3) armature reaction tending to alter the field magnetization.
8. (a) The field current is increased, and (b) the field current is decreased.
9. (a) Percent regulation $= \frac{E_{nL} - E_{fL}}{E_{fL}} \times 100$,

   (b) 11.35 percent.
10. (a) Weakens, (b) decreases, (c) increases, (d) increases, (e) increases, (f) increases.
11. (1) Increases plant capacity above that of a single unit, (2) serves as reserve power for expected demands, and (3) permits service on a unit without interrupting the power supply.
12. (1) The terminal voltages must be the same, (2) the frequencies must be the same, (3) the voltages must be in the proper phase relationship, and (4) the voltage must have the same phase sequence.
13. Frequency.
14. (1) The core, (2) the primary winding, (3) the secondary winding, and (4) the enclosure.
15. Core-type transformers have an iron core surrounded by copper windings. Shell-type transformers have copper windings surrounded by an iron core.
16. To minimize eddy currents in the core.
17. Loss or damage of one of the single-phase transformers would allow the remaining two to be operated in open delta to supply 3-phase power at a reduced capacity. Loss of a composite unit would result in an interruption to service.
18. 6 volts.
19. (a) 16,000 volts, (b) 120 volts.
20. (a) 1 ampere, (b) 10 amperes, (c) 110 volts, (d) 1,000 ampere-turns.
21. In the iron core, and in the primary and secondary windings.
22. The primary current will increase.
23. (a) H_1, (b) H_2, (c) X_1, (d) X_1.
24. (a) 1.5 kva, (b) 13 amperes.
25. (1) Primaries in delta and secondaries in delta, (2) primaries in wye and secondaries in wye, (3) primaries in wye and secondaries in delta, and (4) primaries in delta and secondaries in wye.

26. Test for zero or almost zero voltage between the three secondaries in series before closing the delta.
27. (a) Voltage, (b) current.
28. (a) 120 kilowatts, (b) 1,000 amperes, (c) 1,732 amperes, (d) 73.2 percent, (e) 208 kilowatts.

CHAPTER 15

ALTERNATING-CURRENT MOTORS

1. The number of poles and the frequency of the line supply.
2. By reversing any two line leads.
3. 900 rpm.
4. The revolving field cuts the rotor conductors inducing a voltage in them, which causes the rotor current to flow.
5. It is not affected at all.
6. By the interaction of the rotor current and the magnetic revolving field.
7. Because the induced voltage in the rotor is relatively small and the rotor conductors are relatively large.
8. It is used to increase the rotor circuit resistance during the starting period to increase the starting torque. It is cut out when the rotor has come up to speed, and the motor then operates like a cage-rotor motor.
9. (1) Increases, (2) increases, (3) increases.
10. The inductive reactance is large compared with the resistance because the frequency of the rotor currents is a maximum value at start.
11. Because the revolving field and rotor are revolving in the same direction, and as the rotor speed approaches that of the revolving field, the slip and frequency of the rotor currents approach zero.
12. High rotor reactance and low rotor power factor.
13. The rotor current would be zero.
14. 45 degrees.
15. The counter emf is limited and the stator current becomes dangerously high.
16. Zero.
17. (1) Stator and rotor copper losses, (2) stator and rotor core losses, and (3) friction and windage losses.
18. Constant speed and variable torque.
19. 45°, when the rotor resistance and reactance are equal.
20. The rotor resistance.

21. A high rotor resistance.
22. Rotor resistance and rotor reactance.
23. To provide excitation for the d-c field.
24. (1) Both have separately excited d-c fields, and (2) both operate at constant speed with varying loads.
25. To make it self-starting as an induction motor.
26. To limit the a-c induced voltage to a safe value.
27. The field excitation.
28. The stator current is a minimum value.
29. A synchronous condenser.
30. They are relatively simple, and the ship's generating capacity is adequate to supply the initial current surge.
31. (a) Stator circuit, (b) rotor circuit.
32. It reduces the voltage during acceleration and increases the voltage to full value after acceleration.
33. They are less expensive to manufacture, and they eliminate the need for 3-phase a-c lines.
34. Because of the high rotor reactance.
35. By the interaction of the rotor magnetomotive force and the stator magnetomotive force.
36. By interchanging the starting winding leads.
37. Because the starting-winding current is displaced approximately 90° in time from the running-winding current, the rotating field is of more uniform strength than in the split-phase motor where the currents are displaced by a much smaller angle.
38. The induced current in the shading pole causes a delay in the rise of flux in that portion of the pole face.
39. (1) They have low starting torque, (2) low efficiency, and (3) high noise level.
40. Because the brushes are positioned so that there is a repulsion of like poles between the rotor and stator that causes the motor to start.

CHAPTER 16

A-C INSTRUMENTS

1. A rectifier or thermocouple.
2. It rectifies the current flowing through the moving coil. This action provides a unidirectional force on the coil that is proportional to the average of all the instantaneous values of the pulsating direct current from the rectifier.

3. The sensitivity is improved, and in general the scale has a greater degree of linearity.
4. Errors are caused by the shunting effect of the associated circuit capacitive reactance.
5. Line voltage, line current, and the cosine of the phase angle between the line voltage and current.
6. Unity power factor (phase angle$=0°$).
7. 90°.
8. When the power factor is low, or when the magnitude of the line voltage, or line current exceed the rated values for the meter.
9. The wattmeter indicates the instantaneous rate of power consumption; the watt-hour meter sums up the instantaneous rates to indicate the total energy consumed over a period of time.
10. To provide the opposing torque that varies directly as the speed of the disk and against which the metered energy is balanced.
11. Coil S.
12. I_{ee} would lag E_{line} by an angle of 90°.
13. Load voltage, load current, and load power factor.
14. By a shading disk that is placed under a portion of the potential coil field pole. The resulting shifting field establishes a torque on the disk before any current flows in the current coils.
15. By moving the drag magnets toward the edge of the aluminum disk.
16. Current transformer.
17. Potential transformer.
18. To provide a linear relation between the secondary and primary currents.
19. The high voltage developed by the secondary may be dangerous to life; the magnetic property of the iron may be damaged.
20. Because it would damage the transformer and short-circuit the source.
21. A current transformer with a split core, and a rectifier-type instrument connected to the transformer secondary.
22. By means of earphones. When minimum sound is heard, the closest approach to a balance will be obtained.
23. The greater the inequality in the ratios, the greater the error in R_x and C_x.
24. It will gain 24 minutes, or be 24 minutes fast at the end of the period.
25. Because the current through the coil is a maximum twice during each cycle—once for each direction of current flow.
26. Because the change in currents in the two coils and the resultant change in opposing torques are the same, and there is no net difference in torque to move the indicator disk.

27. Because the current in A lags the line voltage by 90° due to L, and because the current in C is in phase with the line voltage, the currents in A and C are 90° out of phase.
28. (1) The frequencies must be the same. (2) The voltages must be equal. (3) The alternators must be in the proper phase. (4) In polyphase alternators, the phase sequences must be the same.
29. The lamps will go out.
30. (1) The exact point of synchronism, (2) whether the incoming machine is running too fast or too slow, (3) the amount by which the incoming machine is running too fast or too slow.

CHAPTER 17

SYNCHROS AND SERVOMECHANISMS

1. Angular displacement.
2. The receiver uses a damping device; the transmitter does not.
3. The voltages induced in the secondary windings of the transmitter and receiver are equal and opposite and therefore cancel.
4. The Bureau of Ordnance synchro.
5. Through brushes and slip rings from a single-phase 115-volt 60-cycle power supply.
6. To apply a braking effect on the receiver to prevent it from oscillating or spinning.
7. The secondary effective voltage varies as the cosine of this angle.
8. Zero.
9. There are two positions 180° apart in which the rotor may come into alignment with the receiver field, and the rotor torque is reduced.
10. To amplify the weak torque developed by the synchro.
11. The differential synchro rotor has a 3-phase winding distributed in slots around the rotor. There are no salient poles on the rotor. The receiver rotor has a single concentrated winding on a salient-pole rotor.
12. The differential transmitter has two inputs (one mechanical and one electrical) and one electrical output; the differential receiver has two electrical inputs and one mechanical output.
13. 45°.
14. Interchange the $S1$ and $S3$ leads of the differential transmitter and also the $R1$ and $R3$ leads.
15. The $R1$ and $R3$ leads of the differential receiver are interchanged.
16. It indicates by means of a voltage the difference in angular positions of the input and output shafts of a servomechanism.

17. They are in quadrature (displaced 90°).
18. It is the position where minimum (zero) voltage is induced in the rotor by the $S2$ stator coil.
19. The amount by which the shafts of the control transformer and the synchro transmitter are out of correspondence.
20. In the BuOrd type, the rotor is the primary and the stator is the secondary; the reverse is true of the BuShips synchros.
21. By means of slip rings and brushes.
22. To operate a set of electrical contacts that controls the current to a servo motor that drives the load.
23. In the closed-loop system, no type of compensation is required to counteract any variations in the output of the power source or variations in the operating characteristics.
24. The antenna is positioned by the servo system when the handwheel is turned, and the indicator system indicates the position of the antenna.
25. The relative position of the antenna shaft and the position of the handwheel.
26. Zero.
27. By varying the strength of the current in the field coils of the amplidyne generator.
28. An amplidyne generator, a driving motor, and a servomotor.
29. The amplidyne requires only a small fraction of the field power required for a regular d-c generator.
30. The armature acting as an electromagnet.
31. To create an opposing mmf to counterbalance the armature load current mmf.
32. To counterbalance the armature load-current mmf automatically with load change.
33. To remove any residual magnetism along the axis of the control field.
34. To maintain a linear relation between input and output.
35. To amplify and rectify the input signal so as to properly energize the amplidyne control field.
36. An antihunt device.
37. The output drive motor should (1) have the required power, (2) be easily reversible, and (3) be capable of speed control over a fairly wide range.
38. So that its speed will be proportional to the voltage supplied to its armature.
39. The single-phase induction motor.

40. (a) By reversing the connections to one stator coil, or by shifting the capacitor from one coil to the other, and (b) by changing the voltage applied to the motor.
41. Speed control and control of the direction of rotation of the a-c motor.
42. The amplidyne 3-phase drive motor.
43. It has greater accuracy.
44. 10°.
45. 36-to-1.

APPENDIX II

NAVY CABLE TYPE AND SIZE DESIGNATIONS

All cables described herein are identified by NAVY CABLE TYPE AND SIZE DESIGNATIONS, which consists of a letter followed by numerals.

I. The TYPE DESIGNATION indicates the type or construction of the cable and consists of the first letters of the words used in describing the cable:

EXAMPLE: Type SHFA–Single, Heat and Flame-resistance, Armored.

II. The SIZE DESIGNATION relates to the copper conductors. No set rule has been established for application of SIZE DESIGNATIONS since the numerals used may indicate one or more of the following:

(1) Size of copper conductor.
(2) Stranding of conductors.
(3) Number of copper conductors.
(4) Number of twisted pairs.

Examples of the most common uses of SIZE DESIGNATIONS are given below:

(1) To indicate approximate cross-sectional area of the conductor expressed in thousands of circular mils (abbreviated C.M.).
EXAMPLE: The conductor area of SHFA–3 is approximately 3000 C.M. (exactly 2828 C.M.).

(2) To indicate approximate cross-sectional area with stranding shown in parentheses.
EXAMPLE: The conductor area of SRI–2½(26) is approximately 2500 C.M. (exactly 2613 C.M.), and consists of 26 individual wire strands.

(3) To indicate number of conductors.
EXAMPLE: MCOP–7 is a cable comprised of seven conductors.

(4) To indicate number of twisted pairs.
EXAMPLE: TTOP–10 is a 20-conductor cable comprised of 10 twisted pairs.

The following are typical examples of type designations and constructions:

FLA—Four conductor, Lighting, Armored. Varnished cambric or rubber insulation, rubber hose jacket, armored.

MCMB—Multiple Conductor, Marker-Buoy. Rubber insulated identifying colors, steel supporting cable, tape over assembly, tough rubber sheath.

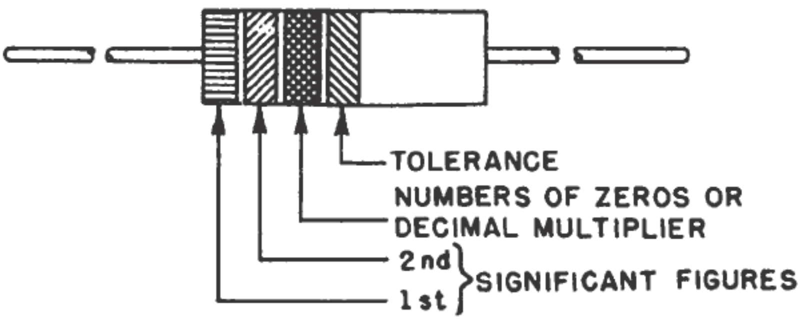
SHFA—Single conductor, Heat and Flame-resistant, Armored. Asbestos-varnished cambric-asbestos insulation, impervious sheath, armored.

THFR—Triple conductor, Heat and Flame-resistant, Radio. Synthetic resin-varnished cambric and felted asbestos insulation, asbestos belt, impervious sheath, armored.

TTHFA—Twisted pairs, Telephone, Heat and Flame-resistant, Armored. Solid conductor, textile wrap or synthetic resin insulation, twisted pair, felted asbestos belt, impervious sheath, armored.

APPENDIX III

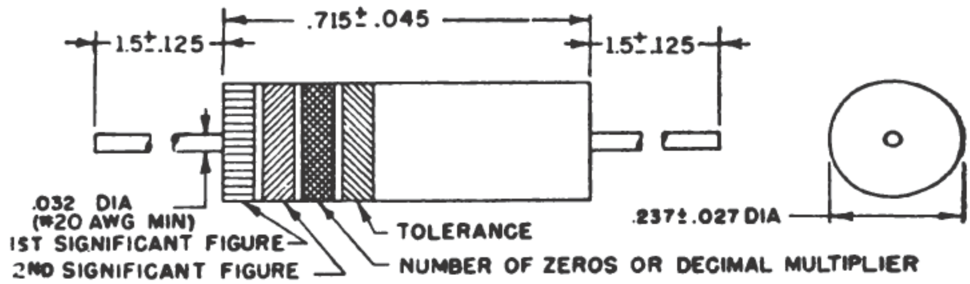
ELECTRONIC COLOR CODING AND SYMBOLS



COLOR	SIGNIFICANT FIGURE OR NUMBER OF ZEROS	DECIMAL MULTIPLIER	RESISTANCE TOLERANCE
			PERCENT ±
BLACK	0	---	---
BROWN	1	---	---
RED	2	---	---
ORANGE	3	---	---
YELLOW	4	---	---
GREEN	5	---	---
BLUE	6	---	---
VIOLET	7	---	---
GRAY	8	---	---
WHITE	9	---	---
GOLD	---	0.1	5(J) *
SILVER	---	---	10(K) *
NO COLOR...	---	---	20(M) *

* SYMBOL DESIGNATION ALTERNATE FOR COLOR

Figure A–1.—Standard resistor color code.



NOTES:

1. All dimensions in inches.
2. Referenced specification shall be of the issue in effect on date of invitation for bids.

TYPE-DESIGNATION EXAMPLE

RC30	BF	153	M
STYLE	CHARACTERISTIC	RESISTANCE	RESISTANCE TOLERANCE

CHARACTERISTIC (MAXIMUM AMBIENT TEMPERATURE)

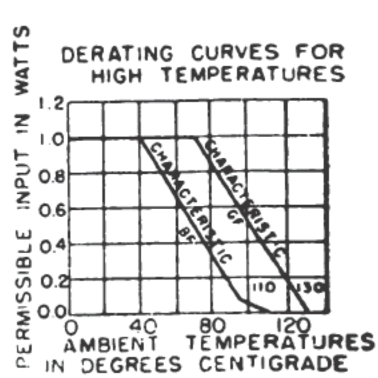
Symbol	Maximum ambient temperature for full-load operation
	° C.
B	40
G	70

RESISTANCE TOLERANCE

Symbol	Resistance tolerance
	Percent (±)
J	5
K	10
M	20

CHARACTERISTIC (RESISTANCE-TEMPERATURE)

Nominal resistance	Maximum allowable change in resistance from resistance at ambient temperature of 25° C. Symbol F	
	At -55° C. (ambient)	At 105° C. (ambient)
Ohms	Percent (±)	Percent (±)
1,000 and under —	6.5	5
1,100 to 10,000 —	10	6
11,000 to 0.1 megohm	13	7.5
Megohms		
0.11 to 1.0 ——	20	10
1.1 to 10 ——	26	18
11 and over ——	35	22



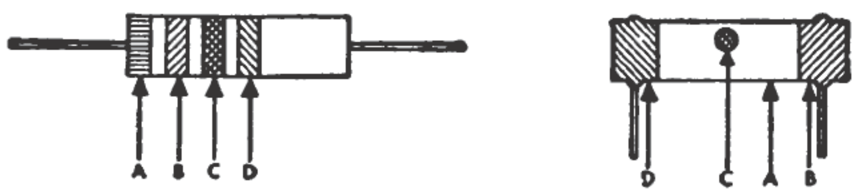
Power rating - 1 watt 1/
Minimum resistance value - 10 ohms 1/
Maximum resistance value - 20 megohms 1/
Continuous working voltage (maximum) 2/ - - - - - - - - - - - - - - - - 500 volts

1 FOR STANDARD RESISTANCE VALUES, SEE MS91374 IN MIL-R-11A.

2 CONTINUOUS WORKING VOLTAGE SHALL BE COMPUTED IN ACCORDANCE WITH THE FOLLOWING FORMULA BUT IN NO CASE SHALL IT EXCEED 500 VOLTS:

$$\text{VOLTAGE} = \sqrt{\text{POWER RATING} \times \text{NOMINAL RESISTANCE}}$$

Figure A–2.—Specifications of composition resistors.

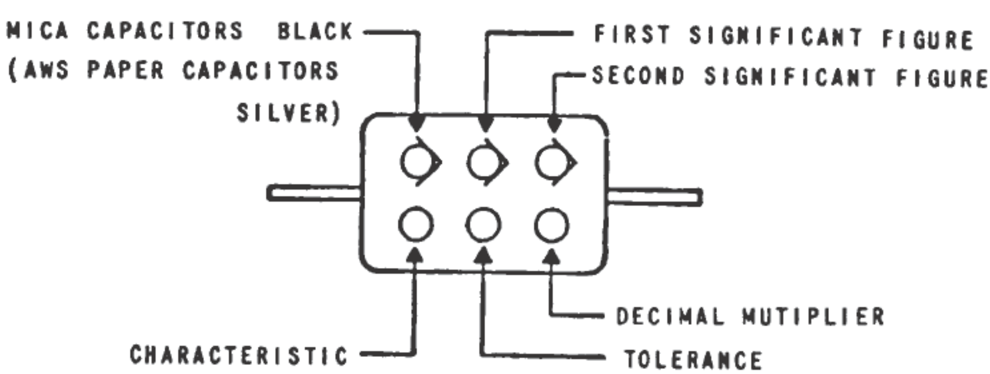


axial leads	color	radial leads
Band A	indicates first significant figure of resistance value in ohms.	Body A
Band B	indicates second significant figure.	End B
Band C	indicates decimal multiplier.	Band C or dot
Band D	if any, indicates tolerance in percent about nominal resistance value. If no color appears in this position, tolerance is 20%.	Band D

Note: low-power insulated wire-wound resistors have axial leads and are color coded similar to the left-hand figure above except that band A is double width.

Courtesy Telecommunication Laboratories, Inc.

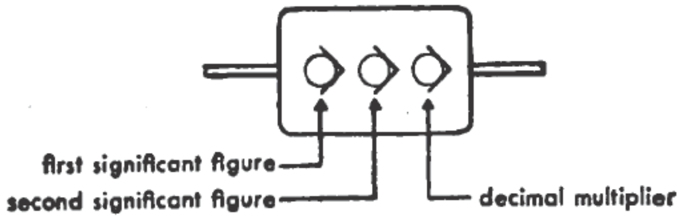
Figure A–3.—Color code for axial- and radial-lead fixed composition resistors.



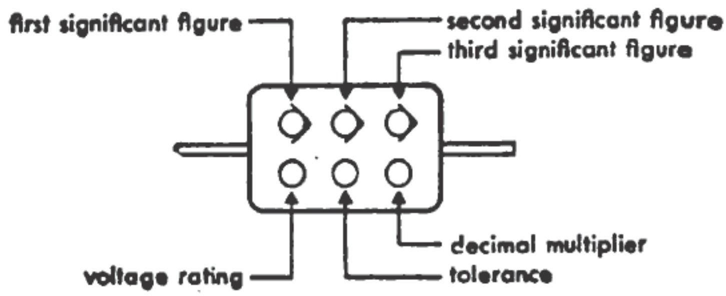
RMA 6-dot

Courtesy Telecommunication Laboratories, Inc.

Figure A–4.—AWS and NME color code for fixed mica capacitors.



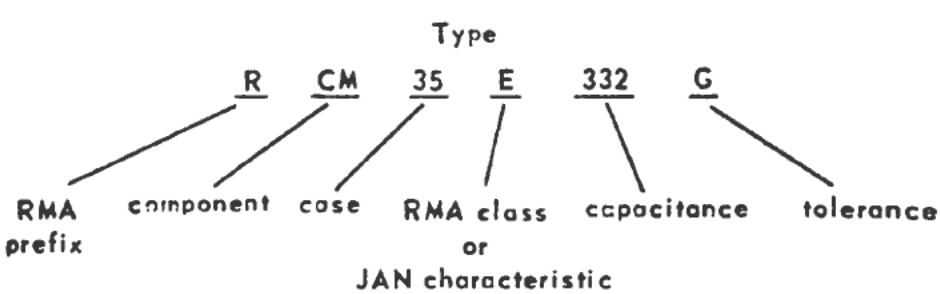
RMA 3-dot **500-volt, ± 20% tolerance only**



RMA 6-dot

Examples

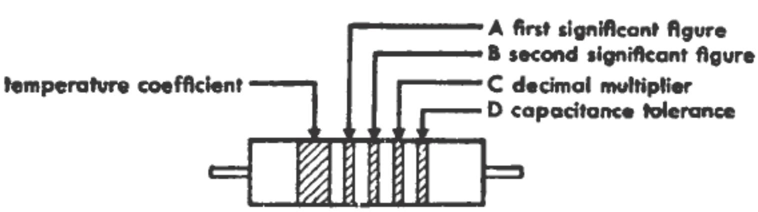
type	top row left	top row center	top row right	bottom row left	bottom row center (tolerance)	bottom row right (multiplier)	description
RMA (3 dot)	red	green	brown	none	none	none	250 μμf ± 20%, 500 volts
RMA	brown	black	black	blue	green	brown	1000 μμf ± 5%, 600 volts
RMA	brown	red	green	gold	red	brown	1250 μμf ± 2%, 1000 volts
CM30B681J	black	blue	gray	brown	gold	brown	680 μμf ± 5%, characteristic B
CM35E332G	black	orange	orange	yellow	red	red	3300 μμf ± 2%, characteristic E



Courtesy Telecommunication Laboratories, Inc

Figure A–5.—RMA 3-dot and 6-dot color code for fixed mica capacitors.

color	significant figure	multiplier	capacitance tolerance in % $c > 10\ \mu\mu f$	capacitance tolerance in $\mu\mu f$ $c \leq 10\ \mu\mu f$	temperature coefficient parts/million/° C
black	0	1	±20	2.0	0
brown	1	10	±1		−30
red	2	100	±2		−80
orange	3	1,000			−150
yellow	4	—			−220
green	5	—	±5	0.5	−330
blue	6	—			−470
violet	7	—			−750
gray	8	0.01		0.25	+30
white	9	0.1	±10	1.0	−330 ±500



Examples

wide band	narrow bands or dots A	B	C	D	description
black	black	red	black	black	2.0 $\mu\mu f$ ± 2 $\mu\mu f$, zero temp coeff
blue	red	red	black	green	22 $\mu\mu f$ ±5%, −470 ppm/° C temp coeff
violet	gray	red	brown	silver	820 $\mu\mu f$ ±10%, −750 ppm/° C temp coeff

Courtesy Telecommunication Laboratories, Inc.

Figure A–6.—Color code for fixed ceramic capacitors.

APPENDIX IV

GREEK ALPHABET

Name	Capital	Lower case	Designates
Alpha.....	Λ	α	Angles
Beta.......	B	β	Angles, flux density
Gamma...	Γ	γ	Conductivity
Delta......	Δ	δ	Variation of a quantity
Epsilon....	E	ϵ	Base of natural logarithms
Zeta......	Z	ζ	Impedance
Eta.......	H	η	Hysteresis coefficient
Theta.....	Θ	θ	Phase angle
Iota.......	I	ι	
Kappa....	K	κ	
Lambda...	Λ	λ	Wavelength
Mu.......	M	μ	Permeability, micro
Nu........	N	ν	Reluctivity
Xi........	Ξ	ξ	
Omicron...	O	o	
Pi.........	Π	π	3.14
Rho.......	P	ρ	Resistivity
Sigma.....	Σ	σ	
Tau.......	T	τ	
Upsilon....	Υ	υ	
Phi.......	Φ	ϕ	Angles, magnetic flux
Chi.......	X	χ	
Psi........	Ψ	ψ	
Omega....	Ω	ω	Ohms (Ω), angular velocity (ω)

APPENDIX V

COMMON ABBREVIATIONS AND LETTER SYMBOLS

Term	*Abbreviation or Symbol*
alternating current (noun)	ac
alternating current (adj)	a-c
ampere	a
area	A
audio frequency (noun)	af
audio frequency (adj)	a-f
capacitance	C
capacitive reactance	X_c
conductance	G
coulomb	Q
current (d-c or rms value)	I
current (instantaneous value)	i
cycles per second	cps
dielectric constant	K,k
difference in potential (d-c or rms value)	E
difference in potential (instantaneous value)	e
direct current (noun)	dc
direct current (adj)	d-c
electromotive force	emf
frequency	f
henry	h
impedance	Z
inductance	L
inductive reactance	X_L
kilowatt	kw
magnetic field intensity	H
megohm	M
microfarad	μf
microhenry	μh
micromicrofarad	$\mu\mu$f
microvolt	μv
millihenry	mh
milliwatt	mw
mutual inductance	M

COMMON ABBREVIATIONS AND LETTER SYMBOLS—Continued

Term	*Abbreviation or Symbol*
power	P
resistance	R
revolutions per minute	rpm
root mean square	rms
time	t
torque	T
volt	v
watt	w

APPENDIX VI

TRIGONOMETRIC FUNCTIONS

(Natural)

Angle	Sine	Cosine	Tangent	Angle	Sine	Cosine	Tangent
0°	0.000	1.000	0.000				
1°	.018	1.000	.018	46°	0.719	0.695	1.036
2°	.035	.999	.035	47°	.731	.682	1.072
3°	.052	.999	.052	48°	.743	.669	1.111
4°	.070	.998	.070	49°	.755	.656	1.150
5°	.087	.996	.088	50°	.766	.643	1.192
6°	.105	.995	.105	51°	.777	.629	1.235
7°	.122	.993	.123	52°	.788	.616	1.280
8°	.139	.990	.141	53°	.799	.602	1.327
9°	.156	.988	.158	54°	.809	.588	1.376
10°	.174	.985	.176	55°	.819	.574	1.428
11°	.191	.982	.194	56°	.829	.559	1.483
12°	.208	.978	.213	57°	.839	.545	1.540
13°	.225	.974	.231	58°	.848	.530	1.600
14°	.242	.970	.249	59°	.857	.515	1.664
15°	.259	.966	.268	60°	.866	.500	1.732
16°	.276	.961	.287	61°	.875	.485	1.804
17°	.292	.956	.306	62°	.883	.470	1.881
18°	.309	.951	.325	63°	.891	.454	1.963
19°	.326	.946	.344	64°	.899	.438	2.050
20°	.342	.940	.364	65°	.906	.423	2.145
21°	.358	.934	.384	66°	.914	.407	2.246
22°	.375	.927	.404	67°	.921	.391	2.356
23°	.391	.921	.425	68°	.927	.375	2.475
24°	.407	.914	.445	69°	.934	.358	2.605
25°	.423	.906	.466	70°	.940	.342	2.747
26°	.438	.899	.488	71°	.946	.326	2.904
27°	.454	.891	.510	72°	.951	.309	3.078
28°	.470	.883	.532	73°	.956	.292	3.271
29°	.485	.875	.554	74°	.961	.276	3.487
30°	.500	.866	.577	75°	.966	.259	3.732
31°	.515	.857	.601	76°	.970	.242	4.011
32°	.530	.848	.625	77°	.974	.225	4.331
33°	.545	.839	.649	78°	.978	.208	4.705
34°	.559	.829	.675	79°	.982	.191	5.145
35°	.574	.819	.700	80°	.985	.174	5.671
36°	.588	.809	.727	81°	.988	.156	6.314
37°	.602	.799	.754	82°	.990	.139	7.115
38°	.616	.788	.781	83°	.993	.122	8.144
39°	.629	.777	.810	84°	.995	.105	9.514
40°	.643	.766	.839	85°	.996	.087	11.43
41°	.656	.755	.869	86°	.998	.070	14.30
42°	.669	.743	.900	87°	.999	.052	19.08
43°	.682	.731	.933	88°	.999	.035	28.64
44°	.695	.719	.966	89°	1.000	.018	57.29
45°	.707	.707	1.000	90°	1.000	.000	∞

INDEX

U. S. GOVERNMENT PRINTING OFFICE: 1956